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Singular Integral Equations for Pulp Production from Non-linear Fibers

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Abstract
A new technology is further improved by applying the *Singular Integral Operators Method (S.I.O.M.).* for the determination of the properties of non-wood cellulosic fibers used for paper manufacturing. Hence, the solution of the anisotropic elastic stress analysis problem is investigated, which defines the basic feature for the mechanical behavior of non-wood cellulosic fibers. Thus, the above modern method depends on the existence and explicit definition of the fundamental solution to the governing partial differential equations. Then, after the determination of the fundamental solution, a real variable boundary integral formula is generated. Besides, the construction of the solution for the composite solids problem is presented as is the derivation of the expression for the surface tractions necessary to maintain the fundamental solution in a bounded region. Many parameters, like intensity factors, incorporate stress kernels, geometry and crack size, may be evaluated by the elastic stress analysis of cracked structures. Consequently, by using the S.I.O.M., then the anisotropic elastic stress of composite solids will be determined.

Key Word and Phrases
Singular Integral Operators Method (S.I.O.M.), Paper Manufacturing, Non-Wood Cellulosic Fibers, Composite Solids, Anisotropic Materials, Anisotropic Elastic Stress Analysis, Somigliana Identity.

1. Introduction

Generally, pulp and paper production is one of the high demand sectors in the industrial world. Currently world paper production is about 500 million tons. Cleaner technology is applied to achieve increased production with minimum effect on the environment and to save, utilize, and recycle expensive and scarce chemicals and raw materials. Hence, the increasing demand for paper has raised the need for low-cost raw materials and also the developing of new process in order to boost production.

Hence, non-wood fibers, for example agricultural residues and annual plants, are considered an effective alternative source of cellulose for producing pulp and paper sheets with acceptable properties in lower cost. There is a growing interest in the use of non-wood such as annual plants and agricultural residues as a raw material for pulp and paper. Consequently, non-wood raw materials account for less than 10% of the total pulp and paper production worldwide.

The benefits of non-wood plants as a fiber resource are their fast annual growth and the smaller amount of lignin in them that bind their fibers together. Another benefit is that non-wood pulp can be produced at low temperatures with lower chemical charges. As the world pulp production is unlikely to increase dramatically in near future, there is a practical need for non-wood pulp to supplement the use of conventional wood pulp. Moreover, the specific physical and chemical characteristics of non-wood fibers have an essential role in the technical aspects involved in paper production.

Also, the production of pulp from non-wood resources has many advantages such as easy pulping capability, excellent fibers for the special types of paper and high-quality bleached pulp. They can be used as an effective substitute for the forever decreasing forest wood resources. Besides to their sustainable nature, other advantages of non-wood pulps are their easy pulping and bleaching capabilities. These allow the production of high-quality bleached pulp by a less polluting process than hardwood pulps and reduced energy requirements.

Traditionally non-wood material is cooked with hybrid chemimechanical and alkali-based chemicals. In chemical pulping, the raw materials are cooked with appropriate chemicals in an aqueous solution at an elevated temperature and pressure. The objective is to degrade and dissolve away the lignin and leave behind most of the cellulose and hemicelluloses in the form of intact fibers. Hence, in practice, chemical pulping methods are successful in removing most of the lignin and they also degrade and dissolve a certain amount of the cellulose and hemicelluloses.

One group of the most promising pulp processes is called the Organosolv processes. Thus, in terms of production technologies, novel and improved processes are proposed. The Organosolv methods are based on cooking with organic solvents such as alcohols or organic acids. Methanol and ethanol are common alcohols used and the organic acids are normally formic acid and acetic acid. High cooking temperatures and associated high pressures are needed when alcohols are used in cooking. The Organosolv process has certain advantages. It makes possible the breaking up of the lignocellulosic biomass to obtain cellulosic fibers for pulp and papermaking, high quality hemicelluloses and lignin degradation products from generated black liquors, thus avoiding emission and effluents. Thus, the Organosolv pulping process is an alternatives to conventional pulping processes, and has environmental advantages. Organosolv pulping features an organic solvent in the pulping liquor which limits the emission of volatile sulfur compounds into the atmosphere and gives efficient chlorine-free bleaching. These processes should be capable of pulping all lignocellulose species with equal efficiency. Another major advantage of the Organosolv process is the formation of useful by-products such as furfural, lignin and hemicelluloses.

By the present research plants like kenaf (Hibiscus cannabinus L.) and giant reed (Arundo donax L.) are proposed as internodes gave very good derived values, especially slenderness ratio, which is directly comparable to some softwood and most hardwood species. Chemical analysis of the raw plant materials revealed satisfactory levels of α-cellulose content (close to 40%) and Klason lignin content (<30%) compared to those of hardwoods and softwoods.

The above non-wood plants offer several advantages including short growth cycles, moderate irrigation and fertilization requirements and low lignin content resulting to reduced energy and chemicals use during pulping. The fiber dimensions are shown in Table 1. As a dicot, kenaf has two distinct kinds of fibers—long bark fibers, which account for 35% of its fibrous part, and short core fibers, which account for the rest. Bark fibers have very good derived values (especially slenderness ratio) compared to those of some softwoods and certainly to most hardwoods. Hence, papers made from kenaf bark fibers are expected to have increased mechanical strength and thus be suitable for writing, printing, wrapping and packaging purposes.

Table 1

Non-wood Fiber Dimensions

Plant Material	Length (mm)	Diameter (μm)	Lumen Diameter(μm)	Cell Wall Thick.(μm)
Kenaf (bark)	2.32	21.9	11.9	4.2
Kenaf (core)	0.74	22.2	13.2	4.3
Kenaf (whole)	1.29	22.1	12.7	4.3
Reed (internodes)	1.22	17.3	8.5	4.4
Reed (nodes)	1.18	18.8	8.6	5.6

Consequently, because of their lower lignin content (compared to wood), non-wood plants can be pulped in one-third of the time needed for softwoods and hardwoods. Pulping of non-wood fibers also demands around 30% less chemical charge, and reduced power consumption in pulp refining. Many homogeneous solids like paper or pulp are often anisotropic (or at least orthotropic from point to point).

During the past years, special effort has gone into studying stress fields in anisotropic solids, because numerous engineering materials under normal or loading conditions show different mechanical properties along certain preferred directions. Among them we shall mention the following authors, following classical lines: S.G.Lekhnitskii [1]-[5], G.N.Savin [6]-[10], M.O.Basheleishvili [11], [12], J.R.Willis [13], H.T.Rathod [14], S.Krenk [15], G.C.Sih and H.Liebowitz [16], G.C.Sih and M.K.Kassir [17] and G.C.Sih et al. [18].

On the other hand, by using an integral transform method obtained by I.N.Sneddon [19], [20] the governing partial differential equation of anisotropic elasticity is solved, while G.E.Tupholme [21], D.D.Ang and M.L.Williams [22], O.L.Bowie and C.E.Freese [23] have studied some fracture mechanics problems of orthotropic media.

Singular integral equation methods for solving two- and three-dimensional problems of cracks and holes in anisotropic bodies have been introduced by F.J.Rizzo and D.J.Shippy [24], S.M.Vogel and F.J.Rizzo [25], M.D.Snyder and T.A.Cruse [26], [27], E.G.Ladopoulos [28], [29], K.S.Parihar and S.Sowdamini [30], T.Mura [31], C.Ouyang and Mei-Zi Lu [32], R.P.Gilbert et al. [33], R.P.Gilbert and M.Schneider [34], R.P.Gilbert and R.Magnanini [35] and U.Zastrow [36] - [38].

Hence, the ***Singular Integral Operators Method (S.I.O.M.)*** [40]-[42] which was used very successfully for the solution of several engineering problems of fluid mechanics, hydraulics, aerodynamics, solid mechanics, potential flows and structural analysis, is further extended by the current study for the solution of problems of non-wood cellulosic fibers for paper manufacturing.

2. Formulation of Anisotropic Elastic Stress Analysis

Let us express the stresses $(\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy})$ in terms of strains $(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy})$ through a set of constants C_{ij}, which are called the moduli of elasticity:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} \tag{2.1}$$

On the contrary, in order to express the strains in terms of stresses, let us use another set of 36 constants a_{ij} $(i, j = 1,2,\ldots,6)$, known as the coefficients of deformation:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} \tag{2.2}$$

Hence, by considering the case where the material is "transversely isotropic", which means, that it possesses an axis of elastic symmetry such that the material is isotropic in the planes normal to this axis, then the following formula is valid between the stresses and strains:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{12} & a_{11} & a_{13} & 0 & 0 & 0 \\ a_{13} & a_{13} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(a_{11}-a_{12}) \end{bmatrix} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} \tag{2.3}$$

in which z is the direction of the elastic symmetry.

The coefficients of deformation in (2.3) are expressed as: [1]

$$a_{11} = \frac{1}{E_1}, \quad a_{12} = -\frac{v_1}{E_1},$$

$$a_{33} = \frac{1}{E_2}, \quad a_{13} = -\frac{v_2}{E_2}, \tag{2.4}$$

$$a_{44} = \frac{1}{G_2}, \quad 2(a_{11}-a_{12}) = \frac{2(1+v_1)}{E_1} = \frac{1}{G_1}$$

where E_1, G_1 and v_1 are the Young's modulus, shear modulus, and Poisson's ratio, respectively, in the plane of isotropy and E_2, G_2 and v_2 are the same quantities in the transverse direction.

Furthermore, in order to express the stress components in terms of strains for a "transversely isotropic" material one obtains the following formula:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11}-C_{12}) \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} \tag{2.5}$$

where the elastic moduli C_{ij} may be expressed as following: [1],[2]

$$C_{11} = 2G_1\left(1 - v_2^2\frac{E_1}{E_2}\right) \Bigg/ \left(1 - v_1 - 2v_2^2\frac{E_1}{E_2}\right)$$

$$C_{12} = 2G_1\left(v_1 + v_2^2\frac{E_1}{E_2}\right) \Bigg/ \left(1 - v_1 - 2v_2^2\frac{E_1}{E_2}\right)$$

$$C_{13} = E_1 v_2 \Bigg/ \left(1 - v_1 - 2v_2^2\frac{E_1}{E_2}\right)$$

$$\tag{2.6}$$

$$C_{33} = E_2(1-\nu_1) \Big/ \left(1-\nu_1-2\nu_2^2\frac{E_1}{E_2}\right)$$
$$C_{44} = G_2$$
$$\frac{1}{2}(C_{11}-C_{12}) = G_1$$

Besides, in the case of isotropic material, $\nu_1 = \nu_2$, $E_1 = E_2$ and $G_1 = G_2$ and so the elastic moduli C_{ij} may be related to the Lamé coefficients λ and μ as:

$$C_{11} = C_{12} = \lambda + 2\mu$$
$$C_{12} = C_{13} = \lambda \tag{2.7}$$
$$C_{44} = \mu$$

The strain components in (2.5) are expressed by the formulas:

$$\varepsilon_x = \frac{\vartheta u_x}{\vartheta x},\ \varepsilon_y = \frac{\vartheta u_y}{\vartheta y},\ \varepsilon_z = \frac{\vartheta u_z}{\vartheta z}$$
$$\gamma_{yz} = \frac{\vartheta u_z}{\vartheta y} + \frac{\vartheta u_y}{\vartheta z}$$
$$\gamma_{zx} = \frac{\vartheta u_x}{\vartheta z} + \frac{\vartheta u_z}{\vartheta x} \tag{2.8}$$
$$\gamma_{xy} = \frac{\vartheta u_x}{\vartheta y} + \frac{\vartheta u_y}{\vartheta x}$$

in which u_x, u_y and u_z are the components of displacements in Cartesian coordinates.

3. Composite Stress Analysis Fundamental Solutions

Let us consider a body in three-dimensional space, which has a bounding surface L. According to Betti's reciprocal theorem and by considering absence of body forces, one obtains: [24],[29]

$$\int_L (u_i T_{ij} - t_i U_{ij})\,\mathrm{d}R + \int_\Gamma (U_i T_{ij} - t_i U_{ij})\,\mathrm{d}R = 0 \tag{3.1}$$

where $\mathrm{d}R$ is an element of surface area at R, which is a point on L. Also Γ is the boundary of the finite or infinite domain of space in coordinates x_1, x_2, x_3, in which exist the anisotropic elastic body. This boundary Γ is a connected closed Lyapounov surface.

In (3.1) u_i and t_i are the displacement and traction components, $U_{ij}(x,y)$ the displacement at point x in response to a concentrated unit body force acting in the j coordinate direction at point y, and T_{ij} the suitable boundary tractions.

Furthermore, Betti's theorem (eq. (3.1)) results in Somigliana's identity [24]:

$$u_i(y) = \frac{1}{a}\int_L \left[t_j(x)U_{ij}(x,y) - u_j T_{ij}(x,y)\right]\cdot \mathrm{d}R \tag{3.2}$$

where the point dependence is explicitly indicated and a is the magnitude of the force components.

The following two limiting formulas have to exist:

$$\lim_{\varepsilon \to 0} \int_{\Gamma} u_i T_{ij} \, \mathrm{d}R = a u_j(y) \tag{3.3}$$

$$\lim_{\varepsilon \to 0} \int_{\Gamma} t_i U_{ij} \, \mathrm{d}R = 0 \tag{3.4}$$

in which ε is the radius of a sphere centre y, with the boundary Γ, and $u_j(y)$ the displacement at the origin corresponding to u_i and t_i on L.

In order to derive the formula of the fundamental solution, we adopt the method of decomposition into plane waves used in [39]. Thus, consider the function g, which is an arbitrary distribution and vanishes outside a finite sphere.

The next formula is a solution of the differential equation:

$$\Delta_y u(y) = g(y) \tag{3.5}$$

$$u(y) = \int_A g(x) \left(-\frac{1}{4\pi |x-y|} \right) \mathrm{d}x \tag{3.6}$$

where Δy denotes the Laplacean with respect to y_i.

The following identity is easily seen to be:

$$\int_{|\zeta|=1} \left| (x_i - y_i)\zeta_i \right| \mathrm{d}R = 2\pi |x-y| \tag{3.7}$$

From (3.5) and (3.7) one obtains the result:

$$\Delta_y |x-y| = \frac{2}{|x-y|} \tag{3.7a}$$

Thus, from (3.5), (3.6) and (3.7a) we have:

$$g(y) = \frac{1}{16\pi^2} \Delta_y^2 \int_A \int_{|\zeta|=1} g(x) \left| (x_i - y_i)\zeta_i \right| \mathrm{d}R \, \mathrm{d}x \tag{3.8}$$

Furthermore, consider the function $h(\zeta, p)$ which is given by the formula:

$$h(\zeta, p) = \int_{(x \cdot \zeta) = p} g(x) \, \mathrm{d}R \tag{3.9}$$

Besides, the following formula is valid:

$$\int_A \int_{|\zeta|=1} g(x)\left|(x_i - y_i)\zeta_i\right| \mathrm{d}\,R\,\mathrm{d}\,x = \int_{|\zeta|=1} \mathrm{d}\,R \int_{-\infty}^{\infty} |p|\,\mathrm{d}\,p \int_{(x-y)\cdot\zeta=p} g(x)\,\mathrm{d}\,x$$

$$= \int_{|\zeta|=1} \mathrm{d}\,R \int_{-\infty}^{\infty} |p| h(\zeta, p + y\cdot\zeta)\,\mathrm{d}\,p \tag{3.10}$$

and:

$$\Delta_y \int_{-\infty}^{\infty} |p| h(\zeta, p + y\cdot\zeta)\,\mathrm{d}\,p \tag{3.11}$$

$$= \Delta_y \left[\int_{(y\cdot\zeta)}^{\infty} (p - y\cdot\zeta) h(\zeta, p) \right] \mathrm{d}\,p - \int_{-\infty}^{(y\cdot\zeta)} (p - y\cdot\zeta) h(\zeta, p)\,\mathrm{d}\,p = 2h(\zeta, y\cdot\zeta)$$

From (3.8), (3.10) and (3.11) one has:

$$g(y) = -\frac{1}{8\pi^2} \Delta_y \int_{|\zeta|=1} h(\zeta, y\cdot\zeta)\,\mathrm{d}\,R \tag{3.12}$$

By considering the case where:

$$g(y) = \delta(y) \tag{3.13}$$

then we obtain:

$$h(\zeta, y\cdot\zeta) = \delta(y\cdot\zeta) \tag{3.14}$$

From (3.12), (3.13) and (3.14) we have the expression for the three-dimensional delta function:

$$\delta(x-y) = -\frac{1}{8\pi^2} \Delta_y \int_{|\zeta|=1} \delta((x-y)\cdot\zeta)\,\mathrm{d}\,R \tag{3.15}$$

Thus, from (3.15) we derive the fundamental solution for the displacements:

$$U_{ij}(x,y) = \frac{1}{8\pi^2} \Delta_y \int_{\zeta=1} W_{ij}(x,y,\zeta)\,\mathrm{d}\,R \tag{3.16}$$

where the function W_{ij} is given by:

$$W_{ij}(x,y,\zeta) = \begin{cases} W_{ij}(x,y,\zeta), & (x-y)\cdot\zeta > 0 \\ 0, & (x-y)\cdot\zeta \le 0 \end{cases} \tag{3.17}$$

According to the Cauchy-Kowalewski theorem we have:

$$W_{ij} = P_{ij}(\zeta)(x_k - y_k)\zeta_k \tag{3.18}$$

So, from (3.16), (3.17) and (3.18) we obtain:

$$U_{ij}(x,y) = \frac{1}{8\pi^2} \Delta_y \int_{\substack{|\zeta|=1 \\ (x-y)\cdot\zeta>0}} P_{ij}(\zeta)\cos\varphi\,\mathrm{d}\,R \tag{3.19}$$

By using (3.7), then (3.19) takes the simpler form:

$$U_{ij}(x,y)=\frac{1}{4\pi^2|x-y|}\int_{\substack{|\zeta|=1\\(x-y)\cdot\zeta>0}} P_{ij}(\zeta)\cos\varphi\,\mathrm{d}R \tag{3.20}$$

where φ is the angle between the vectors x-y and ζ.

From (3.20) we derive a simpler form, if the part of the integration over the unit hemispherical shell of (3.20) involving the azimuthal angle, is carried out:

$$U_{ij}(x,y)=\frac{1}{8\pi^2|x-y|}\int_{\substack{|\zeta|=1\\(x-y)\cdot\zeta>0}} P_{ij}(\zeta)\,\mathrm{d}s \tag{3.21}$$

in which $\mathrm{d}s$ is an element of arc length.

Thus, (3.21) gives the solution for the general case of three-dimensional elasticity.

Beyond the above, $P_{ij}(\zeta)$ in (3.21) is given by:

$$P_{ij}(\zeta)=\frac{1/2\,\varepsilon_{imn}\varepsilon_{jrs}Q_{mr}(\zeta)Q_{ns}(\zeta)}{\det Q} \tag{3.22}$$

and:

$$Q_{ik}(\zeta)=C_{ijkl}\zeta_j\zeta_l$$

where the constants C_{ijkl} are the elasticities, Q_{ij} is the characteristic matrix and the quantities ε_{imn} and $\det Q$ are the alternating symbol and determinant of Q_{ij}, respectively. On the other hand, the suitable boundary tractions T_{ij} are given by the formula:

$$T_{im}(x,y)=C_{ijkl}U_{km}(x,y)_{,l}\,n_j \tag{3.22a}$$

where n_i are the components of the unit outward at the point x on L. Moreover, let us take a new point $\mathbf{x}^1$ relative to the point x. Then for the vectors $\mathbf{x}$, $\mathbf{x}^1$ one has:

$$\mathbf{x}^1=\mathbf{x}+\delta\mathbf{x} \tag{3.23}$$

By the same way, the new point ζ_1 relative to the point ζ is valid as:

$$\boldsymbol{\zeta}^1=\boldsymbol{\zeta}+\delta\boldsymbol{\zeta} \tag{3.24}$$

Thus, from (3.22a) we obtain in an analogous way, the displacement tensor:

$$U_{ij}(x^1,y)=\frac{1}{8\pi^2|x^1-y|}\oint_{|\zeta^1|=1} P_{ij}(\zeta^1)\,\mathrm{d}s \tag{3.25}$$

Hence, from (3.23) and (3.24), eq. (3.25) takes the form:

$$U_{ij}(x^1,y)=\frac{1}{8\pi^2|x+\delta x-y|}\oint_{|\zeta^1|=1} P_{ij}(\zeta+\delta\zeta)\,\mathrm{d}s \tag{3.26}$$

Besides, we introduce the expressions:

$$\lambda_i=\frac{x_i-y_i}{|x-y|} \tag{3.27}$$

and:

$$\lambda_i^1=\frac{x_i^1-y_i}{|x^1-y|}=\frac{x_i+\delta x_i-y_i}{|x+\delta x-y|} \tag{3.28}$$

So, it is easy to show that:

$$\delta\zeta_i = \frac{-\lambda_k^1 \zeta_k}{1+\lambda_k \lambda_k^1}(\lambda_i^1 + \lambda_i) \tag{3.29}$$

The insertion of (3.27), (3.28) and (3.29) into (3.26) results the displacements:

$$U_{ij,k} = -\frac{(x_k - y_k)}{8\pi^2 |x-y|^3} \oint_{|\zeta|=1} P_{ij}(\zeta)\,\mathrm{d}\,s - \frac{1}{8\pi^2 |x-y|^3} \int_{|\zeta|=1} \zeta_k \tag{3.30}$$

$$\times \frac{\lfloor (x_q - y_q)R_{jiq} + (x_r - y_r)R_{jir} + (x_s - y_s)R_{jis} + (x_t - y_t)R_{jit} \rfloor \mathrm{d}\,s}{\det Q} + \frac{1}{8\pi^2 |x-y|^3} \int_{|\zeta|=1} P_{ij}\zeta_k$$

$$\times \frac{\lfloor (x_1 - y_1)W_1 + (x_m - y_m)W_m + (x_n - y_n)W_n + (x_p - y_p)W_p + (x_r - y_r)W_r + (x_s - y_s)W_s \rfloor \mathrm{d}\,s}{\det Q}$$

Consequently, (3.30) gives the solution for the general case of three-dimensional elasticity, while the boundary tractions P_{ij} are given by (3.22).

4. Conclusions

By the present study non-wood cellulosic materials have been proposed for pulp production. The increasing demands for paper and environmental concerns have increased the need for non-wood pulp as a low-cost raw material for papermaking. This has also led to the developing of alternative pulping technologies that are environmentally benign. Annual plants and agricultural residues appear to be well suited for papermaking due to them being an abundant and renewable.

Besides, by the current research plants like kenaf (Hibiscus cannabinus L.) and giant reed (Arundo donax L.) are proposed as internodes gave very good derived values, especially slenderness ratio, which is directly comparable to some softwood and most hardwood species. Besides, chemical analysis of the raw plant materials revealed satisfactory levels of α-cellulose content (close to 40%) and Klason lignin content (<30%) compared to those of hardwoods and softwoods.

Additionally, a mathematical model has been presented as an attempt to determine the properties non-wood cellulosic fibers. The above mentioned problem was reduced to the solution of a singular integral equation, which was numerically solved by using the Singular Integral Operators Method.

Such singular integral equation method will be of increasing interest in future, as these methods are very important for the solution of generalized solid mechanics and fluid mechanics problems. Modern problems of fluid and solid mechanics are much more simplified when solved by general singular integral equation methods.

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