Universal Journal of Structural Analysis **8 (2020), 12-30**
www.papersciences.com

Is the Speed of Light a Limit for the Universal Stress Intensity Factors by using the Next Generation Spacecraft ? What will be Next ?

E.G. Ladopoulos
Interpaper Research Organization
8, Dimaki Str.
Athens, GR - 106 72, Greece
eladopoulos@interpaper.org

Abstract
For the design of the future spacecraft of any speed, the sophisticated theory of *"Universal Mechanics"* is further improved and studied. The proposed theory of *"Universal Mechanics"* consists to the combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*. Hence, according to the above theories there is a considerable difference between the absolute stress tensor and the stress tensor of the airframe even in low speeds. Moreover, for bigger speeds of the new generation spacecraft, like c/3, c/2 or 3c/4 (c=speed of light), then the difference between the two stress tensors is very much increased. Thus, for the new generation spacecraft with very high speeds, the relative stress tensor will be very much different than the absolute stress tensor. Moreover, for velocities near the speed of light, then the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. Our theory will still exist even once in the very future somebody will prove that the speed of light is not the maximum speed in nature. The above new generation spacecraft will be moving by using laser engines. The theory of *"Relativistic Elasticity"* is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results in the *"Universal Equation of Elasticity"*. Additionally, the theory of *"Relativistic Thermo-Elasticity"* is a combination between the theories of "Classical Thermo-Elasticity" and "Special Relativity" and results in the *"Universal Equation of Thermo-Elasticity"*. The "structural design" of super speed vehicles requires the consideration of mass pulsation and energy-mass interaction at high velocity space-time scale, as the relative stress intensity factors are different than the corresponding absolute stress intensity factors. Such theory results in the *"Universal Stress Intensity Factors"*. Thus, the *"Universal Equation of Elasticity"*, the *"Universal Equation of Thermo-Elasticity"* and the *"Universal Stress Intensity Factors"* are parts of the general theory of *"Universal Mechanics"*.

Key Word and Phrases
Next Generation Spacecraft, Relativistic Elasticity, Relativistic Thermo-Elasticity, Relative Stress Tensor, Absolute Stress Tensor, Stationary and Moving Frames, Laser Engines, Energy-Momentum Tensor, Universal Mechanics, Universal Equation of Elasticity, Universal Equation of Thermo-Elasticity, Universal Stress Intensity Factors.

1. Investigations of Universal Mechanics for Next Generation Spacecraft

In the near future the International Space Agencies should effect a competitive technological advantage in several strategic areas of new and rapidly developing advanced technologies. Hence, the scope by the big Space Agencies is to achieve in the future, an absolute spacecraft moving with very high speeds, even approaching the speed of light. So, how far could be this future ? According to the present study and research such future could be much closer than everybody believes.

For the future spacecraft the relative stress tensor will be much different than the absolute stress tensor and so special solid should be used for the construction of the next generation spacecraft.

On the contrary, the suitable choice of the solid which should be used for the construction of the absolute spacecraft is under investigation, but such solid will be very much different than the usual composite materials.

Beyond the above, in order the next generation spacecraft to achieve very high speed, even approaching the speed of light, then such new generation spacecraft should be moving by using laser engines. Laser is light and so their speed is the speed of light. Thus, the use of laser engines for the future spacecraft would be the best device.

Consequently, we will show that there is a significant difference between the absolute stress tensor and the stress tensor of the airframe even for low speeds. Furthermore, for bigger speeds the difference of the two stress tensors will be very much increased. Thus, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is very much different than the absolute one and for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the airframe is included in the theory proposed by E.G.Ladopoulos [30] - [32] under the term *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* and the final formula which results from the above theories is called the *"Universal Equation of Elasticity"* and the *"Universal Equation of Thermo-Elasticity"*, correspondingly. Moreover, both theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* are included in a more general theory under the term *"Universal Mechanics"*.

One more question is the following: What happens with our theory if somebody in the very future proves that the speed of light is not the maximum speed in the whole universe, but there is another type of energy with higher speed ? The answer is that our theory of "Universal Mechanics" will valid over the centuries and the milleniums, as the spacecraft when reaching the speed of light then becomes energy and will not be mass any more. Hence, after the speed of light there is no mass available, but only energy. According to NASA the Large and Small Magellanic clouds were thought to be the closest galaxies to ours, until 1994, when the Sagittarius Dwarf Elliptical Galaxy (SagDEG) was discovered. In 2003, the Canis Major Dwarf Galaxy was discovered - this is now the closest known galaxy to ours. Thus, The Canis Major Dwarf Galaxy is only 25,000 light years from the Sun, and 42,000 light years from the Galactic center. It too, is well-hidden by the dust in the plane of the Milky Way - which is why it wasn't discovered until recently. To get to the closest galaxy to ours, the Canis Major Dwarf, at Voyager's speed, it would take approximately 749,000,000 years to travel the distance of 25,000 light years! If we could travel at the speed of light, it would still take 25,000 years. On the other hand, the galaxy MACS0647-JD appears very young and is only a fraction of the size of our own Milky Way. The galaxy is about 13.3 billion light-years from Earth, the farthest galaxy yet known, and formed 420 million years after the Big Bang. The universe itself is only 13.7 billion years old, so this galaxy's light has been traveling toward us for almost the whole history of space and time.

Also, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et al. [17]-[22] proposed singular integral equation methods applied to elasticity, plasticity and fracture mechanics theories. In the above mentioned publications the *Singular Integral Operators Method (S.I.O.M.)* is proposed for the numerical evaluation of the multidimensional singular integral equations in which the stress tensor analysis of the linear elastic theory is reduced. Furthermore, the theory of linear singular integral equations was extended to non-linear singular integral equations, too. [23]-[29]. So, the theory of *"Universal Mechanics"* and correspondingly the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* will be applied for the design of the elastic stress analysis of the airframes.

Also, the classical theory of elastic stress analysis and thermo-elastic stress analysis began to be analyzed in the early nineteenth century and was further developed during the twentieth century. In the past, several important monographs were published on the classical theory of elasticity and thermo-elasticity. [33]-[52].

Over the past years special attention has been given, by many scientists worldwide, on the theoretical aspects of the special theory of relativity. So, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [53]–[60]. Besides, by the current research we will show that the "*relative stress tensor is not symmetrical*", while, as it is well known, the "*absolute stress tensor is symmetrical*". Such a difference is very important for the design of the future aircraft and spacecraft of very high speeds. Finally, the "structural design" of super speed vehicles requires the consideration of mass pulsation

[61], [62] and energy-mass interaction [63] at high velocity space-time scale.

2. Investigations of Universal Equation of Elasticity for Next Generation Spacecraft by Relativistic Elasticity

Consider the state of stress at a point in the stationary frame S^0, defined by the following symmetrical stress tensor: (Fig.1)

$$\sigma^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.1}$$

where: $$\sigma_{21}^0 = \sigma_{12}^0, \sigma_{31}^0 = \sigma_{13}^0, \sigma_{32}^0 = \sigma_{23}^0 \tag{2.2}$$

Furthermore, consider an infinitesimal face element df with a directed normal, defined by a unit vector **n**, at definite point p in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to df.

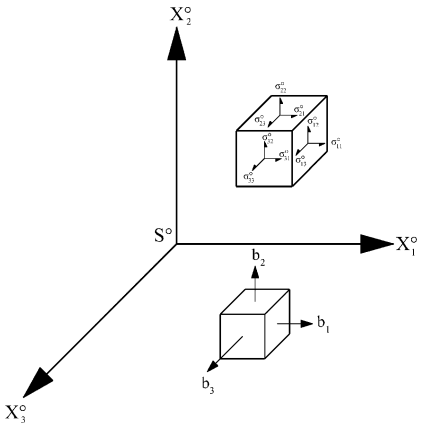

Fig. 1 The state of stress σ_{ik}^0 in the stationary system S^0.

Thus, the force is valid as:

$$\mathrm{d}\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\mathrm{d}f \tag{2.3}$$

The components $\sigma_i(\mathbf{n})$ of $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components n_k of **n**:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k,\ i,k = 1,2,3 \tag{2.4}$$

where σ_{ik} is the elastic stress tensor, also called as the relative stress tensor, in contrast to the space part σ_{ik}^0 of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor. [53], [54} (Fig. 2).

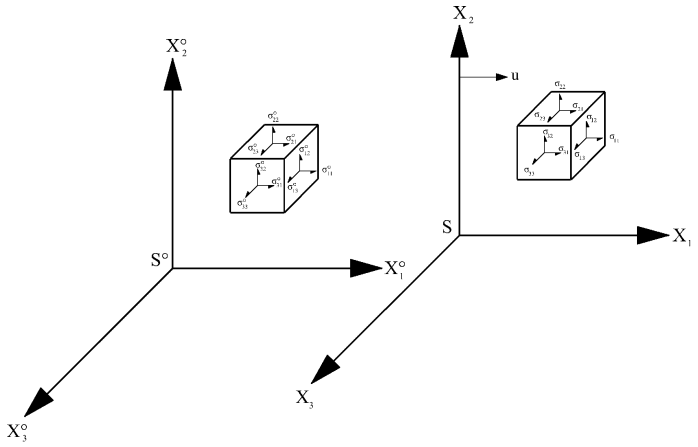

Fig. 2 The state of stress σ_{ik}^{0} in the stationary system S^{0} and σ_{ik} in the airframe system with velocity u parallel to the x_1 - axis.

In addition, the connection between the absolute and relative stress tensors is defined as:

$$\sigma_{ik}^{0} = \sigma_{ik} + g_i u_k,\ i,k = 1,2,3 \tag{2.5}$$

where g_i are the components of the momentum density $\mathbf{g}$ and u_k the components of the velocity $\mathbf{u}$ of the matter.

The connection between $\mathbf{g}$ and the energy flux $\mathbf{s}$, is equal to:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

where c denotes the speed of light (= 300.000 *km*/*sec*).

Besides, the total work done per unit time by elastic forces on the matter inside the closed surface f can be given by the formula:

$$W = \int_f \left(\boldsymbol{\sigma}(\mathbf{n})\cdot\mathbf{u}\right)\mathrm{d}f = \int_f \sigma_{ik} n_k u_i \,\mathrm{d}f = -\int_\upsilon \frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\,\mathrm{d}\upsilon,\ i,k = 1,2,3 \tag{2.7}$$

where the integration in the last integral is extended over the interior υ of the surface f.

Consequently, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is valid as:

$$\delta W = -\frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\delta\upsilon \tag{2.8}$$

Also, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \delta W \tag{2.9}$$

where h denotes the total energy density, including the elastic energy and $\mathrm{d}/\mathrm{d}t$ is the substantial time derivative.

Eq. (2.9) is valid as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon)=\left(\frac{\vartheta h}{\vartheta t}+\frac{\vartheta h}{\vartheta x_k}u_k\right)\delta\upsilon+h\delta\upsilon\frac{\vartheta u_k}{\vartheta x_k}=\left[\frac{\vartheta h}{\vartheta t}+\frac{\vartheta}{\vartheta x_k}(hu_k)\right]\delta\upsilon \tag{2.10}$$

which finally leads to the relation:

$$\frac{\vartheta h}{\vartheta t}+\frac{\vartheta}{\vartheta x_k}(hu_k+u_i\sigma_{ik})=0 \tag{2.11}$$

Thus, the total energy flow is valid as:

$$\mathbf{s}=\mathbf{hu}+(\mathbf{u}\cdot\boldsymbol{\sigma}) \tag{2.12}$$

where $(\mathbf{u}\cdot\boldsymbol{\sigma})$ is a space vector with components $(\mathbf{u}\cdot\boldsymbol{\sigma})_k=u_i\sigma_{ik}$.

So, the total momentum density can be written as:

$$\mathbf{g}=\frac{\mathbf{s}}{c^2}=\mu\mathbf{u}+\frac{(\mathbf{u}\cdot\boldsymbol{\sigma})}{c^2} \tag{2.13}$$

in which $\mu=h/c^2$ denotes the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) we have:

$$\sigma_{ik}-\sigma_{ki}=-g_iu_k+g_ku_i=[-(\mathbf{u}\cdot\boldsymbol{\sigma})_iu_k+(\mathbf{u}\cdot\boldsymbol{\sigma})_ku_i]/c^2\neq 0 \tag{2.14}$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame S^0 the velocity $u^0=0$ and so, from (2.5), (2.12) and (2.13) the following expressions are obtained:

$$\sigma^0_{ik}=\sigma_{ik}=\sigma_{ki}=\sigma^0_{ki}\ (i,k=1,2,3) \tag{2.15}$$

Moreover, the mechanical energy-momentum tensor satisfies the following relation:

$$T_{ik}U_k=-h^0U_i \tag{2.16}$$

where U_i is the four-velocity of the matter, in the Lorentz system and $U^0_i=(0,0,0,ic)$.

Hence, the following scalar can be formed:

$$U_iT_{ik}U_k/c^2=U^0_iT^0_{ik}U^0_k/c^2=-T^0_{44}=h^0(x_1) \tag{2.17}$$

with $h^0(x_1)$ the invariant rest energy density considered as a scalar function of the coordinates (x_i) $(i = 1,2,3)$ in S. (Fig. 2)

Also, by applying the tensor:

$$\Delta_{ik} = \delta_{ik} + U_i U_k / c^2 \tag{2.18}$$

which satisfies the relations:

$$U_i \Delta_{ik} = \Delta_{ik} U_k = 0 \tag{2.19}$$

then, the following symmetrical tensor can be formed:

$$S_{ik} = \Delta_{i1} T_{1m} \Delta_{mk} = S_{ki} \tag{2.20}$$

which is orthogonal to U_i:

$$U_i S_{ik} = S_{ik} U_k = 0 \tag{2.21}$$

By combining eqs. (2.16), (2.17) and (2.20) we have:

$$S_{ik} = T_{ik} - h^0 U_i U_k / c^2 \tag{2.22}$$

Besides, in the stationary system S_0 we obtain:

$$S^0_{ik} = \sigma^0_{ik} = \sigma_{ik}, \; S^0_{i4} = S^0_{4i} = 0 \tag{2.23}$$

Eq. (2.22) may also be written as:

$$T_{ik} = \xi_{ik} + S_{ik} \tag{2.24}$$

where:

$$\xi_{ik} = h^0 U_i U_k / c^2 = \mu^0 U_i U_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^0 = h^0 / c^2 \tag{2.26}$$

is the proper mass density.

We further introduce in every system S the quantity:

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is valid as:

$$\sigma_{ik} = T_{ik} - T_{i4} U_k / U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik}$$

in the stationary system is a real symmetrical matrix. The corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p_{(j)}^0$, the principal stresses, are the three roots of the following algebraic equation, where λ is the unknown:

$$\left| S_{ik}^0 - \lambda \delta_{ik} \right| = \left| \sigma_{ik}^0 - \lambda \delta_{ik} \right| = 0 \tag{2.30}$$

The matrix S_{ik}^0 can be further written in terms of the eigenvalues and eigenvectors as:

$$S_{ik}^0 = \sigma_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

Thus, from eqs. (2.23) and (2.31) we obtain the following form of the stress four-tensor in S^o:

$$S_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

So, in any system S we obtain:

$$S_{ik} = p_{(j)}^0 h_i^{(j)} h_k^{(j)} \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) follow the expressions:

$$T_{ik} = \mu^0 U_i U_k + p_{(j)}^0 h_i^{(j)} h_k^{(j)} \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p_{(j)}^0 h_k^{(j)} \left(h_k^{(j)} + i h_4^{(j)} u_k / c \right) \tag{2.35}$$

By putting:

$$h_i^{(j)} = (\mathbf{h}^{(j)}, h_4^{(j)}) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, then eqn (2.35) can be written for the relative stress tensor $\boldsymbol{\sigma}$ as following:

$$\boldsymbol{\sigma} = p_{(j)}^0 \left[\mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h_4^{(j)} (\mathbf{h}^{(j)} \bullet \mathbf{u}) \right], j = 1,2,3 \tag{2.37}$$

Furthermore, the triad vectors $h_i^{(j)}$ satisfy the tensor relations:

$$h_i^{(j)} h_i^{(\rho)} = \delta^{j\rho} \tag{2.38}$$

$$h_i^{(j)} h_k^{(j)} = \Delta_{ik} \tag{2.39}$$

with Δ_{ik} given by (2.18).

If the stationary system S^0 for every event point is chosen in such a way that the spatial axes in S^0 and in S have the same orientation, one has:

$$\begin{gathered} \mathbf{h}^{(j)} = \mathbf{h}^{(j)0} + \left\{\mathbf{u}(\mathbf{u}\cdot\mathbf{h}^{(j)0})(\gamma-1)\right\}/u^2 \\ h_4^{(j)} = i\mathbf{u}\cdot\mathbf{h}^{(j)0}\,\gamma/c \end{gathered} \tag{2.40}$$

with:

$$\gamma = 1/(1-u^2/c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ follows:

$$h = -T_{44} = -\mu^0 U_4^2 - p_{(j)}^0 (\mathbf{u}\cdot\mathbf{h}^{(j)0})^2 \cdot \gamma^2/c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces to:

$$\boldsymbol{\sigma}^0 = p_{(j)}^0 \left(\mathbf{h}^{(j)0} \bullet \mathbf{h}^{(j)0}\right) \tag{2.43}$$

So, from (2.42) follows the transformation law for the energy density:

$$\begin{gathered} h = \frac{h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^2}{1-u^2/c^2} \\ \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u} = u_i \sigma_{ik}^0 u_k \end{gathered} \tag{2.44}$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^4}{1-u^2/c^2} \tag{2.45}$$

From (2.40) and (2.34) with $k = 4$, we obtain the momentum density $\mathbf{g}$ with the components $g_i = T_{i4}/ic$:

$$\begin{gathered} \mathbf{g} = \mathbf{u}\left[h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}(1-\gamma^{-1})/u^2\right]\gamma^2/c^2 + (\boldsymbol{\sigma}^0\cdot\mathbf{u})\gamma/c^2 \\ (\boldsymbol{\sigma}^0\cdot\mathbf{u})_i = \sigma_{ik}^0 u_k \end{gathered} \tag{2.46}$$

In addition, from (2.40) and (2.35) we have the relative stress tensor:

$$\begin{gathered} \boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u}\bullet(\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)/u^2 - (\boldsymbol{\sigma}^0\cdot\mathbf{u})\bullet\mathbf{u}(\gamma-1)/\gamma u^2 \\ -(\mathbf{u}\bullet\mathbf{u})(\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)^2/\gamma u^4 \end{gathered} \tag{2.47}$$

In the special case **u** = (*u*,0,0), where the notation of the matter at the point considered is parallel to the x_1-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$\begin{aligned} h &= \left(h^0 + \frac{u^2}{c^2}\sigma_{11}^0 \right)\gamma^2 \\ g_{x_1} &= \gamma^2 \left(\mu^0 + \frac{\sigma_{11}^0}{c^2} \right) u \\ g_{x_2} &= \frac{\gamma \sigma_{21}^0}{c^2} u \\ g_{x_3} &= \frac{\gamma \sigma_{31}^0}{c^2} u \end{aligned} \tag{2.48}$$

and the relative stress tensor gives the ***Universal Equation of Elasticity***:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^0 & \gamma\sigma_{12}^0 & \gamma\sigma_{13}^0 \\ \frac{1}{\gamma}\sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \frac{1}{\gamma}\sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.49}$$

where γ is given by (2.41). Finally, as it could be easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

3.Investigations of Universal Equation of Thermo-Elasticity for Next Generation Spacecraft by Relativistic Thermo-Elasticity

In the previous paragraphs the system under investigation, which is the elastic body, was regarded as a purely mechanical system. However, all macroscopic systems are in reality thermo-dynamical systems with properties depending on non-mechanical variables such as the proper temperature T^o, and so the question which arises is to what kind of thermodynamical processes may be described by an energy-momentum tensor.

Thus, it is clear that all properties in which heat energy is transferred from one part of the system to another are excluded, for heat flow in the manner would give rise to a non-vanishing energy current in the rest system.

Consider further a general system of continuously distributed ponderable or visible matter, inside which invisible heat conduction can take place, while the motion of the visible matter is described by the four-velocity U_i. Then the energy-momentum tensor of the general system can be given by the following relation:

$$T_{ik} = M_{ik} + H_{ik} \tag{3.1}$$

where M_{ik} denotes the mechanical part of the energy-momentum tensor and H_{ik} the heat part.

Moreover, the mechanical part M_{ik} is valid by the following formula:

$$M_{ik} = d^0 U_i U_k / c^2 + S_{ik} \tag{3.2}$$

and the heat part:

$$H_{ik} = \left(U_i V_k + V_i U_k\right)/c^2 \tag{3.3}$$

where the four-vector V_i satisfies the relation:

$$V_i = -\Delta_{ik} T_{kj} U_j = -T_{ik} U_k - d^0 U_i \tag{3.4}$$

where d^0 denote the normalized eigenvectors, Δ_{ik} is the tensor given by (2.18) and P_{ik} the potential part of the energy momentum tensor.

The four-vector V_i is orthogonal to U_i:

$$U_i V_i = 0 \tag{3.5}$$

and so we have:

$$V_i = \left(\mathbf{V}, i\left(\mathbf{V},\mathbf{u}\right)/c\right) \tag{3.6}$$

in which $\mathbf{u}$ denotes the velocity of the matter.

So, in the stationary system, (3.6) reduces to:

$$V_i^0 = \left(\mathbf{V}^0, 0\right) \tag{3.7}$$

Moreover, by replacing (2.18) into (2.20) and using (2.17) and (3.4), then we have instead of (2.22):

$$S_{ik} = T_{ik} - d^0 U_i U_k / c^2 - \left(U_i V_k + V_i U_k\right)/c^2 \tag{3.8}$$

Thus, from (3.8) follows the required relation (3.1), instead of (2.24).

Let us further consider the general system of continuously matter described previously inside which invisible heat conduction can take place, while the motion of the matter is described by the four-velocity U_i or by the velocity u_i.

Then, for the connection between the energy-momentum tensor T_{ik} and the relative stress tensor σ_{ik} of the general system, the following relation is valid:

$$T_{ik} = g_i u_k + \sigma_{ik} + u_i \xi_k / c^2 \tag{3.9}$$

with:

$$\xi_k = U_4 \left(V_k - V_4 U_k / U_4\right)/ic \tag{3.10}$$

where V_k denotes the four-vector given by (3.4), g_i the momentum density and c the speed of light.

The quantity ξ_k seems to be the most important part of ξ_{ik}:

$$\xi_{ik} = H_{ik} - H_{i4}\,U_k/U_4 = U_i\left(V_k - V_4\,U_k/U_4\right)/c^2 \tag{3.11}$$

Furthermore, ξ_k can be written by the following form by using (2.41) and (3.6):

$$\xi_k = (\boldsymbol{\xi}, 0) \tag{3.12}$$

with:

$$\boldsymbol{\xi} = \gamma\left[\mathbf{V} - \mathbf{u}(\mathbf{V},\mathbf{u})/c^2\right] \tag{3.13}$$

In the stationary system, $\boldsymbol{\xi}^0$ is equal to the heat current density $\mathbf{V}^0$:

$$\boldsymbol{\xi}^0 = \mathbf{V}^0 \tag{3.14}$$

By combining (3.10) and (3.11), then one obtains:

$$\xi_{ik} = U_i\,\xi_k/\gamma c^2 \tag{3.15}$$

So, by using (2.35), (3.1), (3.2), (3.11) and (3.15), we have:

$$T_{ik} - T_{i4}\,U_k/U_4 = \sigma_{ik} + \xi_{ik} = \sigma_{ik} + U_i\,\xi_k/\gamma c^2 \tag{3.16}$$

which finally reduces to the required formula (3.9).

Moreover, consider the general system of continuously matter, inside which invisible heat conduction can take place. Then the momentum density $\mathbf{g}$ of this system is given by the ***Universal Equation of Thermo-Elasticity:***

$$\mathbf{g} = m\mathbf{u} + \frac{(\mathbf{u},\boldsymbol{\sigma})}{c^2} + \frac{\boldsymbol{\xi}}{c^2} \tag{3.17}$$

where $\mathbf{u}$ denotes the velocity of the matter at the place and time considered, $\boldsymbol{\sigma}$ the relative stress tensor, $\boldsymbol{\xi}$ is given by (3.13) and $m = E/c^2$ is the total mass density.

From (3.9), we obtain for the energy current density:

$$D_k = Eu_k + u_i\sigma_{ik} + \xi_k \tag{3.18}$$

which can be further written as:

$$\mathbf{D} = E\mathbf{u} + (\mathbf{u},\boldsymbol{\sigma}) + \boldsymbol{\xi} \tag{3.19}$$

So, from (3.19) by using the formula of the momentum density $\mathbf{g}$:

$$\mathbf{g} = \mathbf{D}/c^2 \tag{3.20}$$

we obtain the required relation (3.17) which is a generalization, for a general system with heat conduction.

4. Investigations of Universal Mechanics by Elastic Stress Analysis for Next Generation Spacecraft

Consider the stationary frame of Fig. 1 with Γ_1 the portion of the boundary of the body on which displacements are presented, Γ_2 the surface of the body on which the force tractions are employed and Γ the total surface of the body equal to $\Gamma_1+\Gamma_2$.

In addition, for the principal of virtual displacements, for linear elastic problems then the following formula is valid:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma \tag{4.1}$$

where u_k are the virtual displacements, satisfying the homogeneous boundary conditions $\bar{u}_k \equiv 0$ on Γ_1, b_k the body forces (Fig. 1) and p_k the surface tractions at the point k of the body. (Fig. 3)

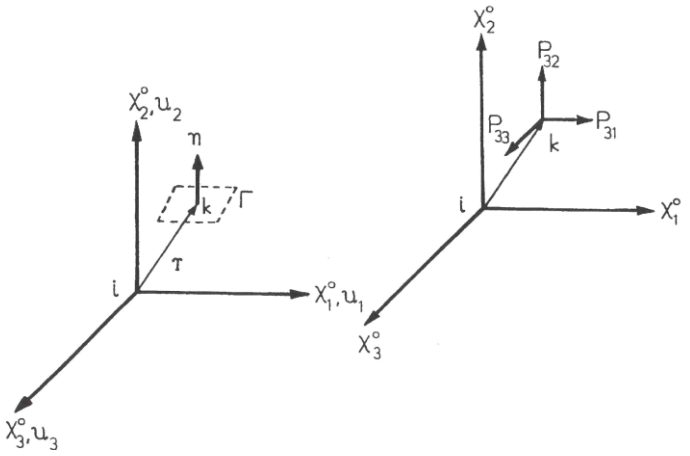

Fig. 3 The stationary system S^0.

Eqn (4.1) can be further written as following if u_k do not satisfy the previous conditions on Γ_1:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{4.2}$$

in which $p_k = n_j \sigma^0_{jk}$ are the surface tractions corresponding to the u_k system.

Then, by integrating (4.2) follows:

$$\int_{\Omega} b_k u_k \, \mathrm{d}\Omega - \int_{\Omega} \sigma^0_{jk} \varepsilon_{jk} \, \mathrm{d}\Omega = -\int_{\Gamma_2} \bar{p}_k u_k \, \mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{4.3}$$

where ε_{jk} are the strains.

By a second integration then (4.3) reduces to:

$$\begin{aligned}&\int_{\Omega} b_k u_k \,\mathrm{d}\Omega + \int_{\Omega} \sigma^0_{jk,j} u_k \,\mathrm{d}\Omega = \\ &-\int_{\Gamma_2} \bar{p}_k u_k \,\mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} \bar{u}_k p_k \,\mathrm{d}\Gamma + \int_{\Gamma_2} u_k p_k \,\mathrm{d}\Gamma\end{aligned} \tag{4.4}$$

Furthermore, a fundamental solution should be found, satisfying the equilibrium equations, of the following type:

$$\sigma^0_{jk,j} + \Delta^i_l = 0 \tag{4.5}$$

in which Δ^i_l denotes the Dirac delta function which represents a unit load at i in the l direction.

The fundamental solution for a three-dimensional isotropic body is: [31]

$$\begin{aligned}u^*_{lk} &= \frac{1}{16\pi G(1-\nu)r}\left[(3-4\nu)\Delta_{lk} + \frac{\partial r}{\partial x_l}\frac{\partial r}{\partial x_k}\right] \\ p^*_{lk} &= -\frac{1}{8\pi(1-\nu)r^2}\left[\frac{\partial r}{\partial n}\left[(1-2\nu)\Delta_{lk} + 3\frac{\partial r}{\partial x_l}\frac{\partial r}{\partial x_k}\right] - \right. \\ &\left. -(1-2\nu)\left[\frac{\partial r}{\partial x_l}n_k - \frac{\partial r}{\partial x_k}n_l\right]\right]\end{aligned} \tag{4.6}$$

where G denotes the shear modulus, ν Poisson's ratio, n the normal to the surface of the body, Δ_{lk} Kronecker's delta, r the distance from the point of application of the load to the point under consideration and n_j the direction cosines (Fig.3).

The displacements at a point are given by the formula:

$$u^i = \int_{\Gamma} up\,\mathrm{d}\Gamma - \int_{\Gamma} pu\,\mathrm{d}\Gamma + \int_{\Omega} bu\,\mathrm{d}\Omega \tag{4.7}$$

Hence, (4.7) takes the following form for the "l" component:

$$u^i_l = \int_{\Gamma} u_{lk} p_k \,\mathrm{d}\Gamma - \int_{\Gamma} p_{lk} u_k \,\mathrm{d}\Gamma + \int_{\Omega} b_k u_{lk} \,\mathrm{d}\Omega \tag{4.8}$$

By differentiating u at the internal points, one obtains the stress-tensor for an isotropic medium:

$$\sigma^0_{ij} = \frac{2G\nu}{1-2\nu}\Delta_{ij}\frac{\partial u_l}{\partial x_l} + G\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4.9}$$

Additionally, after carrying out the differentiation one has:

$$\sigma_{ij}^{0}=\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\partial u_{lk}}{\partial x_l}+G\left(\frac{\partial u_{ik}}{\partial x_j}+\frac{\partial u_{jk}}{\partial x_i}\right)\right]p_k\,\mathrm{d}\Gamma+$$
$$+\int_{\Omega}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\partial u_{lk}}{\partial x_l}+G\left(\frac{\partial u_{ik}}{\partial x_j}+\frac{\partial u_{jk}}{\partial x_i}\right)\right]b_k\,\mathrm{d}\Omega-\tag{4.10}$$
$$-\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\partial p_{lk}}{\partial x_l}+G\left(\frac{\partial p_{ik}}{\partial x_j}+\frac{\partial p_{jk}}{\partial x_i}\right)\right]u_k\,\mathrm{d}\Gamma$$

Eq. (4.10) can be further written as follows:

$$\sigma_{ij}^{0}=\int_{\Gamma}D_{kij}p_k\,\mathrm{d}\Gamma-\int_{\Gamma}S_{kij}u_k\,\mathrm{d}\Gamma+\int_{\Omega}D_{kij}b_k\,\mathrm{d}\Omega\tag{4.11}$$

where the third order tensor components D_{kij} and S_{kij} are:

$$D_{kij}=\frac{1}{8\pi(1-v)r^2}\left[(1-2v)\left[\Delta_{ki}r_{,j}+\Delta_{kj}r_{,i}-\Delta_{ij}r_{,k}\right]+3r_{,i}r_{,j}r_{,k}\right]\tag{4.12}$$

$$S_{kij}=\frac{G}{4\pi(1-v)r^3}\left[3\frac{\partial r}{\partial n}\left[(1-2v)\Delta_{ij}r_{,k}+v(\Delta_{ik}r_{,j}+\Delta_{jk}r_{,i})-5r_{,i}r_{,j}r_{,k}\right]\right.$$
$$\left.+3v(n_ir_{,j}r_{,k}+n_jr_{,i}r_{,k})+(1-2v)(3n_kr_{,i}r_{,j}+n_j\Delta_{ik}+n_i\Delta_{jk})-(1-4v)n_k\Delta_{ij}\right]\tag{4.13}$$

with: $r_{,i}=\dfrac{\partial r}{\partial x_i}$

Finally, because of eqs (2.49) and (4.11) by considering the moving system S of Fig. 2, then the stress-tensor reduces to the following form:

$$\begin{aligned}
\sigma_{11}&=\sigma_{11}^{0}\\
\sigma_{12}&=\gamma\sigma_{12}^{0}\\
\sigma_{13}&=\gamma\sigma_{13}^{0}\\
\sigma_{21}&=\frac{1}{\gamma}\sigma_{21}^{0}\\
\sigma_{22}&=\sigma_{22}^{0}\\
\sigma_{23}&=\sigma_{23}^{0}\\
\sigma_{31}&=\frac{1}{\gamma}\sigma_{31}^{0}\\
\sigma_{32}&=\sigma_{32}^{0}\\
\sigma_{33}&=\sigma_{33}^{0}
\end{aligned}\tag{4.14}$$

where σ_{ij}^{0} are given by. (4.11) to (4.13).

Table 1 shows the values of γ as given by (2.41) for some arbitrary values of the velocity u of the moving aerospace structure:

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	c	∞

So, from Table 1 follows that for small velocities 50,000 *km*/*h* to 200,000 *km*/*h*, the absolute and the relative stress tensor are nearly the same. On the contrary, for bigger velocities like $c/3$, $c/2$ or $3c/4$ (c = speed of light), the variable γ takes values more than the unit and thus, relative stress tensor is very different from the absolute one. Besides, for values of the velocity for the moving structure near the speed of light, the variable γ takes bigger values, while when the velocity is equal to the speed of light, then γ tends to the infinity.

Hence, the Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et al. [22] will be used for the numerical calculation of the stress tensor (3.11), for every specific case.

5. Investigations of Universal Stress Intensity Factors for Next Generation Spacecraft by Theory of Relativistic Fracture Mechanics

Consider a stationary frame for elastic materials in an in-plane loaded plate. Then, the first and second mode stress intensity factors are given by the formulas (Fig.4): [64]

$$K_I^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{5.1}$$

$$K_{II}^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12}^0 \right\} \tag{5.2}$$

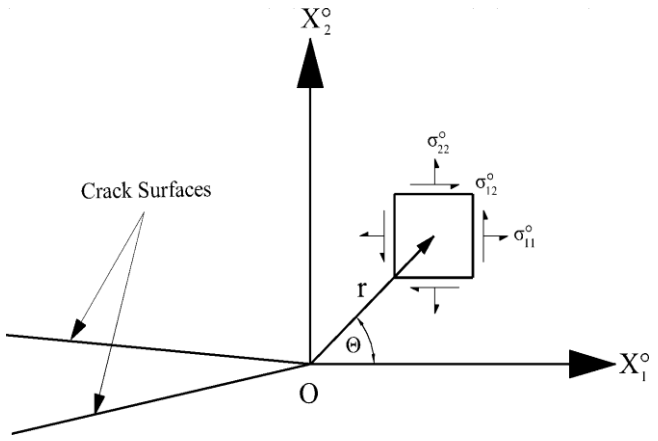

Fig. 4 2-D Coordinates near the crack tip.

Also, the ***relative first and second mode stress intensity factors*** for the airframes are equal to:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22} \right\} \tag{5.3}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12} \right\} \tag{5.4}$$

Thus, because of (4.14), eqs (5.3) and (5.4) can be written as:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{5.5}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \gamma \sigma_{12}^0 \right\} \tag{5.6}$$

Beyond the above, the first, second and third mode stress intensity factors in the stationary frame for elastic materials in a 3-D solid are given by the relations (Fig.5): [65]

$$K_I^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{5.7}$$

$$K_{II}^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12}^0 \right\} \tag{5.8}$$

$$K_{III}^0 = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{23}^0 \right\} \tag{5.9}$$

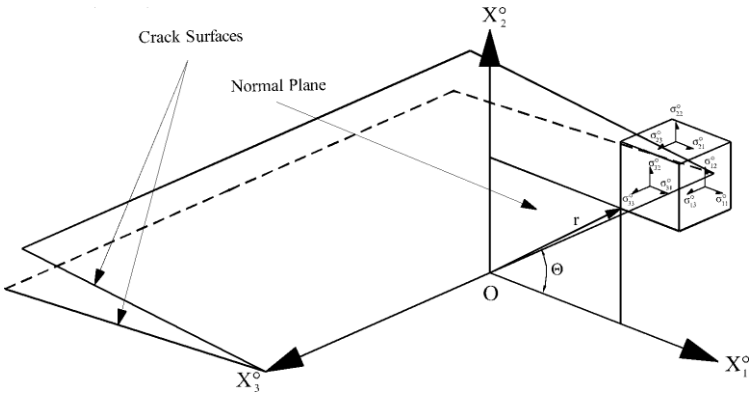

Fig. 5 3-D Coordinates near the crack tip.

Moreover, the ***relative first, second and third mode stress intensity factors*** for the airframes are equal to:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22} \right\} \tag{5.10}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{12} \right\} \tag{5.11}$$

$$K_{III} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{23} \right\} \tag{5.12}$$

So, because of (4.14), eqs (5.10), (5.11) and (5.12) can be written as:

$$K_I = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{22}^0 \right\} \tag{5.13}$$

$$K_{II} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \gamma \sigma_{12}^0 \right\} \tag{5.14}$$

$$K_{III} = \lim_{x_1 \to 0} \left\{ \sqrt{2\pi x_1}\, \sigma_{23}^{0} \right\} \tag{5.15}$$

By eqs (5.13), (5.14) and (5.15) are given the ***Universal Stress Intensity Factors***. So, from eqs (5.13) to (5.15) follows that the relative first and third mode stress intensity factors are the same for both stationary and moving frames, while the relative second mode stress intensity factor is much different in the above frames. All the relative stress intensity factors (first, second and third) are important for the fracture mechanics analysis of the future spacecraft, as for their fracture mechanics analysis a combination of all the three intensity factors should be used [66]. Hence, because of the above difference of the stress intensity factors, follows that the fracture behavior of the new century spacecraft would be much different and thus special materials should be used for their construction.

6. Conclusions

By the current research in the area of aerospace and aeronautical technologies the theory of *"Universal Mechanics"* has been further investigated and applied for the design of the next generation spacecraft moving with very high speeds, even approaching the speed of light, as the plan of the International Space Agencies is to achieve such spacecraft in the future. The future investigation concerns to the determination of the proper composite materials or any other kind of materials for the construction of the next generation spacecraft, as usual composite solids are not suitable for such constructions.

The theory of *"Universal Mechanics"* and correspondingly the *"Universal Equation of Elasticity"* and the *"Universal Equation of Thermo-Elasticity"* show that there is a considerable difference between the absolute stress tensor of the airframe even in the range of speeds of 50,000 km/h. For bigger speeds the difference between the two stress tensors is very much increased. *"Universal Mechanics"* results as a combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*.

Consequently, for the structural design of the new generation spacecraft the stress tensor of the airframe will be used in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical solution the Singular Integral Operators Method (S.I.O.M.) will be used.

References

1. *Ladopoulos E.G.*, 'On the numerical solution of the finite – part singular integral equations of the first and the second kind used in fracture mechanics', *Comp. Meth. Appl. Mech. Engng,* ***65*** *(1987), 253 – 266.*
2. *Ladopoulos E.G.*, 'On the solution of the two – dimensional problem of a plane crack of arbitrary shape in an anisotropic material', *J. Engng Fract. Mech.,* ***28*** *(1987), 187 – 195.*
3. *Ladopoulos E.G.*, 'On the numerical evaluation of the singular integral equations used in two and three-dimensional plasticity problems', *Mech. Res. Commun.,* ***14*** *(1987), 263 – 274.*
4. *Ladopoulos E.G.*, 'Singular integral representation of three – dimensional plasticity fracture problem', *Theor. Appl. Fract. Mech.,* ***8*** *(1987), 205 – 211.*
5. *Ladopoulos E.G.,* 'On a new integration rule with the Gegenbauer polynomials for singular integral equations, used in the theory of elasticity', *Ing. Arch.,* ***58*** *(1988), 35 – 46.*
6. *Ladopoulos E.G.*, 'On the numerical evaluation of the general type of finite-part singular integrals and integral equations used in fracture mechanics', *J. Engng Fract. Mech.,* ***31*** *(1988), 315 – 337.*
7. *Ladopoulos E.G.*, 'The general type of finite-part singular integrals and integral equations with logarithmic singularities used in fracture mechanics', *Acta Mech..,* ***75*** *(1988), 275 – 285.*
8. *Ladopoulos E.G.*, 'On the numerical solution of the multidimensional singular integrals and integral equations used in the theory of linear viscoelasticity', *Int J.Math. Math. Scien.,* ***11*** *(1988), 561 – 574.*
9. *Ladopoulos E.G.*, 'Singular integral operators method for two – dimensional plasticity problems', *Comp. Struct.,* ***33*** *(1989), 859 – 865.*
10. *Ladopoulos E.G.*, 'Finite–part singular integro-differential equations arising in two-dimensional aerodynamics', *Arch.. Mech.,* ***41*** *(1989), 925 – 936.*
11. *Ladopoulos E.G.*, 'Cubature formulas for singular integral approximations used in three-dimensional elastiicty', *Rev. Roum. Sci. Tech..,Mec. Appl.,* ***34*** *(1989), 377 – 389.*

12. *Ladopoulos E.G*., 'Singular integral operators method for three – dimensional elasto – plastic stress analysis', *Comp. Struct., **38** (1991), 1 – 8.*
13. *Ladopoulos E.G*., 'Singular integral operators method for two – dimensional elasto – plastic stress analysis', *Forsch.. Ingen., **57** (1991), 152 – 158.*
14. *Ladopoulos E.G*., 'New aspects for the generalization of the Sokhotski – Plemelj formulae for the solution of finite – part singular integrals used in fracture mechanics', *Int. J. Fract., **54** (1992), 317 – 328*.
15. *Ladopoulos E.G*., 'Singular integral operators method for anisotropic elastic stress analysis', *Comp. Struct., **48** (1993), 965 – 973*.
16. *Ladopoulos E.G*., 'Systems of finite-part singular integral equations in Lp applied to crack problems', *J. Engng Fract. Mech.., **48** (1994), 257 – 266.*
17. *Ladopoulos E.G., Zisis V.A. and Kravvaritis D.,*'Singular integral equations in Hilbert space applied to crack problems', *.Theor.Appl. Fract. Mech., **9** (1988), 271 – 281*.
18. *Zisis V.A. and Ladopoulos E.G*., 'Singular integral approximations in Hilbert spaces for elastic stress analysis in a circular ring with curvilinear cracks', *Indus. Math., **39** (1989), 113 – 134.*
19. *Zisis V.A. and Ladopoulos E.G*., 'Two-dimensional singular inetgral equations exact solutions', *J. Comp. Appl. Math., **31** (1990), 227 – 232*.
20. *Ladopoulos E.G., Kravvaritis D. and Zisis V.A.,*'Finite-part singular integral representation analysis in Lp of two-dimensional elasticity problems', *J. Engng Fract. Mech., **43** (1992), 445 – 454.*
21. *Ladopoulos E.G. and Zisis V.A.,*'Singular integral representation of two-dimensional shear fracture mechanics problem', *Rev.Roum. Sci. Tech., Mec. Appl.., **38** (1993), 617 – 628.*
22. *Ladopoulos E.G., Zisis V.A. and Kravvaritis D.,*'Multidimensional singular integral equations in Lp applied to three-dimensional thermoelastoplastic stress analysis', *Comp. Struct.., **52** (1994), 781 – 788.*
23. *Ladopoulos E.G.,* 'Non-linear integro-differential equations used in orthotropic shallow spherical shell analysis', *Mech. Res. Commun., **18** (1991), 111 – 119.*
24. *Ladopoulos E.G.,* 'Non-linear integro-differential equations in sandwich plates stress analysis', *Mech. Res. Commun., **21** (1994), 95 – 102.*
25. *Ladopoulos E.G.,* 'Non-linear singular integral representation for unsteady inviscid flowfields of 2-D airfoils', *Mech. Res. Commun., **22** (1995), 25 – 34.*
26. *Ladopoulos E.G.,* 'Non-linear multidimensional singular integral equations in 2-dimensional fluid mechanics analysis', *Int. J.Non-Lin. Mech., **35** (2000), 701 – 708.*
27. *Ladopoulos E.G. and Zisis V.A.,*'Existence and uniqueness for non-linear singular integral equations used in fluid mechanics', *Appl. Math., **42** (1997), 345 – 367.*
28. *Ladopoulos E.G. and Zisis V.A.,*'Non-linear finite-part singular integral equations arising in two-dimensional fluid mechanics', *Nonlin. Anal., Th. Meth. Appl., **42** (2000), 277 – 290.*
29. *Ladopoulos E.G. and Zisis V.A.,*'Non-linear singular integral approximations in Banach spaces', *Nonlin. Anal., Th. Meth. Appl., **26** (1996), 1293 – 1299.*
30. *Ladopoulos E.G.,* 'Relativistic elastic stress analysis for moving frames', *Rev. Roum. Sci.Tech., Mec. Appl., **36** (1991), 195 – 209..*
31. *Ladopoulos E.G., 'Singular Integral Equations, Linear and Non-Linear Theory and its Applications in Science and Engineering'*, Springer Verlag, New York, Berlin, *2000*.
32. *Ladopoulos E.G.,* 'Relativistic mechanics for airframes applied in aeronautical technologies', *Adv. Bound. Elem. Tech.., **10** (2009), 395 – 405.*
33. *Muskhelishvili N.I.,'Some Basic Problems of the Mathematical Theory of Elasticity',* Noordhoff, Groningen, Netherlands, *1953.*
34. *Green A.E. and Zerna W., 'Theoretical Elasticity',* Oxford Ubniv. Press, Oxford, *1954*.
35. *Boley B.A. and Weiner J.H., 'Theory of Thermal Stresses',* J.Wiley, New York, *1960.*
36. *Nowacki W.,'Thermoelasticity',* Pergamon Press, Oxford, *1962*.
37. *Drucker D.C. and Gilman J.J., 'Fracture of Solids',* J.Wiley, New York, *1963.*
38. *Lekhnitskii S.G., 'Theory of Elasticity of an Anisotropic Elastic Body',* Holden-Day, San Fransisco, *1963.*
39. *Truesdell C. and Noll W., 'The Non-linear Field Theories of Mechanics',* Handbuch der Physic, Vol. III/3, Springer Verlag, Berlin, *1965*.
40. *Liebowitz H. 'Fracture',* Academic Press, New York, *1968*.
41. *Sneddon I.N. and Lowengrub M., 'Crack Problems in the Classical Theory of Elasticity',* J.Wiley, New York, *1969*.
42. *Lions J.L., 'Quelques Methodes de Resolution des Problemes aux Limites Non Lineaires,* Dunod, Paris, *1969.*
43. *Oden J.T., 'Finite Elements in Nonlinear Continua',* McGraw Hill, New York, *1972*.

44. *Eringen A.C., 'Continuum Physics',* Academic Press, New York, *1972.*
45. *Duvant G. and Lions J.L., 'Les Inequations en Mecanique et en Physique', Dunod, Paris,* 1972.
46. *Fichera G., 'Boundary Value Problems of Elasticity with Unilateral Constraints',* Handbuch der Physik, Vol. VIa/2, Springer Verlag, Berlin, *1972.*
47. *Germain P., 'Mecanique des Milieux Continus',* Masson, Paris, *1972.*
48. *Wang C.C. and Truesdell C., 'Introduction to Rational Elasticity',* Noordhoff, Groningen, Netherlands, *1973.*
49. *Washizu K., 'Variational Methods in Elasticity and Plasticity',* Pergamon Press, Oxford, *1975.*
50. *Kupradze V.D., 'Three-dimensional Problems in the Mathematical Theory of Elasticity and Thermoelasticity',* Nauka, Moscow, *1976.*
51. *Gurtin M.E., 'Introduction to Continuum Mechanics',* Academic Press, New York, *1981.*
52. *Ciarlet P.G., 'Topics in Mathematical Elasticity',* North Holland, Amsterdam, *1985.*
53. *Laue M.von, 'Die Relativitätstheorie',* Vol. 1, Vieweg und Sohn, Braunschweig, *1919.*
54. *Gold T., 'Recent Developments in General Relativity',* Pergamon Press, New York, *1962.*
55. *Pirani F.A.E., 'Lectures on General Relativity',* Vol.1, Prentice-Hall, New Jersey, *1964.*
56. *Gursey F., 'Relativity, Groups and Topology',* Gordon and Breach, New York, *1964.*
57. *Adler R., 'Introduction to General Relativity',* McGraw-Hill, New York, *1965.*
58. *Rindler W.*, '*Special Relativity',* Oliver and Boyd, Edinburgh, *1966.*
59. *Möller C., 'The Theory of Relativity',* Oxford University Press, Oxford, *1972.*
60. *Synge J.L., 'General Relativity',* Clarendon Press, Oxford,. *1972.*
61. *G.C.Sih,* 'Use specification of multiscale materials for life spanned over macro-, micro-, nano-, and pico-scale', *Theor. Appl. Fract. Mech., **53** (2010) 94-112.*
62. *G.C.Sih,* 'Scale shifting laws from pico to macro in consecutive segments by use of transitional functions', *Theor. Appl. Fract. Mech., **53** (2010) 165-179.*
63. *G.C.Sih,* 'Mesomechanics of energy and mass interaction for dissipative systems', *J. Phys. Mesomech., **13** (2010) 233-244.*
64. *Irwin G.R., 'Fracture'* in *'Encyclopaedia of Physics',* (Ed. S.Flugge), Vol. VI, Spinger, Heidelberg, *1958.*
65. *Kassir M.K. and Sih G.C.,* 'Three-dimensional Crack Problems' in *'Mechanics of Fracture',* (Ed. G.C.Sih), Vol. II, Noordhoff, Netherlands, *1975.*
66. *G.C.Sih, 'Mechanics of Fracture Initiation and Propagation',* Kluwer Academic Publishers, Boston, *1991.*
67. *G.C.Sih and K.K.Tang,* 'Assurance of reliable time limits in fatigue depending on choice of failure simulation: energy density versus stress intensity', *Theor. Appl. Fract. Mech., **55** (2011) 39-51.*
68. *G.C.Sih,* 'Multiscale reliability of physical systems based on the principle of least variance', *Theor.Appl. Fract. Mech., **55** (2011) 1-19.*