A PID Tuning Method for Tracking Control of an Underactuated Gantry Crane

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Abstract
By the current research, a PID tuning procedure is developed to control of an overhead crane. To enable the crane to reach a desired position, a controller design strategy must be used. This work presents a simple and efficient enough approach to control of the overhead crane based on designing a PID controller. Ziegler-Nichols method is used to initiate gains setting for PID controller and a tuning procedure is developed to obtain better results. Tuning design of the PID controller is considered for tracking the response of trolley and minimum oscillation of pendulum. Finally, some simulations are performed to illustrate the capability of the presented method for control of the overhead crane.

Key Word and Phrases
PID Tuning, Gantry Crane, Control, Track.

1. Introduction
Gantry cranes have received a lot of interests in transportation and industrial application, due to their low cost, easy assembly and less maintenance. They are composed of a platform moving in a fixed support, while a pendulum is suspended from a point on the platform. For the sake of minimum transportation time, high tracking accuracy, and swing angle, dynamic modeling and motion control of the gantry crane system become as appealing tasks in the field of control science [1-5]. Moustafa et al. [6] proposed optimal control of the overhead cranes. They presented a nonlinear dynamic model of the systems which considered travel, traverse, and hoisting/lowering motions of the crane. Rahn et al. [7] used a feedback law to stabilize the hoisting angle of a flexible overhead crane. Frang et al. [8] used a nonlinear coupling law to control the overhead robot. Moreover, an input-shaping control law was proposed in [9] to control the motion of the crane. In this method, the input control profile is determined as unwanted oscillation during travel and residual pendulations are avoided. Also, a hybrid input-shaping strategy and a PD-type fuzzy logic control scheme are implemented in [10] to control a gantry (overhead) crane system. However, despite of the efficiency of this method, the input-shaping method leaks from being an open loop control scheme, and the method is not robust enough to disturbances and parameter uncertainties [11]. Mahfouf et al. [12] designed a fuzzy logic-based controller to damp the sway angle of overhead crane. Moreover, a nonlinear modeling and anti-swing control method for the overhead cranes was presented in [13]. Yu et al. [14] used a perturbation technique to separate the slow and fast dynamics of the gantry crane model. Then, they used a feedback control strategy including two independent PD controllers to track the pre-defined motion profile and suppress payload pendulations, respectively. Liu et al. [15] proposed an adaptive sliding mode fuzzy control method for the overhead system. Moreover, a neural based method was used in [16] to control the motion of the crane.

By the present research, a PID tuning procedure is developed to control of an overhead crane. Hence, Ziegler-Nichols method is used to start setting of gains of PID controller, but a tuning procedure is developed to obtain better results. The criterion to tune the PID controller is considered the tracking response of trolley and minimum oscillation of pendulum. Finally, some simulation are performed which discussed the capability of the presented method for control of the overhead crane.
2. Dynamic Model of the System

In this section, dynamic model of the gantry crane is presented. The set of dynamic equations of the system is derived using Lagrange principle. Figure 1 shows an overhead crane.

![Fig 1. The gantry crane](image)

Parameters of the overhead system are presented in Table 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cart position</td>
<td>$x$</td>
</tr>
<tr>
<td>Cart velocity</td>
<td>$\dot{x}$</td>
</tr>
<tr>
<td>Pendulum angular displacement</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Pendulum angular velocity</td>
<td>$\dot{\theta}$</td>
</tr>
<tr>
<td>Pendulum length</td>
<td>$l$</td>
</tr>
<tr>
<td>Mass of the cart system</td>
<td>$M$</td>
</tr>
<tr>
<td>Payload mass</td>
<td>$m$</td>
</tr>
<tr>
<td>Gravitational constant of earth</td>
<td>$g$</td>
</tr>
<tr>
<td>Radius of wheels of cart</td>
<td>$r$</td>
</tr>
<tr>
<td>DC motor voltage of cart</td>
<td>$e$</td>
</tr>
<tr>
<td>Force exerted to cart</td>
<td>$f$</td>
</tr>
<tr>
<td>Motor armature resistance</td>
<td>$R$</td>
</tr>
<tr>
<td>Motor torque constant</td>
<td>$k$</td>
</tr>
</tbody>
</table>

By using the Lagrange principle, nonlinear dynamic equations of the system can be presented as [17]:

\[
(M + m)\ddot{x} + m l \ddot{\theta} \cos \theta - m l \dot{\theta}^2 \sin \theta = f
\]

\[
\dot{x} \cos \theta + l \ddot{\theta} + g \sin \theta = 0
\]

Besides, the linear force is originated from the torque of trolley motor as [18]:

\[
T = rf
\]

\[
T = \frac{k}{R} e - \frac{k^2}{R} \omega
\]

\[
\dot{x} = r \omega
\]

where $\omega$ is the angular velocity of DC motor. This velocity is related to the velocity of the cart by
Furthermore, by combination of Eq. 1 and Eq. 2, the nonlinear equation of the overhead crane can be summarized as follows:

\[
(M + m)\ddot{x} + ml \ddot{\theta} \cos \theta - ml \dot{\theta}^2 \sin \theta = \frac{1}{r} \left( k \frac{e - k^2 x}{R r} \right)
\]

(2.3)

\[
\dot{x} \cos \theta + \dot{\theta} + g \sin \theta = 0
\]

In addition, by defining the state vector as \( \mathbf{\dot{X}} = [x \ \dot{x} \ \theta \ \dot{\theta}]^T = [x_1 \ x_2 \ x_3 \ x_4]^T \) and linearizing the obtained equations, the resulted equation can be derived in the state space form as:

\[
\mathbf{\dot{X}} = A \mathbf{X} + B u
\]

\[
y = C \mathbf{X} + D u
\]

where the matrices \( A, B, C \) and \( D \) are:

\[
A = \begin{bmatrix}
0 & \frac{k^2}{R r M} & \frac{mg}{R r M} & 0 \\
0 & 0 & 1 & 0 \\
0 & \frac{k^2}{R r M} & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
\frac{k}{R r M} \\
0 \\
-\frac{k}{R r M}
\end{bmatrix}
\]

(2.5)

3. PID Control Formulation and Simulation Results

In this section, the PID controller is designed. The first try to obtain the gains of the controller are performed based on Ziegler-Nichols method [17]. Then, tuning procedure is continued to obtain desired results. A plan of controller is shown in figure 2:

![Fig 2. The controller plan](image)

In Fig. 2, the transfer function of the PID controller is shown as \( G(s) \), and the transfer function of the system can be obtained from (2.5):

\[
G(s) = C(sI - A)^{-1} B = \frac{X(s)}{E(s)} = \frac{k r l s^2 + k r g}{M R r l s^4 + l k^2 s^3 + (M + m) g R r^2 s^2 + g k^2 s}
\]

(3.1)

Furthermore, the transfer function of controller is [19]:

\[
G_c(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) = K_p + \frac{K_i}{s} + K_d s
\]

(3.2)

where \( K_p, K_i, K_d \) are the gains of proportional, integral, and derivative controls, respectively.

By using the parameter values of the system as presented in Table 2, the transfer function of the system is obtained as:

\[
TF = \frac{1.52 s^2 + 45.15}{3.316 s^4 + 1.943 s^3 + 119.6 s^2 + 57.71 s}
\]

(3.3)
Table 2. Parameter values of the overhead crane

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pendulum length l</td>
<td>$l = 0.3302\text{m}$</td>
</tr>
<tr>
<td>Mass of the cart system M</td>
<td>$M = 1.073\text{kg}$</td>
</tr>
<tr>
<td>Payload mass m</td>
<td>$m = 0.23\text{kg}$</td>
</tr>
<tr>
<td>Gravitational constant of earth g</td>
<td>$g = 9.81\text{m/s}^2$</td>
</tr>
<tr>
<td>Radius of wheels of cart r</td>
<td>$r = 0.006\text{m}$</td>
</tr>
<tr>
<td>Motor armature resistance R</td>
<td>$R = 2.6\Omega$</td>
</tr>
<tr>
<td>Motor maximum voltage $e_{\text{max}}$</td>
<td>$e_{\text{max}} = 12\text{V}$</td>
</tr>
<tr>
<td>Motor torque constant k</td>
<td>$k = 0.00767\text{Vs/\text{rad}}$</td>
</tr>
</tbody>
</table>

The Ziegler-Nichols method is developed to obtain PID gains of a controller. In [17] the detail of the method is presented. Moreover, by using the second method of Ziegler-Nichols technique for the gantry crane systems [17], the first estimation for control gains can be obtained as:

$K_c = 6.94, \ P_c = 13.96, \ K_p = 0.6 \ K_c, \ T_i = 0.5 \ P_c = 6.98, \ K_d = K_p T_d = 7.24,$

$T_d = 0.125 P_c = 1.74, \ K_i = \frac{K_p}{T_i} = 0.61.$

It must be noticed that in this study simulations are performed using MATLAB Simulink. Employing four tuning attempts as it is shown in Table 3, following results are achieved via PID controller:

Table 3. Characteristics of response of the system

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>N</th>
<th>Maximum percent overshoot of $x$</th>
<th>Setting time of $x$</th>
<th>Maximum voltage of motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.16</td>
<td>0.61</td>
<td>1.74</td>
<td>1</td>
<td>55.8028</td>
<td>10.4409</td>
<td>5.9</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>2.2</td>
<td>1</td>
<td>31.6678</td>
<td>5.4799</td>
<td>6.2</td>
</tr>
<tr>
<td>1.2</td>
<td>0.01</td>
<td>3</td>
<td>1</td>
<td>12.0139</td>
<td>4.6423</td>
<td>7.3</td>
</tr>
<tr>
<td>1.2</td>
<td>0.005</td>
<td>3</td>
<td>2.5</td>
<td>2.1715</td>
<td>2.748</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Assuming the unit step for input reference, the trolley displacement of the overhead crane obtained as depicted in Fig. 3:
According to Fig. 3, the first try of PID tuning done via Ziegler-Nichols method has reasonable time response of the plant while it totally eliminates the error at the steady state. But the resulted overshoot and oscillations of the output are high. Thus, for achieving the better response the tuning process is continued. As it is obvious from Fig. 3 after four steps of tuning the process the desirable results is obtained.

4. Conclusions

By the current paper, a PID tuning procedure has been developed to control of an overhead crane. Hence, Ziegler-Nichols method has been used to initiate controller gains, and then a tuning procedure has been developed to obtain better results.

Finally, numerical simulations have been performed to show the capability of the tuning procedure to control of the overhead crane.

References