Relativistic Mechanics for Airframes Applied in Aeronautical Technologies

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Abstract
The theory of *"Relativistic Mechanics"* is proposed for the design of the new generation large aircrafts with turbjet engines and speeds in the range of 50,000 km/h. This theory shows that there is a considerable difference between the absolute stress tensor and the stress tensor of the moving frame even in the range of speeds of 50,000 km/h. For bigger speeds like c/3, c/2 or 3c/4 (c=speed of light), the differnce between the two stress tensors is very much increased. Also, for velocities near the speed of light, the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The proposed theory of *"Relativistic Mechanics"* is a combination between the theories of classical elasticity ans special relativity and results to the *"Universal Equation of Elasticity"*. For the structural design of the new generation of very big aircrafts the stress tensor of the airframe will be used in combination to the singular integral equations method. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the *Singular Integral Operators Method (S.I.O.M.)*.

Key Word and Phrases
Relativistic Mechanics, Airframes, Relative(Airframe) Stress Tensor, Absolute Stress Tensor, Stationary and Moving Frames, Energy-Momentum Tensor, Multidimensional Singular Integral Equations, Singular Integral Operators Method (S.I.O.M.), Universal Equation of Elasticity

1. Future Applications of Aircraft Design

The big evolution of the jet engines and the high performance axial – flow compressor have considerably increased the possibilities of turbomachines applied in aircrafts. The concern for very light weight in the aircraft propulsion application, and the desire to achive the highest possible isentropic efficiency by minimizing parasitic losses, led inevitably to axial-flow compressors with cantilever airfoils of high aspect ratio. Also, the turbojet engines were found to experience severe vibration of the rotor blades at part speed operation. The increasing evolution of aeroelasticity in aircraft turbomachines continues to be under active investigation, driven by the needs of aircraft powerplant and turbine designers.

The target of international Aeronautical Industries is therefore to achieve a competitive technological advantage in certain strategic areas of new and rapidly developing advanced technologies, by which in the longer terms, can be achieved increased market share. This considerably big market share includes the design of a new generation large aircraft with speeds even in the range of 50,000 km/h. The application of new generation turbojet engines makes possible the design of such type of large aircrafts and therefore there is a need of elastic stress analysis for the construction of the total parts of such type of new generation aircrafts.

In the present investigation we will show that there is a difference between the absolute stress tensor and the stress tensor of the airframe even in the range of speeds of 50,000 km/h. On the other hand, for bigger speeds the difference of the two stress tensors is very much increased. Thus, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is very much different than the absolute one, while for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the airframe is included in the theory proposed by E.G.Ladopoulos [30], [31] under the term *"Relativistic Elasticity"* and the final formula which results from the above theory is called the *"Universal Equation of Elasticity"*.

Hence, in the present study the theory of *"Relativistic Elasticity"* will be applied for the elastic stress analysis design of the new generation big aircrafts.

Furthermore, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et all [17]-[22] proposed several linear singular integral equation methods applied to elasticity, plasticity and fracture mechanics applications. In the above studies the Singular Integral Operators Method (S.I.O.M.) is investigated for the numerical evaluation of the multidimensional singular integral equations in which is reduced the stress tensor analysis of the linear elastic or plastic theory. Also, the theory of linear singular integral equations was extended to non-linear singular integral equatiosn, too. [23]-[29]. The theory of *"Relativistic Elasticity"* will be applied for the design of the elastic stress analysis for the airframes. *"Relativistic Elasticity"* is derived as a generalization of the classical theory of elastic stress analysis for stationary frames. For future aerospace applications the difference between the relative and the absolute stress tensors will be of increasing interest. Furthermore,. the classical theory of elastic stress analysis began to be analyzed in the early nineteenth century and was further developed in the twentienth century. In the past were written several important monographs on the classical theory of elastiicty. [32]-[51}.

On the other hand, during the past years special attention has been concentrated on the theoretical aspects of the special theory of relativity. Hence, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [52]–[59].Furthermore, a very important point which will be shown in the present research is that the relative stress tensor is not symmetrical, while, as it is well known, the absolute stress tensor is symmetrical. This difference is very important for the design of the new generation large aircrafts with very high speeds.Thus, the foundations of the theory of *"Relativistic Elasticity"* for airstructures lead to a general theory, in which no restriction is made with regard to the relative motion. This general theory is further reduced to one class of relative motion, uniform in direction and velocity.

2. Relative Stress Tensor Formulation for Airframes

Let us define the state of stress at a point in the stationary frame S^0, by the following stress tensor: (Fig.1)

$$\sigma^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \qquad (2.1)$$

where:

$$\sigma_{21}^0 = \sigma_{12}^0, \sigma_{31}^0 = \sigma_{13}^0, \sigma_{32}^0 = \sigma_{23}^0 \qquad (2.2)$$

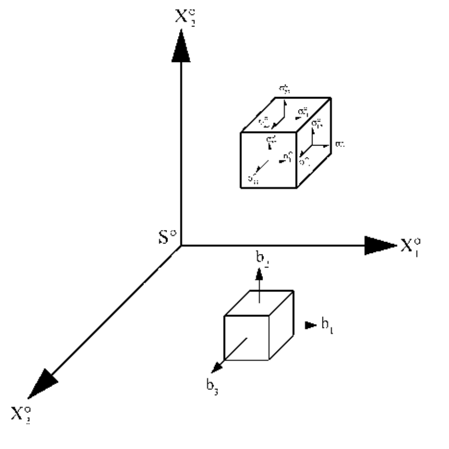

Fig. 1 The state of stress σ_{ik}^0 in the stationary system S^0.

Consider an infinitesimal face element df with a directed normal, defined by a unit vector $\mathbf{n}$, at definite point p in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to df.

Hence, the force is valid as:

$$\mathrm{d}\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}f \tag{2.3}$$

The components $\sigma_i(\mathbf{n})$ of $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components n_k of $\mathbf{n}$:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k ,\ i,k = 1,2,3 \tag{2.4}$$

where σ_{ik} is the elastic tensor, which can be also called the relative stress tensor, in contrast to the space part σ_{ik}^0 of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor. [52], [53} (Fig. 2).

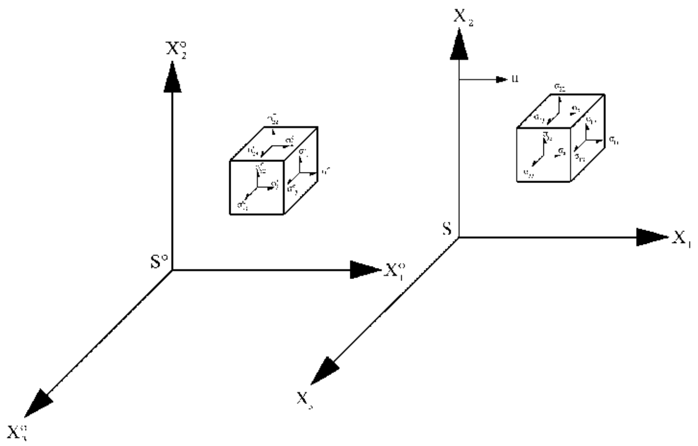

Fig. 2 The state of stress σ_{ik}^0 in the stationary system S^0 and σ_{ik} in the airframe system with velocity u parallel to the x_1 - axis.

The connection between the absolute and relative stress tensors is:

$$\sigma_{ik}^0 = \sigma_{ik} + g_i u_k ,\ i,k = 1,2,3 \tag{2.5}$$

where g_i are the components of the momentum density $\mathbf{g}$ and u_k the components of the velocity $\mathbf{u}$ of the matter.

Also, the connection between $\mathbf{g}$ and the energy flux $\mathbf{s}$, is valid as:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

where c denotes the speed of light (= 300.000 *km*/*sec*).

The total work done per unit time by elastic forces on the matter inside the closed surface f is equal to:

$$W = \int_f (\boldsymbol{\sigma}(\mathbf{n})\cdot\mathbf{u})\,\mathrm{d}f = \int_f \sigma_{ik} n_k u_i\ \mathrm{d}f = -\int_\upsilon \frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\,\mathrm{d}\upsilon ,\ i,k = 1,2,3 \tag{2.7}$$

where the integration in the last integral is extended over the interior υ of the surface f.

Therefore, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is:

$$\delta W = -\frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\,\delta\upsilon \tag{2.8}$$

Moreover, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon) = \delta W \tag{2.9}$$

where h is the total energy density, including the elastic energy and $\mathrm{d}/\mathrm{d}t$ denotes the substantial time derivative.

Eq. (2.9) is valid as:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}(h\delta\upsilon)=\left(\frac{\vartheta h}{\vartheta t}+\frac{\vartheta h}{\vartheta x_k}u_k\right)\delta\upsilon+h\delta\upsilon\frac{\vartheta u_k}{\vartheta x_k}=\left[\frac{\vartheta h}{\vartheta t}+\frac{\vartheta}{\vartheta x_k}(hu_k)\right]\delta\upsilon \tag{2.10}$$

which leads to the relation:

$$\frac{\vartheta h}{\vartheta t}+\frac{\vartheta}{\vartheta x_k}(hu_k+u_i\sigma_{ik})=0 \tag{2.11}$$

Thus, the total energy flow is valid as:

$$\mathbf{s}=\mathbf{hu}+(\mathbf{u}\cdot\boldsymbol{\sigma}) \tag{2.12}$$

where $(\mathbf{u}\cdot\boldsymbol{\sigma})$ is a space vector with components $(\mathbf{u}\cdot\boldsymbol{\sigma})_k=u_i\sigma_{ik}$.

Hence, the total momentum density can be written as:

$$\mathbf{g}=\frac{\mathbf{s}}{c^2}=\mu\mathbf{u}+\frac{(\mathbf{u}\cdot\boldsymbol{\sigma})}{c^2} \tag{2.13}$$

where $\mu=h/c^2$ is the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) we obtain:

$$\sigma_{ik}-\sigma_{ki}=-g_iu_k+g_ku_i=[-(\mathbf{u}\cdot\boldsymbol{\sigma})_iu_k+(\mathbf{u}\cdot\boldsymbol{\sigma})_ku_i]/c^2\neq 0 \tag{2.14}$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame S^0 the velocity $u^0=0$ and hence, from (2.5), (2.12) and (2.13) we obtain the following expressions:

$$\sigma_{ik}^0=\sigma_{ik}=\sigma_{ki}=\sigma_{ki}^0\ \ (i,k=1,2,3) \tag{2.15}$$

Furthermore, the mechanical energy-momentum tensor satisfies the following relation:

$$T_{ik}U_k=-h^0U_i \tag{2.16}$$

where U_i is the four-velocity of the matter, in the Lorentz system and $U_i^0=(0,0,0,ic)$.

Thus, the following scalar can be formed:

$$U_iT_{ik}U_k/c^2=U_i^0T_{ik}^0U_k^0/c^2=-T_{44}^0=h^0(x_1) \tag{2.17}$$

where $h^0(x_1)$ is the invariant rest energy density considered as a scalar function of the coordinates (x_i) $(i=1,2,3)$ in S. (Fig. 2)

By applying further the tensor:

$$\Delta_{ik}=\delta_{ik}+U_iU_k/c^2 \tag{2.18}$$

which satisfies the relations:

$$U_i\Delta_{ik}=\Delta_{ik}U_k=0 \tag{2.19}$$

then, we can form the following symmetrical tensor:

$$S_{ik}=\Delta_{i1}T_{1m}\Delta_{mk}=S_{ki} \tag{2.20}$$

which is orthogonal to U_i:

$$U_iS_{ik}=S_{ik}U_k=0 \tag{2.21}$$

By combining eqs. (2.16), (2.17) and (2.20) we obtain:

$$S_{ik}=T_{ik}-h^0U_iU_k/c^2 \tag{2.22}$$

Also, in the stationary system S_0 we have:

$$S_{ik}^0=\sigma_{ik}^0=\sigma_{ik}\,,\ S_{i4}^0=S_{4i}^0=0 \tag{2.23}$$

Eq. (2.22) may also be written as:

$$T_{ik}=\xi_{ik}+S_{ik} \tag{2.24}$$

where:

$$\xi_{ik}=h^0U_iU_k/c^2=\mu^0U_iU_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^0 = h^0 / c^2 \tag{2.26}$$

is the proper mass density.

Furthermore, we introduce in every system S the quantity:

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is valid as:

$$\sigma_{ik} = T_{ik} - T_{i4} U_k / U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S^0_{ik} = \sigma^0_{ik} = \sigma_{ik}$$

in the stationary system is a real symmetrical matrix. The corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p^0_{(j)}$, the principal stresses, are the three roots of the following algebraic equation, where λ is the unknown:

$$\left| S^0_{ik} - \lambda \delta_{ik} \right| = \left| \sigma^0_{ik} - \lambda \delta_{ik} \right| = 0 \tag{2.30}$$

The matrix S^0_{ik} may also be written in terms of the eigenvalues and eigenvectors as:

$$S^0_{ik} = \sigma^0_{ik} = p^0_{(j)} h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

From eqs. (2.23) and (2.31) we obtain the following form of the stress four-tensor in S^o:

$$S^0_{ik} = p^0_{(j)} h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

Thus, in any system S one has

$$S_{ik} = p^0_{(j)} h_i^{(j)} h_k^{(j)} \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) we obtain the following expressions:

$$T_{ik} = \mu^0 U_i U_k + p^0_{(j)} h_i^{(j)} h_k^{(j)} \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p^0_{(j)} h_k^{(j)} \left(h_k^{(j)} + i h_4^{(j)} u_k / c \right) \tag{2.35}$$

By putting:

$$h_i^{(j)} = (\mathbf{h}^{(j)}, h_4^{(j)}) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, we may write (2.35) for the relative stress tensor $\boldsymbol{\sigma}$ as:

$$\boldsymbol{\sigma} = p^0_{(j)} \left[\mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h_4^{(j)} (\mathbf{h}^{(j)} \bullet \mathbf{u}) \right], j = 1,2,3 \tag{2.37}$$

Moreover, the triad vectors $h_i^{(j)}$ satisfy the tensor relations:

$$h_i^{(j)} h_i^{(\rho)} = \delta^{j\rho} \tag{2.38}$$

$$h_i^{(j)} h_k^{(j)} = \Delta_{ik} \tag{2.39}$$

with Δ_{ik} given by (2.18).

If the stationary system S^0 for every event point is chosen in such a way that the spatial axes in S^0 and in S have the same orientation, we obtain:

$$\mathbf{h}^{(j)} = \mathbf{h}^{(j)0} + \left\{ \mathbf{u}(\mathbf{u} \cdot \mathbf{h}^{(j)0})(\gamma - 1) \right\} / u^2$$

$$h_4^{(j)} = i \mathbf{u} \cdot \mathbf{h}^{(j)0} \gamma / c \tag{2.40}$$

with:

$$\gamma = 1 / (1 - u^2 / c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ one obtains:

$$h = -T_{44} = -\mu^0 U_4^2 - p_{(J)}^0 (\mathbf{u} \cdot \mathbf{h}^{(J)0})^2 \cdot \gamma^2 / c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces to:

$$\boldsymbol{\sigma}^0 = p_{(J)}^0 \left(\mathbf{h}^{(J)0} \bullet \mathbf{h}^{(J)0}\right) \tag{2.43}$$

Thus, from (2.42) we obtain the following transformation law for the energy density:

$$h = \frac{h^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u}/c^2}{1 - u^2/c^2}$$

$$\mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} = u_i \sigma_{ik}^0 u_k \tag{2.44}$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u}/c^4}{1 - u^2/c^2} \tag{2.45}$$

From (2.40) and (2.34) with $k = 4$, one obtains the momentum density $\mathbf{g}$ with the components $g_i = T_{i4}/ic$:

$$\mathbf{g} = \mathbf{u}\left[h^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u}(1 - \gamma^{-1})/u^2\right]\gamma^2/c^2 + (\boldsymbol{\sigma}^0 \cdot \mathbf{u})\,\gamma/c^2$$

$$(\boldsymbol{\sigma}^0 \cdot \mathbf{u})_i = \sigma_{ik}^0 u_k \tag{2.46}$$

Also, from (2.40) and (2.35) we obtain the relative stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u} \bullet (\boldsymbol{\sigma}^0 \cdot \mathbf{u})(\gamma - 1)/u^2 - (\boldsymbol{\sigma}^0 \cdot \mathbf{u}) \bullet \mathbf{u}(\gamma - 1)/\gamma u^2$$

$$- (\mathbf{u} \bullet \mathbf{u})(\mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u})\,(\gamma - 1)^2/\gamma u^4 \tag{2.47}$$

In the special case $\mathbf{u} = (u,0,0)$, where the notation of the matter at the point considered is parallel to the x_1-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$h = \left(h^0 + \frac{u^2}{c^2}\sigma_{11}^0\right)\gamma^2$$

$$g_{x_1} = \gamma^2\left(\mu^0 + \frac{\sigma_{11}^0}{c^2}\right)u$$

$$g_{x_2} = \frac{\gamma\sigma_{21}^0}{c^2}u \tag{2.48}$$

$$g_{x_3} = \frac{\gamma\sigma_{31}^0}{c^2}u$$

and the relative stress tensor:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^0 & \gamma\sigma_{12}^0 & \gamma\sigma_{13}^0 \\ \frac{1}{\gamma}\sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \frac{1}{\gamma}\sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.49}$$

where γ is given by (2.41). Finally, as it is easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

3. Elastic Stress Analysis for Stationary Frames and Airframes

Consider the stationary frame of Fig. 1 with Γ_1 the portion of the boundary of the body on which displacements are presented, Γ_2 the surface of the body on which the force tractions are employed and Γ the total surface of the body equal to Γ_1+Γ_2.

The following formula is valid for the principal of virtual displacements, for linear elastic problems:

$$\int_{\Omega} (\sigma^{0}_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma \tag{3.1}$$

where u_k are the virtual displacements, which satisfy the homogeneous boundary conditions $\bar{u}_k \equiv 0$ on Γ_1, b_k the body forces (Fig. 1) and p_k the surface tractions at the point k of the body. (Fig. 3)

Furthermore, (3.1) takes the following form if u_k do not satisfy the previous conditions on Γ_1:

$$\int_{\Omega} (\sigma^{0}_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \bar{p}_k) u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{3.2}$$

where $p_k = n_j \sigma^{0}_{jk}$ are the surface tractions corresponding to the u_k system.

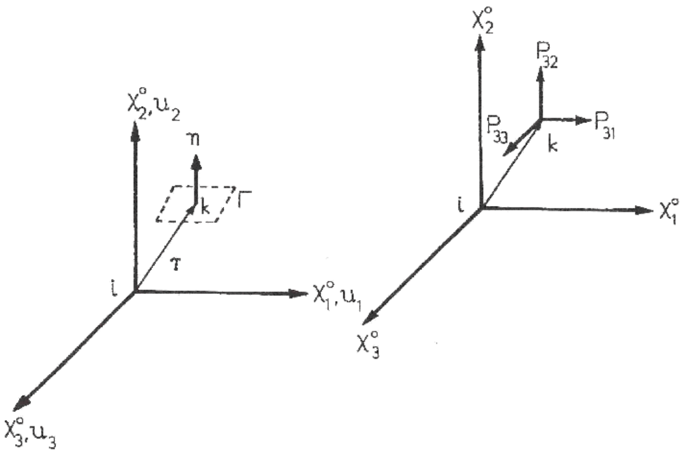

Fig. 3 The stationary system S^0.

By integrating (3.2) follows:

$$\int_{\Omega} b_k u_k \, \mathrm{d}\Omega - \int_{\Omega} \sigma^{0}_{jk} \varepsilon_{jk} \, \mathrm{d}\Omega = -\int_{\Gamma_2} \bar{p}_k u_k \, \mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} (\bar{u}_k - u_k) p_k \, \mathrm{d}\Gamma \tag{3.3}$$

where ε_{jk} are the strains.

By a second integration (3.3) reduces to:

$$\begin{gathered} \int_{\Omega} b_k u_k \, \mathrm{d}\Omega + \int_{\Omega} \sigma^{0}_{jk,j} u_k \, \mathrm{d}\Omega = \\ -\int_{\Gamma_2} \bar{p}_k u_k \, \mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \, \mathrm{d}\Gamma + \int_{\Gamma_1} \bar{u}_k p_k \, \mathrm{d}\Gamma + \int_{\Gamma_2} u_k p_k \, \mathrm{d}\Gamma \end{gathered} \tag{3.4}$$

Also, let us find a fundamental solution, which satisfies the equilibrium equations, of the type:

$$\sigma^{0}_{jk,j} + \Delta^{i}_{l} = 0 \tag{3.5}$$

where Δ^{i}_{l} is the Dirac delta function which represents a unit load at i in the l direction.

The fundamental solution for a three-dimensional isotropic body is: [30]

$$u_{lk}^{*}=\frac{1}{16\pi G(1-v)r}\left[(3-4v)\Delta_{lk}+\frac{\vartheta r}{\vartheta x_{l}}\frac{\vartheta r}{\vartheta x_{k}}\right]$$

$$p_{lk}^{*}=-\frac{1}{8\pi(1-v)r^{2}}\left[\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{lk}+3\frac{\vartheta r}{\vartheta x_{l}}\frac{\vartheta r}{\vartheta x_{k}}\right]-\right.$$
$$\left.-(1-2v)\left[\frac{\vartheta r}{\vartheta x_{l}}n_{k}-\frac{\vartheta r}{\vartheta x_{k}}n_{l}\right]\right] \quad (3.6)$$

where G is the shear modulus, v Poisson's ratio, n the normal to the surface of the body, Δ_{lk} Kronecker's delta, r the distance from the point of application of the load to the point under consideration and n_j the direction cosines (Fig.3).

The displacements at a point are given by the formula:

$$u^{i}=\int_{\Gamma}up\,\mathrm{d}\Gamma-\int_{\Gamma}pu\,\mathrm{d}\Gamma+\int_{\Omega}bu\,\mathrm{d}\Omega \quad (3.7)$$

Thus, (3.7) takes the following form for the "l" component:

$$u_{l}^{i}=\int_{\Gamma}u_{lk}\,p_{k}\,\mathrm{d}\Gamma-\int_{\Gamma}p_{lk}u_{k}\,\mathrm{d}\Gamma+\int_{\Omega}b_{k}u_{lk}\,\mathrm{d}\Omega \quad (3.8)$$

By differentiating u at the internal points, we obtain the stress-tensor for an isotropic medium:

$$\sigma_{ij}^{0}=\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{l}}{\vartheta x_{l}}+G\left(\frac{\vartheta u_{i}}{\vartheta x_{j}}+\frac{\vartheta u_{j}}{\vartheta x_{i}}\right) \quad (3.9)$$

Moreover, after carrying out the differentiation we have:

$$\sigma_{ij}^{0}=\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_{l}}+G\left(\frac{\vartheta u_{ik}}{\vartheta x_{j}}+\frac{\vartheta u_{jk}}{\vartheta x_{i}}\right)\right]p_{k}\,\mathrm{d}\Gamma+$$
$$+\int_{\Omega}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_{l}}+G\left(\frac{\vartheta u_{ik}}{\vartheta x_{j}}+\frac{\vartheta u_{jk}}{\vartheta x_{i}}\right)\right]b_{k}\,\mathrm{d}\Omega- \quad (3.10)$$
$$-\int_{\Gamma}\left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta p_{lk}}{\vartheta x_{l}}+G\left(\frac{\vartheta p_{ik}}{\vartheta x_{j}}+\frac{\vartheta p_{jk}}{\vartheta x_{i}}\right)\right]u_{k}\,\mathrm{d}\Gamma$$

Eq. (3.10) can be further written as following:

$$\sigma_{ij}^{0}=\int_{\Gamma}D_{kij}\,p_{k}\,\mathrm{d}\Gamma-\int_{\Gamma}S_{kij}u_{k}\,\mathrm{d}\Gamma+\int_{\Omega}D_{kij}b_{k}\,\mathrm{d}\Omega \quad (3.11)$$

where the third order tensor components D_{kij} and S_{kij} are:

$$D_{kij}=\frac{1}{8\pi(1-v)r^{2}}\left[(1-2v)\left[\Delta_{ki}r_{,j}+\Delta_{kj}r_{,i}-\Delta_{ij}r_{,k}\right]+3r_{,i}r_{,j}r_{,k}\right] \quad (3.12)$$

$$S_{kij}=\frac{G}{4\pi(1-v)r^{3}}\left[3\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{ij}r_{,k}+v(\Delta_{ik}r_{,j}+\Delta_{jk}r_{,i})-5r_{,i}r_{,j}r_{,k}\right]\right.$$
$$\left.+3v(n_{i}r_{,j}r_{,k}+n_{j}r_{,i}r_{,k})+(1-2v)(3n_{k}r_{,i}r_{,j}+n_{j}\Delta_{ik}+n_{i}\Delta_{jk})-(1-4v)n_{k}\Delta_{ij}\right] \quad (3.13)$$

whith: $r_{,i}=\frac{\vartheta r}{\vartheta x_{i}}$

Finally, by considering the moving system S of Fig. 2, then the stress-tensor reduces to the following form, because of (2.49) and (3.11):

$$
\begin{aligned}
\sigma_{11} &= \sigma_{11}^0 \\
\sigma_{12} &= \gamma\sigma_{12}^0 \\
\sigma_{13} &= \gamma\sigma_{13}^0 \\
\sigma_{21} &= \frac{1}{\gamma}\sigma_{21}^0 \\
\sigma_{22} &= \sigma_{22}^0 \\
\sigma_{23} &= \sigma_{23}^0 \\
\sigma_{31} &= \frac{1}{\gamma}\sigma_{31}^0 \\
\sigma_{32} &= \sigma_{32}^0 \\
\sigma_{33} &= \sigma_{33}^0
\end{aligned}
\tag{3.14}
$$

where σ_{ij}^0 are given by. (3.11) to (3.13).

The following Table 1 shows the values of γ as given by (2.41) for some arbitrary values of the velocity u of the moving aerospace structure:

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	c	∞

From the above Table follows that for small velocities 50,000 *km/h* to 200,000 *km/h*, the absolute and the relative stress tensor are nearly the same. On the other hand, for bigger velocities like $c/3$, $c/2$ or $3c/4$ (c = speed of light), the variable γ takes values more than the unit and thus, relative stress tensor is very different from the absolute one. Finally, for values of the velocity of the moving structure near the speed of light, the variable γ takes bigger values, while when the velocity is equal to the speed of light, then γ tends to the infinity.

The Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et all [22] will be used for the numerical evaluation of the stress tensor (3.11), for every specific case.

4. Conclusions

As a conclusion to the previous outlined analysis, our proposal for investigation in the objective of the aeronautics technologies is the following: The application of the theory of *"Relativistic Mechanics"* for the design of a new generation large aircraft with turbojet engines and speeds in the range of 50,000 km/h. Such a design and construction of the new generation aircraft will be applied to an increased market share of international Aeronautical Industries. The theory of *"Relativistic Mechanics"* and the *"Universal Equation of Elasticity"* show that there is a considerable difference between the absolute stress tensor of the airframe even in the range of speeds of 50,000 km/h. For bigger speeds the difference between the two stress tensors is very much increased. *"Relativistic Mechanics"* is a combination of the theories of classical elasticity and special relativity.

For the strucutural design of the new generation aircrafts will be used the stress tensor of the airframe in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.).

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