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Universal Capillary Pressure-Saturation Model Performed Under Reservoir Conditions Using Pressure Transient Analysis

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Abstract
An analytical model for predicting the capillary pressure-saturation profile developed across the interface that separates two immiscible fluids has been derived from reservoir pressure transient analysis. The model reflects the entire interaction between the contacted fluids and rock properties measured under downhole reservoir conditions; since, it can be considered the most reliable and universal method to estimate the capillary pressure-saturation profile under reservoir conditions. This model retains the natural coupling of oil reservoirs with the displacing fluid zones and treats them as an explicit-region composite system; where each explicit fluid phase zone takes its own fluid and rock properties. The contacted fluid zones are linked by a capillary transition zone that reflects the pressure difference across the contacted fluids. The principle of superposition theorem has been applied to perform this link throughout reflecting the pressure drop behavior created in the oil zone across the free-level interface of the displacing fluids to estimate the reflected pressure drop behavior that is holds the fluid contacts in their equilibrium positions. The results of capillary pressure-saturation generated by the proposed model have been compared with some laboratory measurements taken for core samples representing different oil fields; the results proved the dynamic effect of capillary pressure; at high values of non-wetting phase saturation, the capillary pressure under dynamic effects characterized by lower saturations than observed in static capillary pressures; in contrast higher saturations are measured in the dynamic curves than in static curves at high values of wetting phase. Moreover, results have been generated for several hypothetical systems covering a wide range of variation in rocks and fluids properties for both of displaced and displacing fluids; this shows full agreement with the physics of capillary pressure and its direct affect with all rocks and the fluids that filling the pores. It can be concluded that this model can be considered a universal, reliable, accurate and most cost effective tool than coring measurements which is still one of the capital investment in exploration and production industry.

Keywords and Phrases
Universal Capillary Pressure, Pressure Transient Analysis, Pressure-Saturation Model.

1. Introduction

Capillary pressure is the main controlling parameter affecting on multiphase fluid distribution in porous media. At the static conditions, the capillary pressure is simply defined as the pressure difference across the interface that separates two immiscible fluids. According to this definition, the capillary pressure can be expressed as follows:

$$P_c = P_o - P_l \tag{1.1}$$

where; (P_o, P_l) are the pressures exerted at the fluid interface by the displaced (non wetting) and displacing fluid (wetting) phases, respectively.

This relationship assumes that capillary pressure is a function of saturation of the displaced wetting fluid phase only, and thus uniquely describes the capillary pressure-saturation relationship. However, this simple relationship cannot uniquely describe all real parameters affecting on capillary pressure, in addition to the fact of extremely difficult to measure the capillary pressure under reservoir conditions.

As a result of this potential problem, many authors tried to study the factors that are controlling the capillary pressure measurements; Labstie et.al .[1] indicated that capillary pressure depends on the flow rate and wettability of the porous medium. The experiments made in this study show a positive capillary pressure may exist during the oil drainage phase and becomes negative immediately behind the displacing front. The negative capillary pressure increases as the flooding rate increases. Therefore, it may be impossible to correctly interpret the flooding experiments using the simple expression of static capillary pressure measurements, and then it is necessary to use the experiments performed under actual reservoir conditions. Anderson [2] indicated that the capillary pressure-saturation measurements depend on the wettability degree interaction with rock pore structure. He confirmed that no simple relationship exists that relates the capillary pressure determined at two different wettabilities. Since, the accurate capillary pressure-saturation measurements should be made at native reservoir wettability. On the other hand, Greder et.al. [3], presented results taken from forty comparative tests which have been conducted over several years on consolidated core samples. They showed that rock heterogeneity was found to be the main factor affecting on the capillary pressure-saturation differences, in addition to many other parameters such as rock-fluid and fluid-fluid interactions, temperature, confining and pore pressures proving the necessity of measuring capillary pressures under actual reservoir conditions.

Muccino et al. [4], show that the simple approach such as given in (1.1) does not account for interaction of pore-scale properties, like, fluid-fluid interfacial area, interfacial tension, wettability, geometry of the pore space, in addition to that of fluid properties. Hassanizadeh S.M. et al. [5], indicated that this empirical relationship of capillary pressure-saturation lacks a firm of theoretical foundation. Since, this simple model is lumped implicitly to account for all effects and processes that influence the equilibrium distribution of fluids, such as surface tension, presence of fluid–fluid interfaces, wettability of reservoir rocks, grain size uniformity, and rock heterogeneities. They concluded that there is theoretical and experimental evidence that this simple relationship is not unique, but it depends on the flow dynamics. Hassanizadeh S.M. et al. [6], prevailed that there is experimental evidence that capillary pressure is not only a function of saturation but may also depends on the rate of flow. Current theories of multiphase flow use the capillary pressure-saturation relationships that are commonly measured under static conditions. Recently, new multiphase flow theories have been proposed that include a new capillary pressure-saturation relationship that is valid under dynamic conditions.

Moreover, Kewen Li [7], investigated a number of existing capillary pressure models; Cory-1954, Thomeer-1960, Brooks and Corey-1966, Van Genuchten-1980-, Skelt and Harrison-1995, Jing and Van Wunnik-1998, and Li and Horne-2001, he prevailed that all of these capillary pressure models were proposed empirically; and almost parameters involved in these models do not have a physical significance and did not reflect the real interactions between the fluids and rocks.

Shawkat G. Ghaidan et.al. [8], indicated that the capillary pressure reflects the interaction of rock and the fluids filling these pores. They prevailed that capillary pressure measurements must ideally performed in reservoir conditions to reflect the actual reservoir fluids distribution in fields analysis, but unfortunately one extremely difficult and usually done in laboratory by injecting mercury at ambient conditions. However, the experience showed that either capillary pressure measured in the laboratory or capillary pressure converted to reservoir conditions may not match the log derived Sw - Pc data which also has an accuracy of ± 10% in best and may be as height as ± 20% .

J.O. Helland et al. [9], present three-phase capillary pressure correlation that could be used to simulate the three-phase transition zones in mixed-wet reservoirs. The capillary pressures are expressed as a sum of two terms. One term is a function of decreasing phase saturation and the other term a function of an increasing phase saturation. The correlation uses an adjustable parameters to ensure that the correlation account for different wettability degree conditions. Daniel R. Maloney [10], showed that the capillary pressure-saturation profile is important for estimating hydrocarbon reserves and optimizing its recovery strategies. He prevailed that a limitation of commonly employed methods used for measuring capillary pressure-saturation aren't the same as those of the reservoir conditions, so lab capillary pressure results have to be converted to the real reservoir conditions which reflect the effects of (stress, pore pressure, composition of fluids, temperature). Kewen Li. et al. [11]; indicated that resistivity and capillary pressure are a function of fluid saturation in a porous medium and are influenced by reservoir heterogeneity. They speculated that capillary pressure may be derived from resistivity index that could be obtained from well logs in real time measurements. Therefore, they developed new model that may be useful to evaluate capillary pressure function from resistivity logs to represent capillary pressure under reservoir conditions.

Other studies in capillary pressure topics have been made by Hassanizadeh S.M. et al. [12], Schultze et al. [13], Wildenschild et al. [14], Geremy Camps, et al. [15]; they prevailed that almost experimental and theoretical evidence indicates that the capillary pressure-saturation curves are not unique, but depend upon the flow process, whether a porous medium is being drained or imbibed and also on the degree of wettability. They indicate that it cannot be used the capillary pressure measurements made at static conditions (saturation changes very slowly) to represent the real dynamic reservoir conditions (large changes in saturations). Kihong Kim, et al. [16], indicated that it is difficult to precisely formulate a capillary pressure-saturation relationship in reservoirs having high degree of heterogeneity. They prevailed that an erroneous results encountered while using the previously developed capillary pressure models like Leverett J-function (1941), Cuddy et al. (1993), Johnson (1987), Skelt and Harrison (1995) and Thomeer (1960) methods that are consider the pore as a single pore system in heterogeneous pore. Therefore, they developed a capillary pressure model that can handle the pore system as macropore and micropore terms separately to be suitable in application in high heterogeneity cores.

According to the above mentioned literature survey; follows that a more general capillary pressure-saturation model should be employed to reflect the real dynamic effects between rocks and fluids performed under reservoir conditions. This can be derived theoretically from reservoir pressure transient analysis to make exactly description of the fluid flow dynamics in porous media.

Consequently the objective of the current research is to derive a comprehensive analytical dynamic capillary pressure model which reflects the entire interaction between the contacted fluids and rock properties measured under downhole reservoir conditions; and providing methodology to eliminate the coring works which is still one of the largest capital cost in exploration and production industry and the departure from the complicated laboratory measurements.

2. Theory and Work Development

The review of the presented literature survey prevailed that the processes that determine the distribution of fluid phases in porous media are extremely complicated due to the real complexity to involve all parameters affecting on capillary pressure-saturation relationship taken under actual reservoir conditions. The main theoretical and practical tool currently used to quantify the capillary pressure function is only an empirical simple function that is lacks to interview all parameters affecting on capillary pressure-saturation relationship.

By the current research, a suggestion in deriving a capillary pressure-saturation formula using reservoir pressure transient analysis theories that is based on applying Darcy’s law extensions which involves almost hydrodynamic parameters affecting on estimation the reservoir pressure drops encountered as a results of fluids flowing through porous mediums.

However, it is evident that the extension to Darcy’s law does not concern the forces that are actually driving for two-phase flow in porous media. Other forces which are known to be present in two phase flow (such as interfacial and capillary forces) are absent in extended Darcy’s law Navraj S. Hanspal. et al. [17]. In fact, this impact may be related to the basic assumptions used in deriving the diffusivity equation that is consider the porous media is filled with single-phase of constant fluid saturation. Since, any solutions derived from the diffusivity equation should be exactly valid only for single-phase fluid saturation.

Therefore, it is essential to formulate capillary pressure model in an appropriate way due its importance in reservoir simulation and engineering applications; in order to reduce the complexity, the formulation of an accurate analytical multiphase fluid flow model in which the created reservoir pressure drops moving between different fluid mediums, could be analyzed by treating each fluid-phase zone as explicit single-phase zones (displaced and displacing fluids) taking their own individual fluid and rock properties, as already adapted by Layne M.A. et.al. [18]; hence, the contacted fluid zones are linked across the free level of displacing fluids (zero capillary pressure level) by a capillary transition zone that is balancing the pressure difference across the free level of displacing fluids.

It is well known that the capillary pressure holds the fluid interfaces at their equilibrium positions unless a pressure disturbance may occur at the contacted free fluid interfaces. Thus, the new theory of measuring the in-situ dynamic capillary pressure under reservoir conditions suggests creating reservoir pressure drop pulse (drawdown) that is usually performed throughout conventional well test analysis as shown in schematic drawing in Fig. 1.

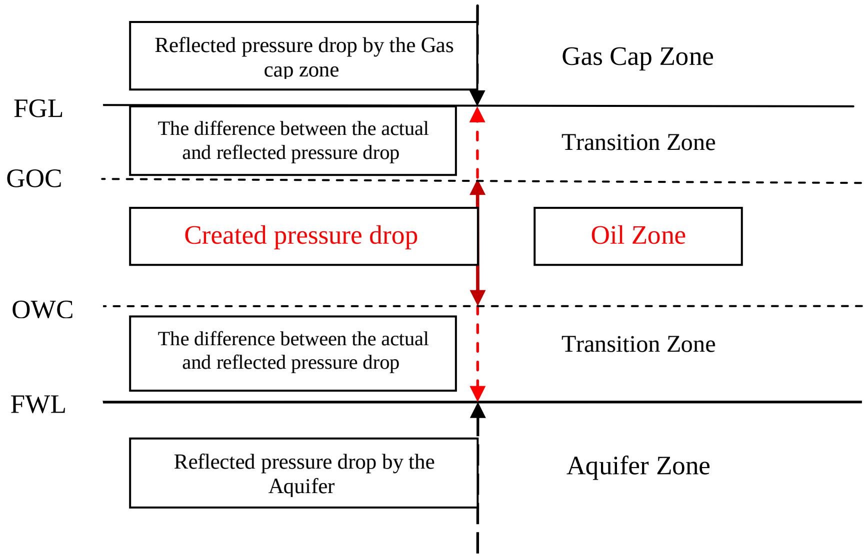

Fig. 1 Schematic drawing for the created and reflected pressure drops at the fluid interface levels.

Saleh Saad T. [19] suggested a method that involved static and dynamic corrections for the pressure drops at the fluid interfaces obtained by Van-Everdingen and Hurst model that is conventionally used for estimating average reservoir pressure drops. He found that the pressures at GWC under different conditions are always less than that estimated by Van-Everdingen and Hurst model. He concluded that errors as much as 14% may result by conventional method when the average reservoir pressures represent the pressures at the GWC. The errors increase as the aquifer size increases and with decreasing of reservoir permeability. Based on Saleh saad's conclusions; Layne M.A. et.al. [18], proposed an analytical model included pressure predictions along the reservoir-aquifer interface, the average pressure in the gas reservoir and the average aquifer pressure are taking their own individual properties and the multiphase systems are treated as explicit phases. This fact is true if the system is linked by the compatibility and continuity conditions on the fluid interface boundary separating the two media with neglecting the capillary pressure that can be developed between the contacted fluids.

The displacing fluid zones (water and/or gas) will response to the created reservoir pressure drops by a reflected pressure depends on their own fluid and rock properties. This response starts as soon as the created reservoir pressure drops reach the fluid interfaces Al-Khalifah A.J.A. and Odeh A.S. [20], thus, both of the created and reflected pressure drops reflect the interaction between the contacted fluids and rock properties.

Hence, the fluid interfaces could be considered as a no-flow boundaries and the reservoir may behave as an infinite acting reservoir as well as these interfaces still not affected by the created reservoir pressure drops. Consequently, the duration of the created reservoir pressure drop pulse must not exceed the infinite acting reservoir (transient pressure drop behavior) to avoid any pressure disturbance in originally fluid levels.

One of the most powerful techniques in reservoir engineering is the principle of superposition theorem that can be applied to remove the restrictions that have been imposed on various forms of solution to the transient flow equation to account for the effects of multiple wells, rate change, boundary and pressure change. This approach makes it possible to construct reservoir response functions in complex situations, using basic models. Superposition is especially useful in well test. The principle of superposition says that the response of the system to a number of perturbations is exactly equal to the sum of the responses to each of the perturbations as if they were present by themselves.

Thus, the exact solution could be generated using the principle of superposition throughout reflecting the created reservoir pressure drops at the free-level of the displacing fluids to estimate the reflected pressure drops in a manner exactly identical to the procedure used to estimate the reservoir pressure drops for a well bounded by a no-flow boundary; therefore, this solution is valid only during the pressure transient period. Hence, both of the created and the reflected pressure drops should be exhibit transient flow behavior as well as the created pressure drops still not reach the fluid interfaces.

Since, based on the principle of superposition theorem, the exact solution to estimate the pressure difference across the fluid interfaces is exactly equal to the sum of the responses for each of the fluid zones, taking into consideration that Van-Everdingen A.F. and Hurst W. [21] solution for infinite-radial flow will represent the created reservoir pressure drops encountered due to oil production activity.

3. Dynamic Capillary Pressure-Saturation Model

Building capillary pressure-saturation model consist of two parts; the first, estimation of dynamic capillary pressure based on rocks and fluids characteristics. While, the second part, estimation of fluid saturation based on the aquifer response function towards a created reservoir pressure drop, the two parts can be modeled as follows.

A- Estimation the Dynamic Capillary Pressure Model

The numerical results for a computer program prepared to estimate the pressure difference across the contacted fluid interfaces for many different hypothetical systems show constant pressure difference behavior between the created and the reflected pressure drops across the free-level of the displacing fluids during the pressure transient solutions as shown in Fig. 2; however, the values of pressure difference depend entirely between the contacted rock and fluid properties (Darcy's law).

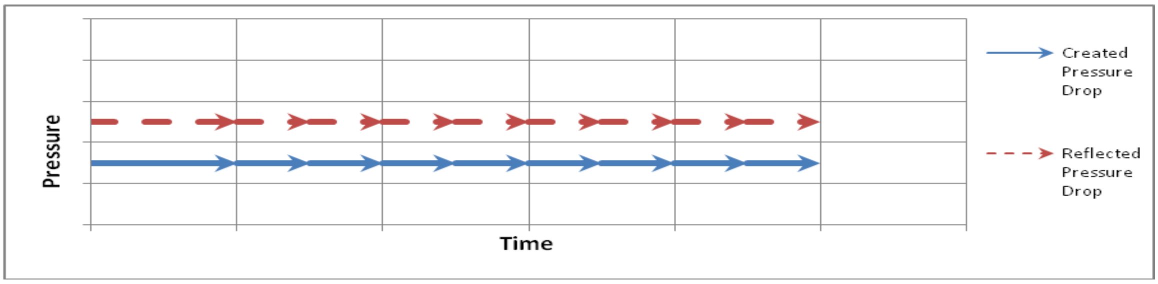

Fig. 2 Schematic drawing shows the constant difference between the created and reflected pressure drops denoting the static capillary pressure across the fluid interface.

Hence, the capillary pressure definition that is already given by (1.1) could be generalized for oil-water and oil-gas systems, throughout consider the positive (P_c) values for upward vertical fluid contact movement (OWC), while the negative (P_c) values for downward vertical fluid contact movement (GOC).

Thus, adding and subtracting the initial reservoir pressure (P_i) to the right hand side of (1.1) and arranging it, which could be rewritten as follows:

$$P_c = (P_i - P_l) - (P_i - P_o) \tag{3.1}$$

The terms $(P_i - P_o)$ and $(P_i - P_l)$ represent the created and reflected pressure drops during reservoir pressure transient flow period which could be estimated using Van-Everdingen, A.F. and Hurst,W. [21] solution. Thus, the pressure difference between the created and reflected pressure drops across the fluid interfaces may determine the dynamic capillary pressure across the contacted fluid interface.

As commonly used in reservoir performance analysis, the pressure drop behavior could be represented in dimensionless form. Thus, the capillary pressure term could also be converted to dimensionless capillary pressure form. So, (3.1) can be expressed in dimensionless form as follows:

$$P_{cD} = P_{Dl} - P_{Do} \tag{3.2}$$

where, (P_{cD}); the dimensionless dynamic capillary pressure.

Hence, the capillary pressure is zero at the free-level of displacing fluids; thus (3.1) could be written as follows:

$$P_{Dl} = P_{Do} \tag{3.3}$$

By using Van-Everdingen A.F. and Hurst W. [21] for infinite homogeneous-radial flow solution for both of the created and reflected pressure drops expressed by the following expressions respectively,

$$P_{Do} = 0.5\,[ln(T_{Do}) + 0.809] \tag{3.4}$$

$$P_{Dl} = 0.5\,[ln(T_{Dl}) + 0.809] \tag{3.5}$$

where; $(T_{Do}\ ,T_{Dl})$ represent the dimensionless time provided by the following expressions which is written in both of displaced and displacing fluid zones respectively;

$$T_{Do} = \frac{0.0002637\ K_o.t}{\varphi_o \mu_o C_t r_w^2} \tag{3.6}$$

$$T_{Dl} = \frac{0.0002637\ K_l.t}{\varphi_l \mu_l C_t r_w^2} \tag{3.7}$$

Therefore, by substituting. (3.4}, {3.5}, {3.6} and {3.7) in (3.2), follows the expression:

$$P_{cD} = \left[ln\left(\frac{0.0002637\ K_l.t}{\varphi_l \mu_l C_l r_w^2}\right) + 0.809\right] - \left[ln\left(\frac{0.0002637\ K_o.t}{\varphi_o \mu_o C_o r_w^2}\right) + 0.809\right] \tag{3.8}$$

Simplifying.(3.8) yields the following:

$$P_{cD} = ln\left(\frac{0.0002637\ K_l.t}{\varphi_l \mu_l C_l r_w^2} * \frac{\varphi_o \mu_o C_o r_w^2}{0.0002637\ K_o.t}\right) \tag{3.9}$$

and:

$$P_{cD} = ln\left(\frac{K_l}{K_o}\frac{\varphi_o \mu_o C_o}{\varphi_l \mu_l C_l}\right) \tag{3.10}$$

For further simplification, assume the displaced and displacing fluid zones have same porosities; thus, (3.10) could be rewritten as follows.

$$P_{cD} = ln\left(\frac{K_l}{K_o}\frac{\mu_o C_o}{\mu_l C_l}\right) \tag{3.11}$$

where, *(l)* denotes to the existing displacing fluids (water or gas).

Equations (3.10) and (3.11) provide general expressions for dimensionless dynamic capillary pressure measurements performed under reservoir conditions. The expressions can be converted to its dimensional form throughout multiplying the dimensionless values by ($\Delta P/P_D$), thus the capillary pressure term could be written as follows:

$$P_c = P_l - P_o = 141.2 \frac{Q_o \mu_o B_o}{K_o H_o} [P_{cD}] \tag{3.12}$$

Hence, it is useful to state that the generated dimensionless dynamic capillary model performed throughout using Van-Everdingen, A.F. and Hurst,W. [21] single-phase, infinite-radial flow solution for both of the created and reflected pressure drops; since, the solution represents (100% oil saturation) for the created pressure drops, and (100% water saturation) for the reflected pressure drops; while, the pressure difference between the created and reflected pressure drops represent the capillary transition region laying between (0-100% oil saturation).

Therefore, it is necessary to develop (3.11) in further analysis to estimate the saturation distribution along the entire determined capillary transition zone; this could be done by using new term definition called the "Response Function" denoted by (B), which reflects the fraction of the pressure support by the displacing fluid zones against the created reservoir pressure drops.

Thus, the entire length of the capillary zone lying between ($S_w = 100\%$) at the free level of the displacing fluids, and ($S_w = 0\%$) or connate water saturation in the oil phase zone could be represented by a "Total Response Function denoted by (B_t)".

Hence, the numerical results for all hypothetical systems show that this term could be exactly expressed by the following formulas:

$$B_t = \frac{T_{Dl} - T_{Do}}{T_{Dl}} \tag{3.13}$$

Successive simplifying of (3.13), and using (3.6) and (3.7) gives the following terms:

$$B_t = \frac{\frac{0.0002637 K_l.t}{\varphi_l \mu_l C_l r_w^2} - \frac{0.0002637 K_o.t}{\varphi_o \mu_o C_o r_w^2}}{\frac{0.0002637 K_l.t}{\varphi_l \mu_l C_l r_w^2}} \tag{3.14}$$

and:

$$B_t = 1 - \frac{K_o \mu_l C_l}{K_l \mu_o C_o} \tag{3.15}$$

Substituting (3.11) in (3.15); yields the following expression:

$$B_t = 1 - e^{-P_{cD}} \tag{3.16}$$

Since, the transition zone thickness is function of the pressure difference across the fluid interfaces and the density difference between the contacted fluids, is commonly expressed as follows:

$$H_{fc} = \frac{144 P_c}{g_c(\rho_l - \rho_o)} \tag{3.17}$$

where, (H_{fc}) and (P_c) represent the transition zone thickness and the capillary pressure respectively. Also, (g_c) is conversion parameters equals= $32.17\ ft.ib_{mass}/ib_{force}.sec^2$.

B- Saturation Distribution Model

The total length of the transition zone that is controlled by the values of capillary pressure and the density difference between the contacted fluids is already determined using the total response function (B_t) laying between water saturations of (zero to 100%).

Thus, the saturation distribution in the transition zone could be estimated by defining the "Unit Response Function (B_i)" of the displacing fluids against each saturation unit change (ΔS_l), as shown in schematic Fig.3, the values of the unit response function (B_i) change with water saturation to reach the total response function (B_t) at (S_w=100%). Consequently, it could be useful to join the unit response function $(B_i)$with the total response function (B_t) as follows:

$$\Delta B = B_t . \Delta S_w \tag{3.18}$$

Integrating both sides of (3.18):

$$\int_{Bi}^{Bt} dB = B_t \int_{Swi}^{Sw=1} dS_w \tag{3.19}$$

This integration yields the following expression:

$$(B_t - B_i) = B_t . (1 - S_{wi}) \tag{3.20}$$

where, (S_{wi}); the displaced fluid saturation at any specified unit response function (B_i).

Moreover, the difference between the total response function (B_t) and the unit response function (B_i), which could be called the "Remaining Response Function- (B_r)" gives the capillary pressure holds the transition zone above the saturation change at (S_{wi}) and could be represented by the following equation:

$$B_r = (B_t - B_i) \tag{3.21}$$

Since, (3.20) could be rewritten as follows:

$$B_r = B_t . (1 - S_{wi}) \tag{3.22}$$

Substituting (3.22) in.(3.16), (3.20) and (3.21); will give respectively the following subsequent forms:

$$\frac{B_r}{(1-S_{wi})} = 1 - e^{-P_{cD}} \tag{3.23}$$

$$S_{wi} = 1 - \frac{B_r}{B_t} \tag{3.24}$$

$$S_{wi} = \frac{B_i}{1-e^{-P_{cD}}} \tag{3.25}$$

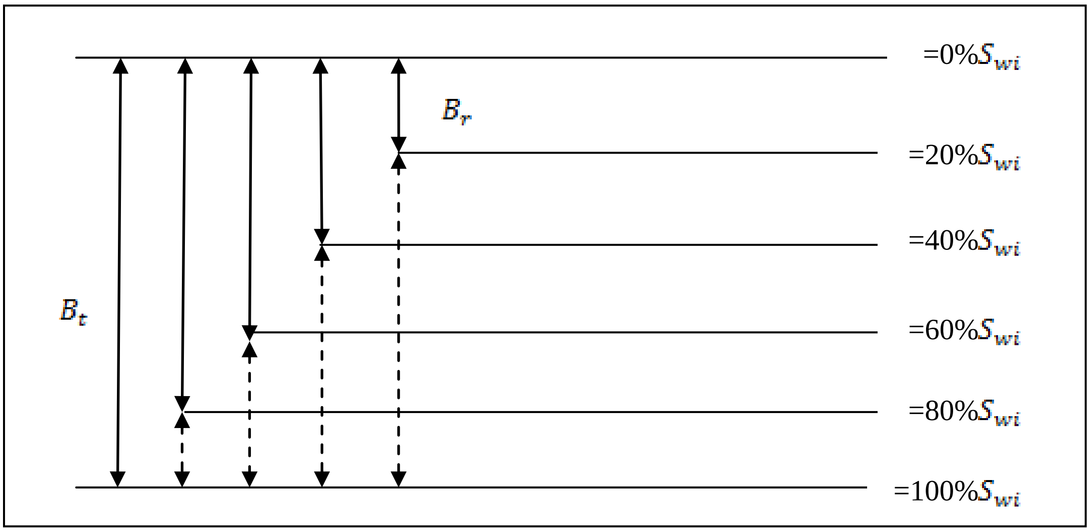

Fig. 3 Schematic drawing shows the change of total response function Reflecting the displacing fluid saturation

4. Three Phase Capillary Pressure Systems

The existing of two displacing fluid zones (water and gas) in contact with the displaced fluid zone (oil) provide combined effect working simultaneously against the created reservoir pressure drops, the explicit effect magnitude for each of the displacing fluid zones depends on their individual rock and fluid properties; in other word, the existing of two displacing fluid zones (water and gas) provide effect on each other against that created reservoir pressure drop.

Hence, Helland et al. [9], prevailed that it is commonly predict the three-phase capillary pressure-saturation curves using the corresponding two phase measurements that is conveniently formulated as a simple correlation with adjustable parameters. They indicated that this practice may not be valid and the fluid distribution and the displacement mechanisms at the pore scale may be more complex than for two phases.

Thus, the principle of treating the three fluid phase system as an explicit two-phase (oil-water and oil-gas) systems require a correction parameter to estimate the fraction of total response functions for each of the individual displacing fluid zones.

Therefore, the total pressure support index in a combined drive mechanism is the sum of partial pressure indices supported by each explicit two-phase system. This can be written by the following expression:

$$I_w + I_g = 1 \tag{4.1}$$

where, I_w and I_g represent the aquifer and gas cap pressure support indices respectively.

The model's derivation shows that the pressure support indices provided by the displacing fluid zones is directly proportional with the ratio of effective displacing fluid motility over the effective displaced fluid mobility, as follows:

$$M_w = \frac{K_w \mu_o C_o}{K_o \mu_w C_w} \tag{4.2}$$

$$M_g = \frac{K_g \mu_o C_o}{K_o \mu_g C_g} \tag{4.3}$$

where, M_w and M_g represent respectively the effective motility ratios of water and gas over oil phase mobility.

Therefore, the fraction of pressure support index that may be provided by each individual zone of the displacing fluids at the displaced fluid interface could be written as follows:

$$I_w = \frac{M_w}{M_w + M_g} \tag{4.4}$$

$$I_w = \frac{\frac{K_w \mu_o C_o}{K_o \mu_w C_w}}{\frac{K_w \mu_o C_o}{K_o \mu_w C_w} + \frac{K_g \mu_o C_o}{K_o \mu_g C_g}} = \frac{1}{1 + \frac{K_g \mu_o C_o}{K_o \mu_g C_g}} \tag{4.5}$$

Similarly, the fraction of pressure support index may be provided by the gas cap zone within combined drive mechanism system could be written as following:

$$I_g = (1 - I_w) = \frac{M_g}{M_w + M_g} = \frac{\frac{K_g \mu_o C_o}{K_o \mu_g C_g}}{\frac{K_w \mu_o C_o}{K_o \mu_w C_w} + \frac{K_g \mu_o C_o}{K_o \mu_g C_g}} = \frac{1}{1 + \frac{K_w \mu_o C_o}{K_o \mu_w C_w}} \tag{4.6}$$

The values of I_w and I_g represent the correction factors used to estimate the fraction of response functions for each individual displacing fluid zones. Since, the corrected total response function of the displacing fluids (B_t) for water and gas phase zones could be written respectively using the following relations:

$$B_{tw(cor)} = B_{tw}.I_w \tag{4.7}$$

$$B_{tg(cor)} = B_{tg}.I_g \tag{4.8}$$

where, B_{tw} *and* B_{tg}; the total response functions of the water and gas phase zones respectively which could be calculated by (3.13) or (3.15).

5. Model Procedure

The following steps can be used to estimate the capillary pressure-saturation profile:

A- Estimate the dimensionless capillary pressure given by (3.10).

B- Estimate the total response function (B_t) by using (3.16).

C- Apply (3.16) and (3.25) to estimate a capillary pressure values and displaced fluid saturations respectively; this can be performed throughout assuming different values of unit response function range from (0 - B_t).

D- Convert all dimensionless capillary pressure values to dimensional form by using (3.12).

E- Draw the profile of capillary pressure versus displaced fluid saturation values.

6. Results and Discussion

The developed model presented in (3.10) which is derived from the simple definition of capillary pressure given in (1.1), involves all rock and fluid properties for both of displaced and displacing fluid zones. Moreover, all of the properties can be measured under reservoir conditions; as, the reservoir permeability can be obtained from well test analysis; while, the displaced fluid's zones permeability can be estimated from injectivity test or using Voigt's [22] model. Then, both of displaced and displacing fluids viscosity and compressibility can be estimated from PVT data at actual reservoir pressure and temperature.

Since, the developed model reflects the real interaction between all rock and fluid properties, and can be considered the wanted target denoted by all authors already reviewed in introduction section of this article. Moreover, this model will eliminate the coring works which is still one of the largest capital cost in exploration and production industry and the departure from some complicated laboratory measurements.

In order to validate the developed model, the results compared against some experimental laboratory data obtained for core samples taken from some oil fields having different lithology, porosity, permeability, depth, temperature and the displaced contacted fluid characteristics, as shown in Table 1. A computer program has been prepared for the calculation of dynamic capillary pressure-saturation profile based on the suggested eqs. (3.12) and (3.25).

The created results obtained by this model versus experimental laboratory data have been stated for oil-water and for gas-oil systems as shown in Tables 2 and 3 respectively; these data have also been drawn in Figs 4, 5, 6, 7, 8 and 9 for oil-water system, and Figs 10, 11, 12 and 13 for gas-oil systems. Investigating these figures, it can be stated that; at high values of non-wetting phase saturation, the capillary pressure under dynamic effects characterized by lower saturations than observed in static capillary pressures; in contrast higher saturations are measured in the dynamic curves than in static curves at high values of wetting phase.

The extensive works of the model show that the wetting phase zone properties affect on the curvature degree of capillary pressure profile in the non-wetting phase saturation; while, the non-wetting phase zone properties affect on the position for the entire curve of capillary pressure.

It is also noticed that the dynamic capillary pressure-saturation profile generated by the presented model gives smooth curvature along the entire profile rather than jumped positions that may be noticed in the static curves; such results gives the logic analysis for the gradual variation in fluids distribution along the transition zone. However, the jumped position in the static capillary curves may be attributed for the errors made while performing the experimental works.

Thus, generating the dynamic capillary pressure curves will assist in evaluating the originally hydrocarbon in place more accurately throughout determining the accurate transition zone intervals and saturation distribution, especially that is concern in high heterogeneous reservoirs.

However, some of these results have already been prevailed by several authors (as already introduced in the literature survey of this article) like Greder et.al. [3], Schultze et al. [13], Wildenschild et al. [14], Shawkat G. Ghaidan et.al. [8], Hassanizadeh S.M. et al. [6], and Geremy Camps et al. [15], who have mentioned different causes for this variation in capillary pressures versus saturation of both wetting and non-wetting phases; the authors tried to develop some solutions accounting this dynamic effect throughout involving only one or some rock properties that may affect on capillary pressure-saturation profile; but without given any complete solution which involves almost rock and fluid parameters affecting on the in-situ measurements of capillary pressure-saturation relationship. It is noticed that all rocks and fluids characteristics involved in the dynamic capillary pressure-saturation model given in eqs (3.12) and (3.25) controls for the capillary pressure values and the curvature degree for the entire capillary pressure profile; since, it exactly determines the fluids saturation distribution within the capillary transition zones.

Hence, Fig. 14 shows the effect of reservoir permeability variation on capillary pressure saturation trends; it can be noticed that the decreasing of reservoir permeability causes increasing in capillary pressure across the fluid contacts and then yields for thicker transition zones. Therefore, such results may response for the reasons beyond the existing of different fluid contacts in some complicated lithology and also for the tilted OOWC contacts in some carbonate reservoirs which is still under investigation.

7. Conclusions

The advantages of the new model in estimating the dynamic capillary pressure-saturation profile than other previously introduced models can be summarized as follows:

A- Reflects the entire actual interaction between the contacted fluid and rock properties measured under the in-situ downhole conditions and can be applicable in heterogeneous reservoirs. On the other hand, almost of the previous models reflect only the effects of porosity and permeability and consider the pore as a single pore system, which led for erroneous results especially in highly heterogeneous pore system.

B- Proved theoretically and has thorough physical mean; moreover, it does not contain any adjustable parameters which restrict the circumstances of application. This may enhance the traditionally experimental capillary pressure expression with theoretical basis.

C- Directly applied using reservoir pressure transient analysis and no experimental laboratory works needed, since it is a cost effective procedure than performing some complicated experiments.

D- The model can be considered the most reliable, accurate and most cost effective tool than coring measurement which is still one of the capital investment in exploration and production industry.

E- The model can be considered an easy tool in estimating the capillary pressures in reservoir simulation studies.

F- The model will assist in accurately evaluating the originally hydrocarbon in place throughout determining the transition zone intervals and saturation distribution.

G- The model may give a thorough explanation for the tilted OOWC in some carbonate reservoirs which is still under investigation.

Table 1

Sample No.	$\Delta P/P_D$	Displaced Fluid Permeability (md)	Displacing Fluid Permeability (md)	Displaced Fluid Viscosity (cp)	Displacing Fluid Viscosity (cp)	Displaced Fluid density (Ib.Cuft)	Displacing Fluid density (Ib/Cuft)	Displaced Fluid Compressibility Psi^{-1}	Displacing Fluid Compressibility Psi^{-1}	OFVF B/Stb
HU-1	46.020	302	514.23	2.5	1.02	52	63.7	0.0000097	0.0000011	1.3
HU-2	13.132	827	1172	2.23	0.85	52	64	0.0000088	0.0000011	1.45
HU-3	24.106	262	1714	2.23	1.0	52	63.6	0.0000157	0.0000011	1.3
HU-4	35.727	522	1227	2.23	1.15	52	63.7	0.0000188	0.0000011	1.45
HU-5	43,997	692	1887	3.12	1.05	52	63.55	0.0000172	0.0000025	1.23
R-1	75.506	182	417	4.23	1.05	52.8	63.9	0.0000094	0.0000021	1.23
HU-1	46.020	302	2977	2.5	0.0315	52	13.2	0.0000097	0.00292	1.3
HU-2	13.132	827	1822	2.23	0.031	52	13	0.0000088	0.00041	1.45
HU-3	24.106	262	2874	2.23	0.027	52	11.5	0.0000157	0.00066	1.3
HU-4	35.727	522	1197	2.23	0.025	52	11.76	0.0000188	0.0020	1.45

Table 1 shows rocks and fluids properties for core samples taken to construct the dynamic capillary pressure-saturation profile.

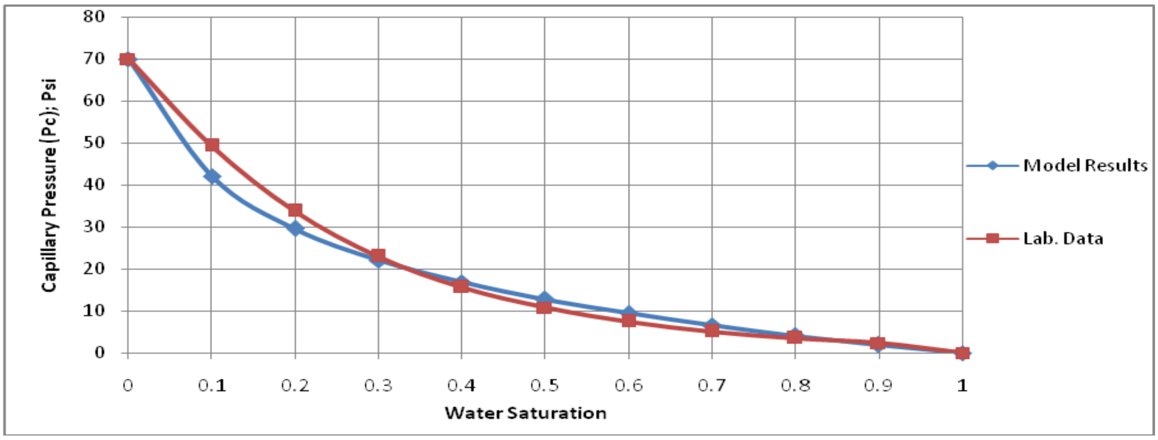

Fig. 4 Capillary pressure-saturation profile for well (HU-1)

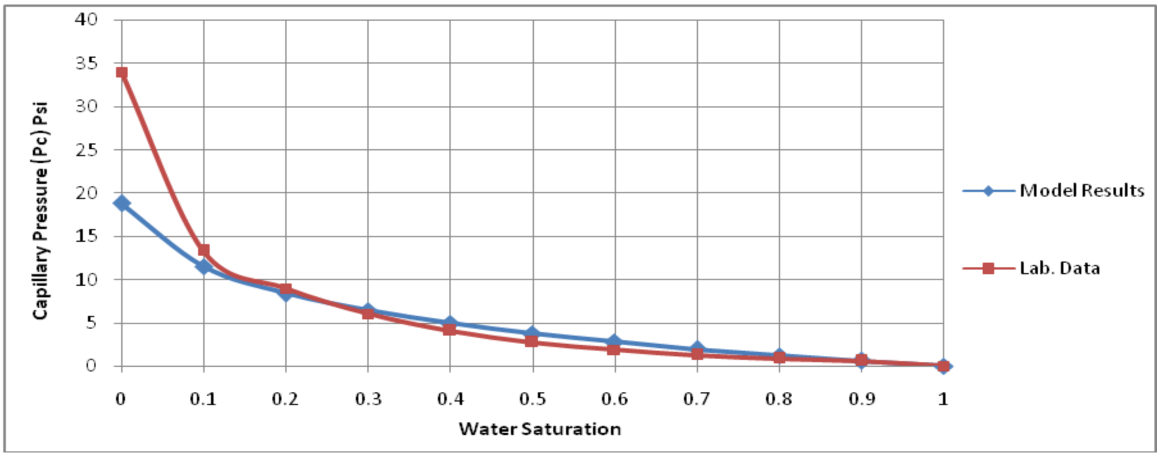

Fig. 5 Capillary pressure-saturation profile for well (HU-2)

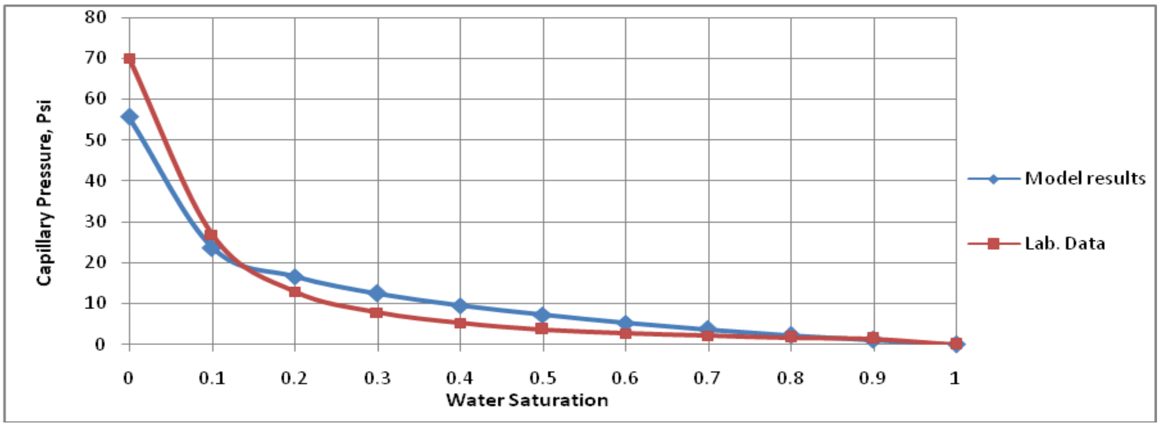

Fig. 6 Capillary pressure-saturation profile for well (HU-3)

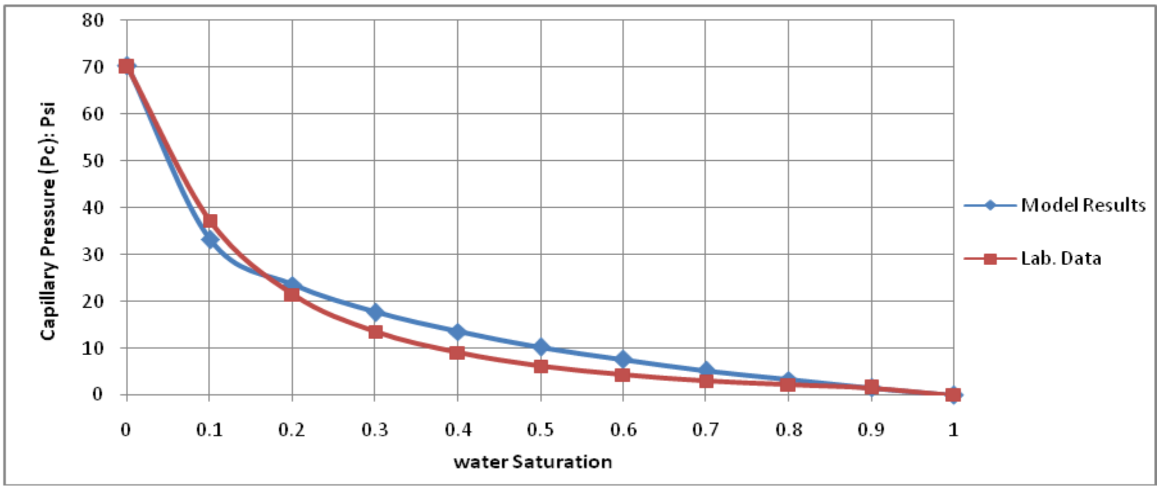

Fig. 7 Capillary pressure-saturation profile for well (HU-4)

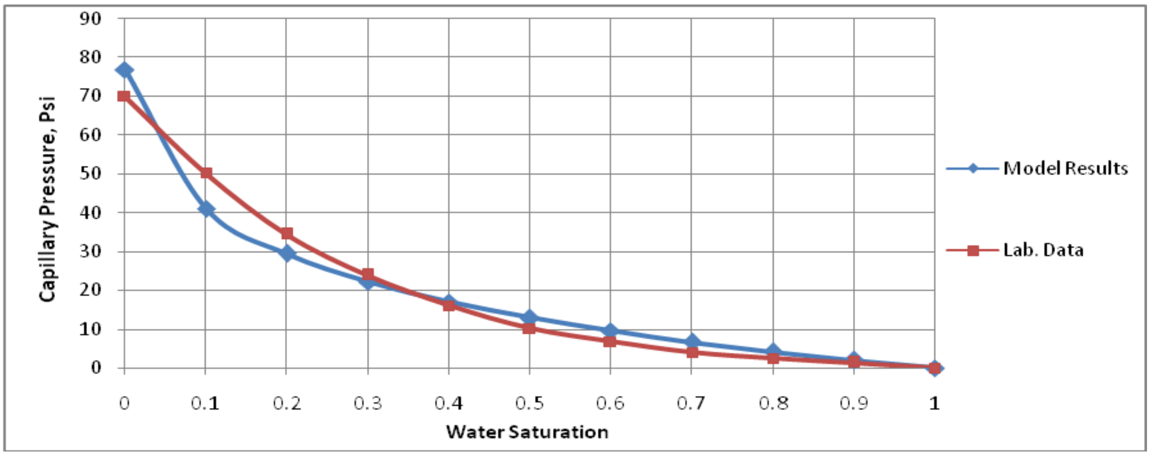

Fig. 8 Capillary pressure-saturation profile for well (HU-5)

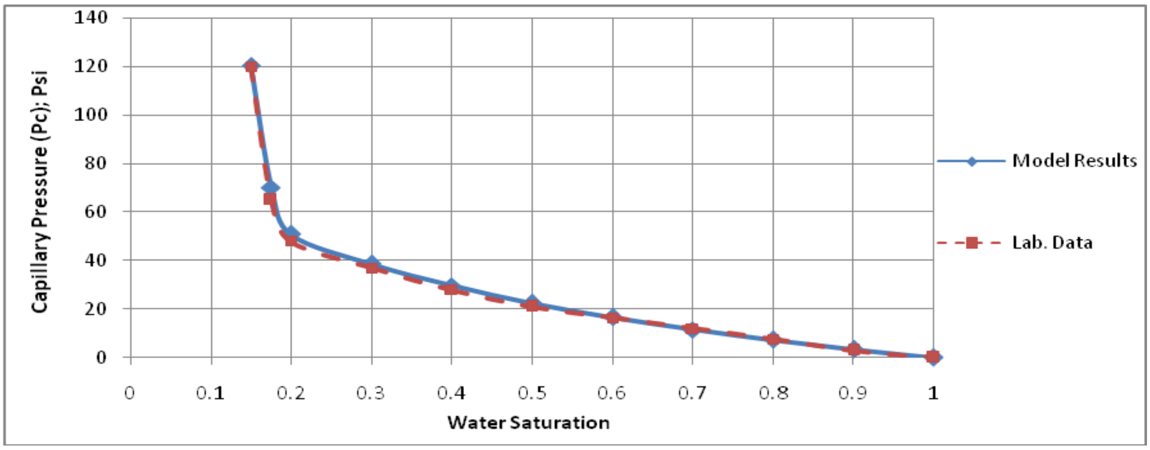

Fig. 9 Capillary pressure-saturation profile for well (HU-R1)

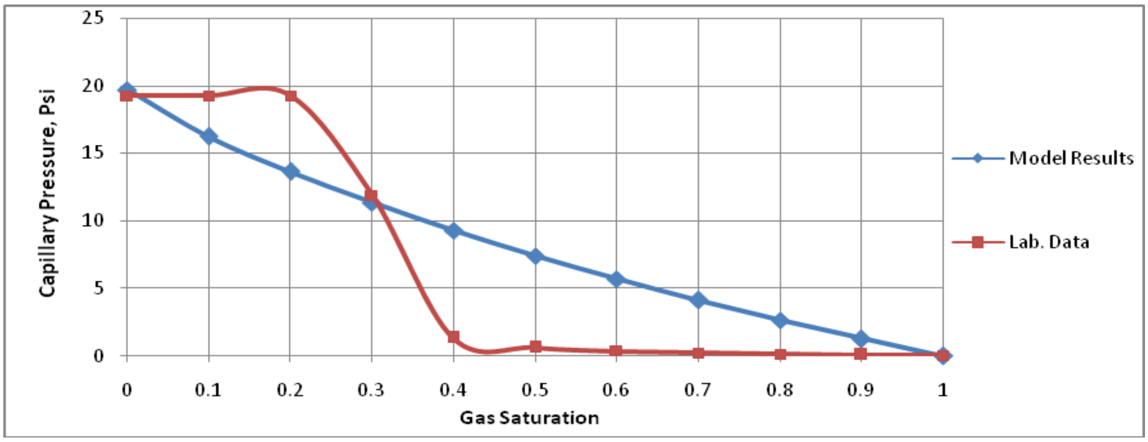

Fig. 10 Capillary pressure-saturation profile for well (HU-1)

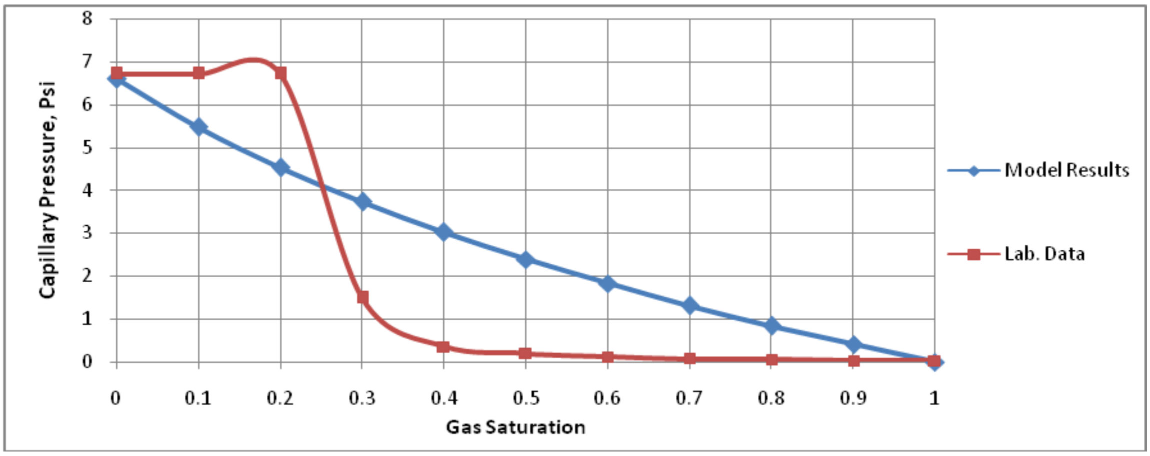

Fig. 11 Capillary pressure-saturation profile for well (HU-2)

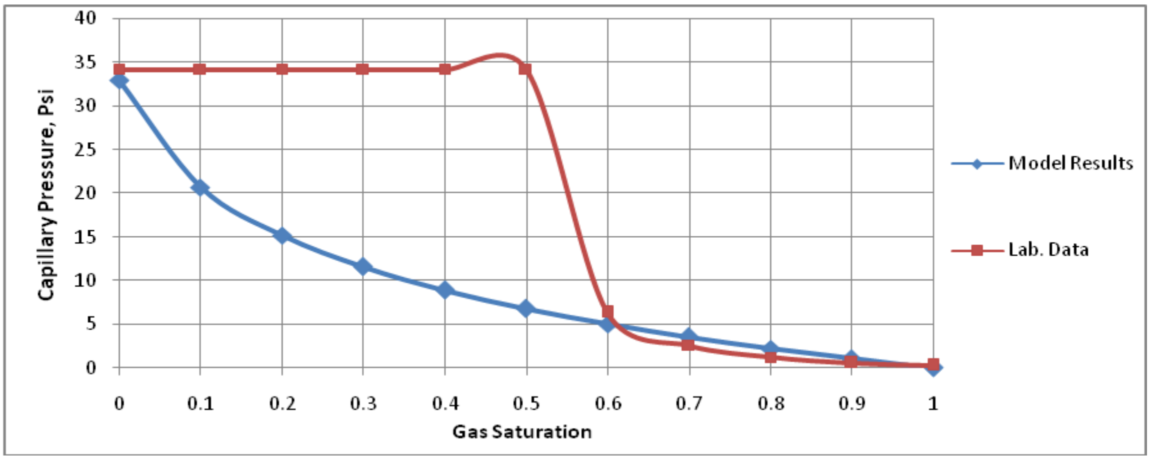

Fig. 12 Capillary pressure-saturation profile for well (HU-3)

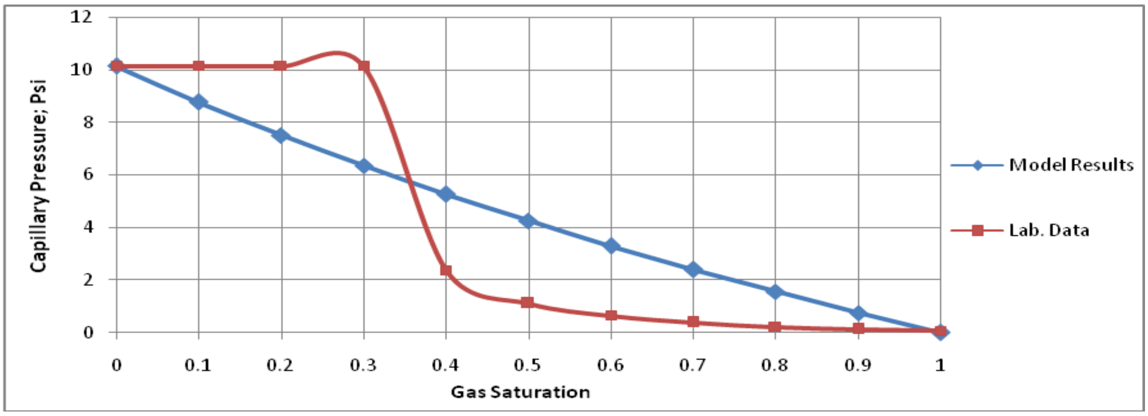

Fig. 13 Capillary pressure-saturation profile for well (HU-4)

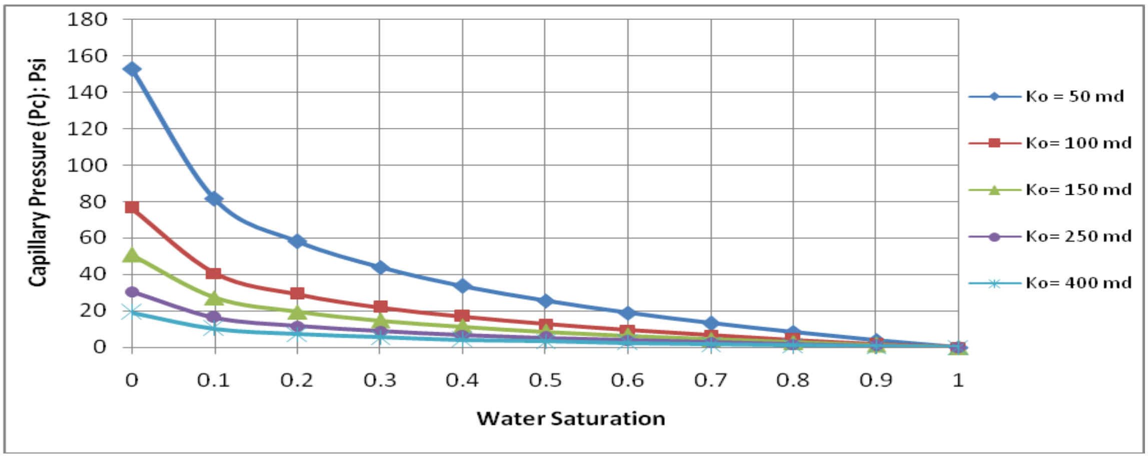

Fig. 14 Capillary pressure-saturation profiles generated by the presented model for different reservoir permeability values; taken $Q_o = 500\, STB/D$, r_e=15000 ft, $\mu_o = 2\, cp$, μ_w=1 cp, $\rho_w = 63.4\, Ib/ft^3$, $\rho_o = 53\, Ib/ft^3$

Table 2

	Well No. HU-1		Well No. HU-2		Well No. HU-3		Well No. HU-4		Well No. HU-5		Well No. R-1	
S_w	Estimated Dynamic Pc- Psi	Static Pc Lab. Data Psi	Estimated Dynamic Pc- Psi	Static Pc Lab. Data Psi	Estimated Dynamic Pc- Psi	Static Pc Lab .Data Psi	Estimated Dynamic Pc- Psi	Static Pc Lab .Data Psi	Estimated Dynamic Pc Psi	Static Pc Lab .Data Psi	Estimated Dynamic Pc- Psi	Static Pc Lab. Data Psi
0	72.039	70	18.821	34	55.674	70	67.567	70	76.875	70		
0.1	41.638	49.27	11.496	13.28	23.650	26.9	34.024	37.28	41.138	50.2		
0.2	30.098	33.77	8.3934	8.93	16.643	12.85	24.191	21.4	29.427	34.43	49.737	48
0.3	22.829	23	6.3951	6	12.483	7.777	18.219	13.45	22.22	23.87	37.672	37
0.4	17.511	15.6	4.9182	4.04	9.5146	5.21	13.918	9.01	16.999	16.1	28.872	28
0.5	13.315	10.75	3.7462	2.71	7.2043	3.7	10.555	6.2	12.903	10.23	21.941	21
0.6	9.8488	7.46	2.7745	1.83	5.3129	2.75	7.7924	4.3	9.5325	6.87	16.223	16.3
0.7	6.8958	5.13	1.9445	1.23	3.7115	2.12	5.4481	3.02	6.6681	3.97	11.355	12
0.8	4.3236	3.51	1.2201	0.827	2.3229	1.688	3.4120	2.16	4.1777	2.54	7.1183	7.6
0.9	2.0451	2.387	0.5775	0.556	1.0971	1.37	1.6124	1.568	1.9749	1.34	3.3663	2.9
1	0	0	0	0	0	0	0	0	0	0	0	0

Table 2, shows a comparison analysis between the dynamic capillary pressure-saturation estimated using the presented model given in (3.11), (3.12 and (3.25) versus the static capillary pressure obtained by laboratory core measurements for oil-water system.

Table 3

	Well No. HU-1		Well No. HU-2		Well No. HU-3		Well No. HU-4	
S_g	Estimated Dynamic Pc Psi	Static Pc Lab.Data Psi	Estimated Dynamic Pc Psi	Static Pc Lab.Data Psi	Estimated Dynamic Pc Psi	Static Pc Lab.Data Psi	Estimated Dynamic Pc Psi	Static Pc Lab.Data Psi
1	0	0.05	0	0.017	0	0.28	0	0.0833
0.9	1.2788	0.085	0.40466	0.029	1.0531	0.59	0.76295	0.1417
0.8	2.64434	0.142	0.84024	0.047	2.22412	1.24	1.56536	0.2344
0.7	4.10924	0.228	1.31187	0.074	3.54285	2.6	2.41156	0.3983
0.6	5.68912	0.367	1.82605	0.115	5.05199	6.36	3.30659	0.6598
0.5	7.40356	0.623	2.39123	0.193	6.81596	34.2	4.25644	1.1189
0.4	9.27764	1.32	3.01865	0.355	8.93868	34.2	5.26826	2.3672
0.3	11.3442	11.87	3.72373	1.48	11.6043	34.2	6.3507	10.1429
0.2	13.6472	19.265	4.52846	6.71	15.1896	34.2	7.51438	10.1429
0.1	16.2481	19.265	5.46571	6.71	20.6857	34.2	8.77248	10.1429
0	19.7054	19.265	6.5879	6.71	32.9646	34.2	10.1417	10.1429

Table 3, shows the comparison analysis between the dynamic capillary pressure-saturation estimated using the presented model given in (3.11), (3.12 and (3.25) versus the static capillary pressure obtained by laboratory core measurements for oil-gas system.

<u>Field Example:</u> Field examples show the calculating capillary pressure - Saturation values.

An oil field producing from two main pay zones under water drive, the reservoir zones have the properties shown in Table 4. On the other hand, the aquifer characteristics differs from west to east regions as shown in Table 5 below, while Oil and water properties are as follows; (C_o=9.1*10^{-6} Psi^{-1} , μ_o=0.605 Cp, B_o=1.473 RB/STB) and (C_w=2.7*10^{-6} Psi^{-1}, μ_w=0.65 Cp, B_w=1.025 RB/STB) ,app. Oil density=53.4 Ib/ft^3, app. Water density=63 Ib/ft^3. approximate reservoir radius=90000 ft.

Four wells have been selected in this test representing the west and east sides of the main producing units (A and D); these wells produce different oil flow rates as shown in Table 6; Required; Estimation of Capillary transition zone thickness versus water saturation.

Table 4

Unit	A	D
Gross thick-(ft)	22-45	166-219
Net thickness (ft)	16-34	100-155
φ	0.207	0.22
K- md	590	1050
Sw	0.11	0.096

Table 4, shows reservoir zone characteristics in units (A and D)

Table 5

	West		East	
Unit	A	D	A	D
K_w – md	1395	2300	305	1176
φ	0.3	0.24	0.13	0.17
Thick- ft	90	100	40	100

Table 5, aquifer characteristics from west to east that is contacted with units (A and D)

Table 6

Reservoir	West Region	East Region
A	8300	4100
D	7875	5155

Table 6, shows well production of oil in (BOPD) in units (A and D)

Solution:

A computer program on Excel sheet has been prepared to estimate the dimensionless capillary pressures, capillary pressures and the transition zone thickness versus water saturation distribution based on the derived analytical model represented by Eqs. (3.11, 3.12, 3.17 and 3.25); the results have been listed below in Tables 7, 8, 9 and 10 as follows.

Table 7

Sw	B_r	P_{CD}	$P_c, (Psi)$	$H_{fc}, (ft)$
1	0.86518	0	0	0
0.9	0.77866	0.03929	2.31852	1.081064
0.8	0.69214	0.0825	4.86792	2.269778
0.7	0.60563	0.13048	7.6993	3.589975
0.6	0.51911	0.18444	10.8829	5.074401
0.5	0.43259	0.24606	14.519	6.769798
0.4	0.34607	0.31789	18.7577	8.74623
0.3	0.25955	0.40402	23.8396	11.11574
0.2	0.17304	0.51156	30.1851	14.07452
0.1	0.08652	0.65482	38.6387	18.01617
0.1	1.1E-16	0.87009	51.3404	23.93864

Table 7, shows the transition zone saturation distribution in west region of unit (A)

Table 8

Sw	B_r	P_{CD}	$P_c, (Psi)$	$H_{fc}, (ft)$
1	0.38336	0	0	0
0.9	0.34502	0.01697	0.494738	0.230683
0.8	0.30669	0.03464	1.009606	0.470752
0.7	0.26835	0.05305	1.546309	0.721002
0.6	0.23002	0.07228	2.106784	0.982336
0.5	0.19168	0.0924	2.693233	1.255782
0.4	0.15334	0.1135	3.308183	1.542516
0.3	0.11501	0.13567	3.954544	1.843897
0.2	0.07667	0.15904	4.6357	2.161501
0.11	0.03834	0.18121	5.281749	2.497176

Table 8, shows the transition zone saturation distribution in east region of unit (A)

Table 9

Sw	B_r	P_{CD}	$P_c, (Psi)$	H_{fc}, (ft)
1	0.85447	0	0	0
0.9	0.76903	0.03878	0.244016	0.113778
0.8	0.68358	0.08138	0.511985	0.238725
0.7	0.59813	0.1286	0.809126	0.377273
0.6	0.51268	0.1816	1.142574	0.532751
0.5	0.42724	0.24198	1.522455	0.70988
0.4	0.34179	0.31213	1.963825	0.915678
0.3	0.25634	0.39584	2.490506	1.161256
0.2	0.17089	0.49964	3.143576	1.465764
0.1	0.08545	0.63632	4.003504	1.866725
0	2.2E-16	1.44465	9.089314	4.23809

Table 9, shows the transition zone saturation distribution in west region of unit (D)

Table 10

Sw	B_r	P_{CD}	$P_c, (Psi)$	$H_{fc}, (ft)$
1	0.71538	0	0	0
0.9	0.64384	0.03223	0.199111	0.09284
0.8	0.57231	0.06705	0.414194	0.193127
0.7	0.50077	0.1049	0.648038	0.302163
0.6	0.42923	0.14637	0.904231	0.421618
0.5	0.35769	0.19222	1.187501	0.553699
0.4	0.28615	0.24349	1.504254	0.701393
0.3	0.21461	0.30164	1.863481	0.868891
0.2	0.14308	0.3688	2.278364	1.062339
0.1	0.07154	0.44828	2.769366	1.29128
0	0	0.54564	3.370838	1.57173

Table 10, shows the transition zone saturation distribution in east region of unit (D)

The results of this field data exactly show the titled of originally oil-water contact from west to east, especially in reservoir unit (A) due to the big variation in rock and fluid properties with the aquifer, compared with that is observed in reservoir unit (D). The results confirm the actual field data in this carbonate field; therefore, the new methodology may explain the reasons beyond this phenomenon which is still under investigation by some authors, like Hsueh [23].

NOMENCLATURE

B_i Unit response function
B_r Remaining response function
B_t Total response function
B_{tg} Total response function of Gas cup zone
B_{tw} Total response function of Aquifer zone
$B_{tg(cor)}$ Corrected total response function of Gas cap zone
$B_{tw(cor)}$ Corrected total response function of Aquifer zone
BOPD Barrel oil per day.
C_l Total compressibility in the displacing fluid zones (water and/or gas), Psi^{-1}
C_o Total compressibility in the displaced fluid (oil) zone, Psi^{-1}
g_c Conversion constant = $32.17\ ft.ib_{mass}/ib_{force}.sec^2$

H_o Oil reservoir column thickness, ft
H_{fc} Transition zone thickness, ft
I_g Fraction of pressure response provided by Gas cap zone
I_w Fraction of pressure response provided by Aquifer zone
K_o Absolute reservoir zone permeability, md
K_l Absolute permeability of the displacing fluid zone, md
M_g Ratio of effective gas phase motility over the effective oil phase mobility; $M_g = \frac{K_g \mu_o C_o}{K_o \mu_g C_g}$
M_w Ratio of effective water motility over the effective oil phase mobility; $M_w = \frac{K_w \mu_o C_o}{K_o \mu_w C_w}$
OOWC Originally oil water contact
P_c Capillary pressure, Psi
P_{cD} Dimensionless capillary pressure
P_i Initial reservoir pressure, Psi
P_l Pressure at the interface of displacing fluid zones, Psi
P_o Pressure at the interface of displaced (oil) zone, Psi
P_D Dimensionless pressure drop
P_{Dl} Dimensionless reflected pressure drop occur in the displacing fluid zones
P_{Do} Dimensionless created reservoir pressure drop
Q_o Oil flow rate, STB/Day
r_w Well radius, ft
Swi Initial Water Saturation
Sw Water Saturation
t Production time, Hours
T_D Dimensionless time
T_{Dl} Dimensionless time calculated using displacing zone properties
T_{Do} Dimensionless time calculated using displaced zone properties

Greek Symbols:

ΔB Change in response function
φ Porosity
μ_l Displacing fluid viscosity, Cp
μ_o Oil viscosity, Cp

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