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Solving a Class of Single Degree of Freedom Nonlinear Vibration Problems using the Reproducing Kernel

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Abstract
The most distinguishing feature of nonlinear problems in engineering is load and the response is no longer proportional relationship, superposition principle no longer apply, we can not use the method dealing with linear problem to deal with it. As we know, reproducing kernel method is an accuracy method to solve some nonlinear problems. In this paper, we use the reproducing kernel method to solve the single degree of freedom nonlinear vibration problems. There are many vibration problems in engineering , we take Duffing vibration control equations and Duffing-Van der Pol vibration equations for example to demonstrate the accuracy of the the reproducing kernel method to solve some vibration problems in engineering. Results obtained by the reproducing kernel method indicate it has the following advantages: small computational work, fast convergence speed and high precision.

Key Word and Phrases
Reproducing Kernel, Single Degree of Freedom Nonlinear Vibration Problems, Duffing Vibration Control Equations, Duffing-Van der Pol Vibration Equations.

1. Introduction

Some variables of engineering problems present a cyclical change with the evolution of time, this class of problem named as vibration problem. Vibration problem include machinery vibration problem and circuitry vibration problem. So, vibration problem always use Newton’s second law set up the governing differential equation, where governing equation is the differential equation or equation set about the second derivative of time. The most distinguishing feature of nonlinear problems in engineering is load and the response is no longer proportional relationship, superposition principle no longer apply. If considering about nonlinear effect in governing system, the function is always the second order ordinary differential equation initial value problem. Degree of freedom is in a no constraint power or other system. In order to completely determine independent variables in the state of the system at a given moment. Single-degree-of-freedom system is the system that at any time as long as a generalized coordinate system can completely determine its position.

Duffing vibration control equations general form is:

$$\ddot{x} + c\dot{x} + ax(t) + bx^3(t) = F\cos(wt),\ t > 0 \tag{1.1}$$

Initial conditions is: $x(0) = A$, $\dot{x}(0) = V$.

where c is damping parameter, $w_0 = \sqrt{a}$, is the natural frequency of the linear system and b is nonlinear coefficient.

Duffing-Van der Pol vibration equations general form is:

$$\ddot{x} + (\alpha + \beta x^2)\ddot{x} + \gamma x(t) + \lambda x^3(t) = 0 \tag{1.2}$$

Initial conditions is: $x(0) = A$, $\dot{x}(0) = V$.

where $\alpha, \beta, \gamma, \lambda$ is arbitrary constant, if $\alpha = -\mu$, $\beta = \mu$, $\gamma = \mu$, $\lambda = 0$, in this time (1.2) changed as classical Van der Pol vibration function.

2. The Reproducing Kernel Method

2.1 Practise Homogenization for Duffing Vibration Control Equations and Duffing-Van der Pol Vibration Equations

In order to use reproducing kernel method to solve (1.1), we need to practise homogenization for (1.1), previously, we found that:

$$\begin{cases} (\ell u)(x) = F(x,u), x \in [0,1] \\ u(0) = A, u'(0) = V \end{cases} \tag{2.1}$$

where: $\ell u(x) = u''(x) + cu'(x) + au(x)$, $F(x,u) = -bu^3(x)$. Obviously, the solution of (2.1) is the solution of (1.1), So we only need to gain the solution of (2.1). The question (2.1) with nonhomogeneous boundary value conditions is equivalent to the problem of having a function $v(x)$ satisfying:

$$\begin{cases} (\ell v)(x) = \overline{F}(x,v), x \in [0,1] \\ v(0) = 0, v'(0) = 0 \end{cases} \tag{2.2}$$

where: $\overline{F}(x,v) = -bu^3(x) - Vx - A$.

In order to use reproducing kernel method to solve (1.2), we need to practise homogenization for (1.2), previously, we found that:

$$\begin{cases} (\ell u)(x) = F(x,u), x \in [0,1] \\ u(0) = A, u'(0) = V \end{cases} \tag{2.3}$$

where: $\ell u(x) = u''(x) + \gamma u(x)$, $F(x,u) = -(\alpha + \beta u^2(x))u''(x) - \lambda u^3(x)$. Obviously, the solution of (2.1) is the solution of (1.2), So we only need to gain the solution of (2.1). The question (2.1) with nonhomogeneous boundary value conditions is equivalent to the problem of having a function $v(x)$ satisfying:

$$\begin{cases} (\ell v)(x) = \overline{F}(x,v), x \in [0,1] \\ v(0) = 0, v'(0) = 0 \end{cases} \tag{2.4}$$

where: $\overline{F}(x,v) = -(\alpha + \beta u^2(x))u''(x) - \lambda u^3(x) - Vx - A$.

2.2 Construct Reproducing Kernel Space

Be aimed with the purpose of solving (2.2) and (2.4), we need to introduce the reproducing kernel space, previously, let's introduce the concept of the reproducing kernel space. For each of $x \in X$, there is a function of two variables $K_x(y) \in H$, where *H* is Hilbert space, *X* is a set abstraction. If we can get:

$$\langle u(y), k_x(y) \rangle = u(x), \quad u(y) \in H \tag{2.5}$$

we say *H* is the reproducing kernel Hilbert space, $K_x(y)$ is the reproducing kernel of *H*.

We introduce a linear space $W_2^3[0,b]$:

$$W_2^3[0,b] = \{u \mid u, u', u'' \text{ is one - variable absolutely continuous function, } u''' \in L^2[0,b], u(0) = 0, \\ u'(0) = 0\}$$

According to [4], [5], we have the inner product:

$$\langle u(y), v(y) \rangle = u''(0)v''(0) + \int_0^b u'''(y)v'''(y)dy \tag{2.6}$$

and according to [6], we can prove that $W_2^3[0,b]$ is a reproducing kernel space, it's reproducing kernel $R(x,y)$ is:

$$R(x,y) = \begin{cases} \frac{1}{120}(120 + x^5 + 120xy - 5x^4 y) + 30x^2 y^2 + 10x^3 y^2, x < y \\ \frac{1}{120}(120 + y^5 + 10x^2 y^3 (3 + y) - 5xy(y^3 - 24)), y < x \end{cases} \tag{2.7}$$

In order to use reproducing kernel method to solve (2.2) (2.4)and refers to [7][8], we can get $\psi_i(x)$:

$$\psi_i(x) = \begin{cases} (\frac{1}{24} x(4x(3+x)y + n(24 - x^3 + 12xy + 4x^2 y))) |_{y=x_i}, x < y \\ (\frac{1}{24}(4y(-3xy^2 + y^3 + 3x^2(1+y)) + n(y^4 + 6x^2 y(2+y) - 4x(-6+y^3)))) |_{y=x_i}, y < x \end{cases} \tag{2.8}$$

where $i = 1,2,3...$

Then practise Gram-Schmidt orthogonalization $\{\psi_i(x)\}_{i=1}^{\infty}$, according to [9], [10] , we get:

$$\begin{cases} \overline{\psi}_1(x) = \beta_{11}\psi_1(x) \\ \overline{\psi}_2(x) = \beta_{21}\psi_1(x) + \beta_{22}\psi_2(x) \\ \overline{\psi}_3(x) = \beta_{31}\psi_1(x) + \beta_{32}\psi_2(x) + \beta_{33}\psi_3(x) \\ \vdots \\ \overline{\psi}_i(x) = \beta_{i1}\psi_1(x) + \beta_{i2}\psi_2(x) + \beta_{i3}\psi_3(x) + \cdots + \beta_{ii}\psi_i(x) \end{cases} \tag{2.9}$$

where β_{ik} are coefficients of Gram-Schmidt orthogonalization.

If $\{x_i\}_{i=1}^{\infty}$ is distinct point dense in [0,b], and ℓ^{-1} is existent, we get:

$$u(x) = \sum_{i=1}^{\infty}\sum_{k=1}^{i} \beta_{ik}\overline{F}(x_k, v(x_k))\overline{\psi}_i(x) + \lambda_1 + \lambda_2 x \tag{2.10}$$

is the solution of (2.2) and (2.4). The proof of it refers to [11], [12] If the equations are linear ones, $\overline{F}(x,v) = \overline{F}(x)$, we can solve the problems directly. If they are nonlinear equations, we have to use iteration method to solve them, the specific methodology refers to [13], [14].

2.3 The Approximate Solution

We denote the approximate solution of $u_m(x)$ by:

$$u(x) = \sum_{i=1}^{\infty}\sum_{k=1}^{i} \beta_{ik}\overline{F}(x_k, v(x_k))\overline{\psi}_i(x) + \lambda_1 + \lambda_2 x \tag{2.11}$$

According to the proof of [15] we can easily to get that $\|u_m(x) - u(x)\| \to 0$, and:

$$u_m^{(k)}(x) \to u^{(k)}(x), k = 0,1,2 .$$

3. Numerical Experiment

Example 1[2] Let's consider the following Duffing vibration control equation.

There, $c=0, a=1, b=-\frac{1}{6}, w=0.7, F=0, A=0, V=1.62376$, take them into (1.1), we get:

$$\begin{cases} u''(x)+u(x)=\frac{1}{6}u^3(x), x\in[0,1] \\ u(0)=0, u'(0)=1.62376 \end{cases} \tag{3.1}$$

The exact solution is $u_T(x)=2.058\sin(0.7x)+0.0816\sin(2.1x)+0.00337\sin(3.5x)$ The result of reproducing kernel method (RKM) are shown in Figure 1, Table 1.

Example 2[3] Considering the following Duffing-Van der Pol vibration equation.

$$\begin{cases} u''(x)+(\frac{4}{3}+3u^2(x))+u'(x)+\frac{2}{3}u(x)+u^3(x)=0 \\ u(0)=-0.28868, u'(0)=0.12 \end{cases} \tag{3.2}$$

The exact solution is:

$$u_T(x)=-0.000026166\ e^{-4.09188\ x}-0.00212777\ e^{-2.16516\ x}-0.0266831\ e^{-1.06495\ x}-0.259843\ e^{-0.334316\ x}$$

The result of reproducing method are shown in Figure 2, Table2 .

Table 1. The Numerical Results of Example 1

x	$u_T(x)$	$u_{30}(x)$	$\lvert u_T(x)-u_{30}(x)\rvert$	$\lvert u_T'(x)-u_{30}'(x)\rvert$	$\lvert u_T''(x)-u_{30}''(x)\rvert$
0	0	0	0	5E—06	5.21968E—02
0.1	-0.162108	-0.162031	7.70035E—05	8.66236E—04	7.70035E—05
0.2	-0.322624	-0.322461	1.63083E—04	8.53025E—04	1.63083E—04
0.3	-0.48008	-0.479761	2.47129E—04	8.25701E—04	2.47123E—04
0.4	-0.632824	-0.632496	3.27782E—04	7.85365E—04	3.27736E—04
0.5	-0.779781	-0.779378	4.03813E—04	7.33511E—04	4.03602E—04
0.6	-0.919771	-0.919296	4.74156E—04	6.71923E—04	4.73446E—04
0.7	-1.05118	-1.05135	5.37937E—04	6.02569E—04	5.35992E—04
0.8	-1.17542	-1.17483	5.94480E—04	5.27504E—04	5.89918E—04
0.9	-1.28991	-1.28927	6.43317E—04	4.48780E—04	6.33824E—04
1	-1.39506	-1.39432	6.84183E—04	3.68382E—04	6.66236E—04

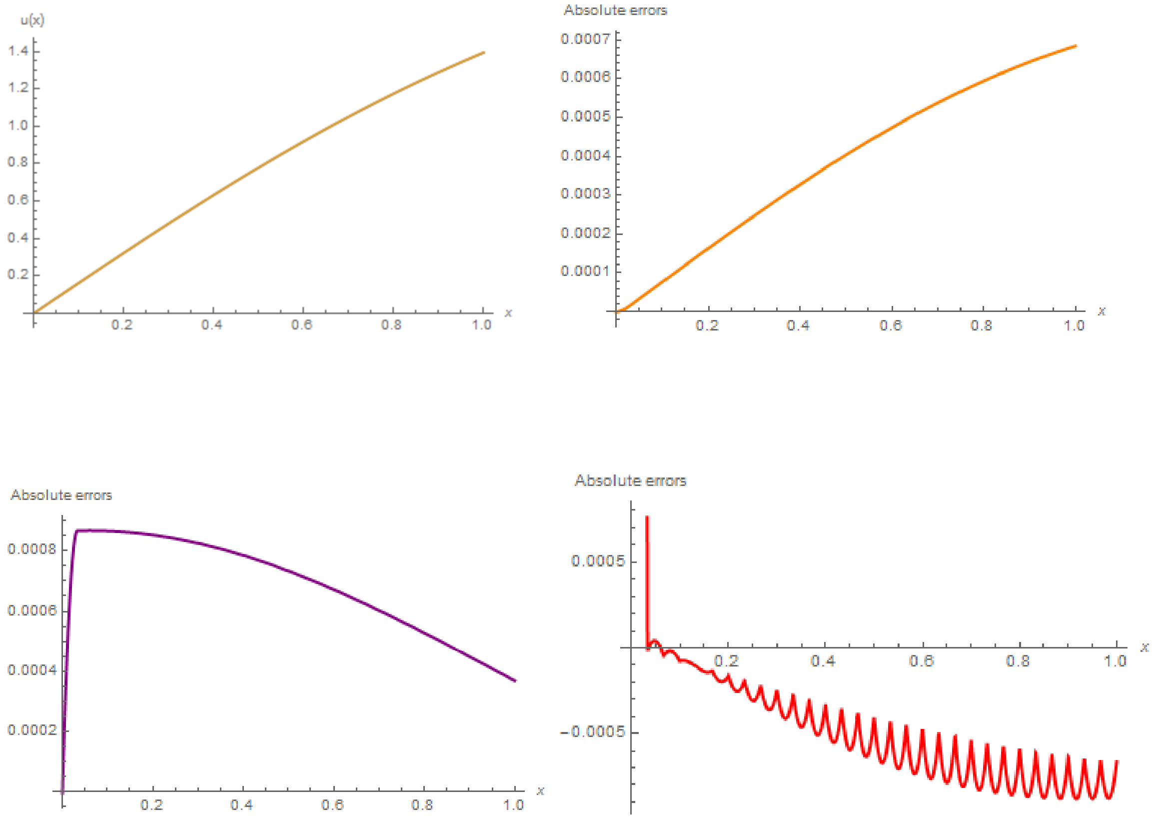

Fig. 1 The reproducing kernel method for Example 1, the first picture is $u_T(x)$ and $u_{30}(x)$, the second picture is $|u_T(x)-u_{30}(x)|$, the third picture is $|u_T'(x)-u_{30}'(x)|$, the fourth picture is $|u_T''(x)-u_{30}''(x)|$.

Table 2. The Numerical Results of Example 2

x	$u_T(x)$	$u_{20}(x)$	$\lvert u_T(x)-u_{20}(x)\rvert$	$\lvert u_T'(x)-u_{20}'(x)\rvert$	$\lvert u_T''(x)-u_{20}''(x)\rvert$
0	-0.28868	-0.28868	3.6E—08	1.29642E—07	6.12137E—03
0.1	-0.277018	-0.277006	1.20708E—05	1.34581E—04	2.40296E—04
0.2	-0.265993	-0.265969	2.40283E—05	1.01609E—04	4.19587E—04
0.3	-0.25551	-0.25552	3.13486E—05	3.68751E—05	8.96352E—04
0.4	-0.245646	-0.245617	2.91487E—05	9.5888E—05	1.79844E—03
0.5	-0.236235	-0.236227	8.3041E—06	3.45004E—04	3.23939E—03
0.6	-0.227283	-0.227329	4.56861E—05	7.69444E—04	5.31882E—03
0.7	-0.218756	-0.21891	1.53695E—04	1.43747E—03	8.12302E—03
0.8	-0.210625	-0.210968	3.43833E—04	2.42532E—03	1.17256E—02
0.9	-0.202863	-0.203515	6.5218 E—04	3.81599E—03	1.61882E—02
1	-0.195446	-0.19657	1.12341E—05	5.69808E—03	2.15613E—02

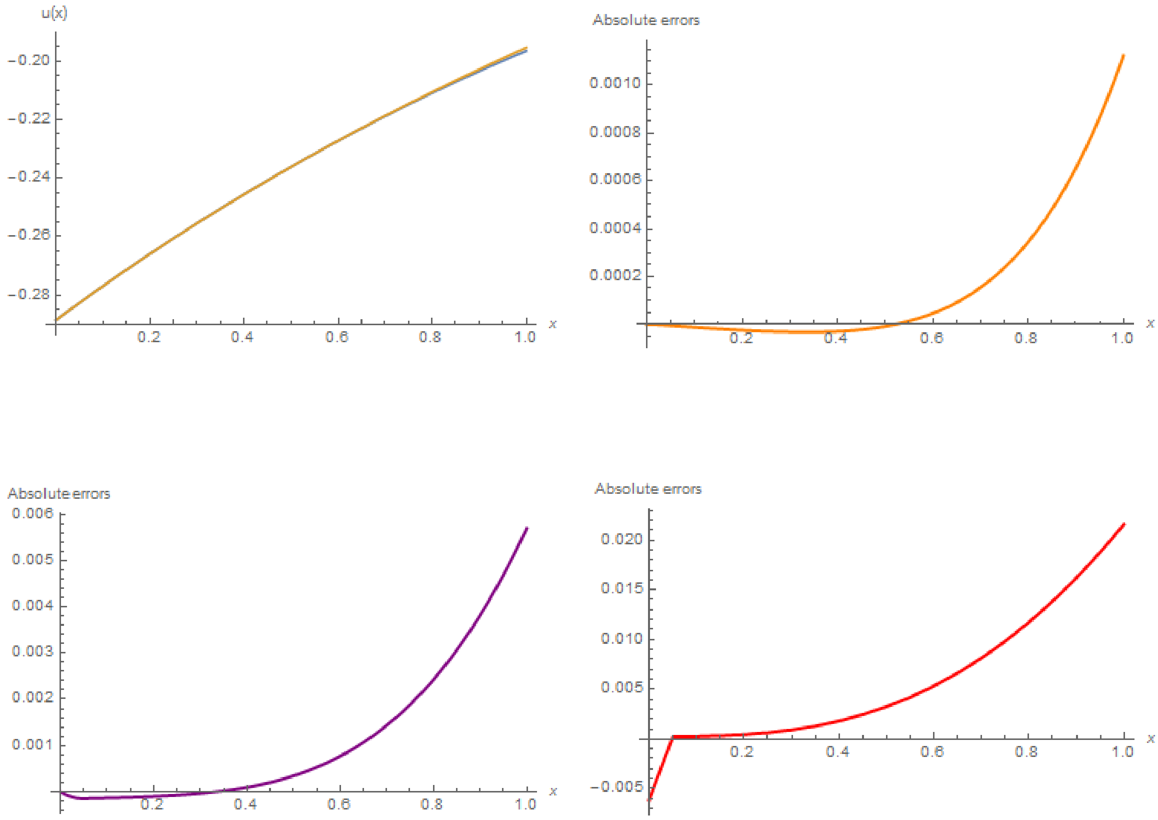

Fig. 2 The reproducing kernel method for Example 1, the first picture is $u_T(x)$ and $u_{20}(x)$, the second picture is $|u_T(x)-u_{20}(x)|$, the third picture is $|u_T'(x)-u_{20}'(x)|$, the fourth picture is $|u_T''(x)-u_{20}''(x)|$.

4. Conclusions and Remarks

Nonlinear problems widely exist in engineering problems, because it is difficult to get accurate solution, so the numerical algorithm for solving such problems is necessary. As scientists focus on these problems, many numerical methods have been proposed. By the current research, in order to solve a class of single degree of freedom nonlinear vibration problem, we take Duffing vibration control equations and Duffing-Van der Pol vibration equations for example. We studied two examples, the numerical results demonstrate that the reproducing kernel method is quite accurate and efficient for some vibration problems in engineering. Reproducing kernel can effectively deal this kind of nonlinear problems in engineering. Our work provides an effective method for the future investigations in some nonlinear problem in engineering. As we know, many practical problems facing more complex problems than we now discuss, after completion of this article, naturally we will focus on the combination of several classes of problems in engineering. All computations are performed by the Mathematica 10.0 software package.

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