Universal Journal of Structural Analysis **2 (2014), 36-51**
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Universal Mechanics for Non-linear Frames

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Abstract
The theory of "*Relativistic Elasticity"* for moving (nonlinear) frames is introduced and investigated by combining the theories of elasticity and special relativity. Furthermore, the theory of *"Relativistic Thermo-Elasticity"* for moving (nonlinear) frames is investigated by combining the theories of thermo-elasticity and special relativity. So, the proposed theory of *"Universal Mechanics"* consists of the combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*. Consequently, the theory of special relativity, known mainly for its theoretical aspects, leads to more applicable forms, by investigating the thermo-elastic stress behavior for moving structures with velocities which can approximate nearly the speed of light. Therefore, we shall show that for small velocities 50,000 km/h to 200,000 km/h the relative and the absolute stress tensors are nearly the same, while for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light), the relative stress tensor differs from the absolute one. Finally, for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The combination of the theories of elasticity and special relativity results to the ***Universal Equation of Elasticity*** for moving frames, while the combination of the theories of thermo-elasticity and special relativity results to the ***Universal Equation of Thermo-Elasticity.*** Hence, the *"Universal Equation of Elasticity"*, and the *"Universal Equation of Thermo-Elasticity"* are parts of the general theory of *"Universal Mechanics"*.

Key Word and Phrases
Relativistic Elasticity, Relativistic Thermo-Elasticity, Relativistic Thermoelastic Stress Analysis, Relative Stress Tensor, Absolute Stress Tensor, Stationary and Moving (nonlinear) Frames, Energy-Momentum Tensor, Universal Mechanics, Universal Equation of Elasticity, Universal Equation of Thermo-Elasticity.

1. Universal Mechanics

The mechanical foundations of the theory of thermo-elastic stress analysis for stationary frames began to be analyzed in the early nineteenth century and were further developed during the twentieth century [1] - [33]. Also, the modern resources for computer calculations have brought within the realm of the practical the means of solving integral or differential equations and of making available a great number of useful solutions which were previously considered to be unmanageable. Past attempts to obtain solutions for several thermo-elasticity problems for stationary systems have led to a large variety of methods of solution, like singular integral equation methods, finite elements, boundary elements, integral transformation methods, etc.

In addition, the previous methods dealing with the thermo-elastic stress behavior for stationary structures, shall be extended by the present study to the distribution of the thermo-elastic stress behavior for moving structures. Hence, a combination of the theories of thermoelasticity and special relativity is presented, which leads to the investigation of the relativistic thermo-elastic stress analysis for moving (non-linear) frames.

During the past years special attention has been concentrated on the theoretical aspects of the special theory of relativity. Hence, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity [34] – [43].

On the other hand, by the current research the combination of the theories of thermoelasticity and special relativity leads to some applicable relativistic forms. Therefore, the stress field is investigated for moving structures under a big level of velocities. This level begins from very small velocities up to the approximation of the speed of light.

Consequently, we shall show that for small velocities 50,000 km/h to 200,000 km/h, the absolute and the relative stress tensors are nearly the same. Also, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is much different from the absolute one, while for velocities near the speed of light the values of the relative stress tensor are very bigger than the corresponding values of the absolute stress tensor. For future aerospace applications the difference between the relative and the absolute stress tensors would be of very big interest.

Also, a very important point which will be shown in the present research is that the relative stress tensor is not symmetrical, while, as it is well known, the absolute stress tensor is symmetrical. This difference is of big interest for future applications of aerospace structures.

So, the foundations of the new theory of relativistic thermo-elasticity for non-linear structures lead to a general theory, in which no restriction is made with regard to the relative motion. This general theory is further reduced to one class of relative motion, uniform in direction and velocity.

The combination of the theories of elasticity and special relativity results to the ***Universal Equation of Elasticity*** for moving frames, while the combination of the theories of thermo-elasticity and special relativity results to the ***Universal Equation of Thermo-Elasticity.*** Hence, the *"Universal Equation of Elasticity"*, and the *"Universal Equation of Thermo-Elasticity"* are parts of the general theory of *"Universal Mechanics"*.

2. Relativistic Elastic Stress Analysis for Non-linear Frames

Theorem 2.1

Consider an infinitesimal face element $\mathrm{d}s$ with a directed normal, defined by a unit vector $\mathbf{n}$, at definite point p in the three-space of a Lorenz system. The solid on either side of this face element experiences a force defined by:

$$\mathrm{d}\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}s \tag{2.1}$$

in which the components $\sigma_i(\mathbf{n})$ of the force $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components n_k of $\mathbf{n}$:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k\,, \quad i,k = 1,2,3 \tag{2.2}$$

where σ_{ik} is the relative stress tensor of the moving system S, in contrast to the space part of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor σ_{ik}^0 of the stationary system S^0 (Figs 1 and 2).

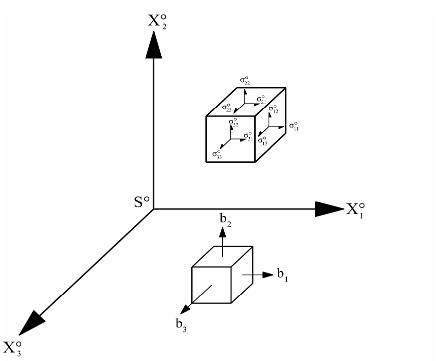

Fig. 1 The state of stress σ_{ik}^0 in the stationary system S^0

Then, the connection between the absolute σ_{ik}^{0} and relative σ_{ik} stress tensors is given by the relation:

$$\sigma_{ik}^{0} = \sigma_{ik} + g_i u_k , \quad i,k = 1,2,3 \tag{2.3}$$

where g_i denotes the momentum density and u_k the components of the velocity $\mathbf{u}$ of the solid in the moving system S at the place and time considered.

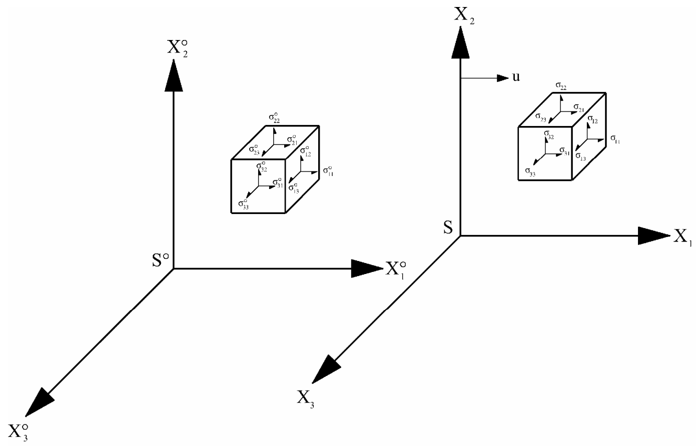

Fig. 2 The state of stress σ_{ik}^{0} in the stationary system S^0 and σ_{ik} in the moving system with velocity u parallel to the x_1 - axis.

Proof.
The total elastic force $\mathbf{F}$ acting on the solid inside a closed surface s is valid as:

$$\int_s \boldsymbol{\sigma}(\mathbf{n}) ds = \int_s \mathrm{d}\boldsymbol{\sigma}(\mathbf{n}) \tag{2.4}$$

in which $\mathbf{n}$ is the inward normal to the surface element ds. By using Gauss's theorem and eqn (2.2), then the components F_i of this force are equal to:

$$F_i = \int_s \sigma_{ik} n_k \,\mathrm{d}s = -\int_V \frac{\partial \sigma_{ik}}{\partial x_k} \mathrm{d}V \tag{2.5}$$

where V denotes a domain in three-dimensional space bounded by the closed surface s.

Consequently, consider an elastic force density $\mathbf{f}$ such that:

$$F_i = \int_V f_i \mathrm{d}V \tag{2.6}$$

Then, by using (2.5) and (2.6) we obtain the following relation between the relative stress tensor σ_{ik} and the elastic force density f_i:

$$f_i = -\frac{\partial \sigma_{ik}}{\partial x_k} \tag{2.7}$$

Beyond the above, the motion of an infinitesimal piece of matter with the volume δV is determined by the following equations of motion:

$$\frac{\mathrm{d}}{\mathrm{d}t}(g_i \delta V) = f_i \delta V = -\frac{\partial \sigma_{ik}}{\partial x_k}\delta V \tag{2.8}$$

where $\mathbf{g}$ denotes the momentum density and d/dt the substantial time derivative.

Consequently, eqn (2.8) may be written as following:

$$\frac{\mathrm{d}g_i}{\mathrm{d}t}\delta V + g_i \frac{\mathrm{d}}{\mathrm{d}t}\delta V = (\frac{\partial g_i}{\partial t} + \frac{\partial g_i}{\partial x_k} u_k)\delta V + g_i \delta V \frac{\partial u_k}{\partial x_k} \tag{2.9}$$

which is finally equal to:

$$(\frac{\partial g_i}{\partial t} + \frac{\partial_i (g_i u_k)}{\partial x_k} u_k)\delta V \tag{2.10}$$

So, by combining (2.8), (2.9) and (2.10) one obtains:

$$\frac{\partial g_i}{\partial t} + \frac{\partial}{\partial x_k}(g_i u_k + \sigma_{ik}) = 0 \tag{2.11}$$

In addition, consider the law of conservation of energy and momentum:

$$\frac{\partial T_{ik}}{\partial x_k} = 0 \tag{2.12}$$

where T_{ik} denotes the total energy momentum tensor.

Hence, eqn (2.12) for i, k = 1,2,3 can be also written as:

$$\frac{\partial \sigma^o_{ik}}{\partial x_k} + \frac{\partial g_i}{\partial t} = 0 \tag{2.13}$$

and thus, from (2.12) and (2.13) follows the required relation (2.3).

Theorem 2.2
Consider the momentum density $\mathbf{g}$ defined by the relation (2.3) for the moving frame shown in Figure 1. Then, this density is given by the formula:

$$\mathbf{g} = m\mathbf{u} + \frac{(\mathbf{u}, \boldsymbol{\sigma})}{c^2} \tag{2.14}$$

where m is the total mass density:

$$m = E / c^2 \tag{2.15}$$

with E the total energy density, including the elastic energy, $\boldsymbol{\sigma}$ the relative stress tensor, **u** the velocity of the solid at the place and time considered and c the speed of light.

Proof.
The connection between the momentum density **g** and the energy flux **D** is given by the relation:

$$\mathbf{g} = \mathbf{D}/c^2 \tag{2.16}$$

in which c denotes the speed of light.

Furthermore, the total work per unit time done by the elastic forces on the solid, inside a closed surface s is valid as:

$$W = \int_s (\boldsymbol{\sigma}(\mathbf{n}), \mathbf{u}) \mathrm{d}s = \int_s \sigma_{ik} n_k u_i \, \mathrm{d}s = -\int_V \frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \mathrm{d}V \,, \quad i,k = 1,2,3 \tag{2.17}$$

where **n** denotes the inward normal to the surface element *ds* and the integration in the last integral is extended over the interior V of the surface s.

Thus, from eqn (2.17) we obtain the work done on an infinitesimal piece of matter of volume δW :

$$\delta W = -\frac{\partial (u_i \sigma_{ik})}{\partial x_k} \delta V \tag{2.18}$$

Moreover, eqn (2.18) must be equal to the increase per unit time of the energy inside δV:

$$\frac{\mathrm{d}}{\mathrm{d}t}(E \delta V) = \delta W \tag{2.19}$$

in which E denotes the total energy density, including the elastic energy. Eqn (2.9) can be further written as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(E \delta V) = \left(\frac{\partial E}{\partial t} + \frac{\partial E}{\partial x_k} u_k \right) \delta V + E \delta V \frac{\partial u_k}{\partial x_k} \tag{2.20}$$

which is equal to:

$$\left[\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_k}(E u_k) \right] \delta V \tag{2.21}$$

and finally one obtains:

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x_k}(Eu_k+u_i\sigma_{ik})=0 \tag{2.22}$$

In addition, the general continuity equation for energy has the form:

$$div\mathbf{D}+\frac{\partial E}{\partial t}=0 \tag{2.23}$$

So, by combining (2.22) and (2.23) follows:

$$\mathbf{D}=E\mathbf{u}+(\mathbf{u},\boldsymbol{\sigma}) \tag{2.24}$$

where $(\mathbf{u},\boldsymbol{\sigma})$ is the inner product with components $(\mathbf{u},\boldsymbol{\sigma})_k=u_i\sigma_{ik}$ and therefore, from (2.16) and (2.24) follows the required (2.14).

Corollary 2.1
The momentum density vector **g** does not in general have the same direction as the motion of the matter:

$$g_iu_k\neq g_ku_i \tag{2.25}$$

in which u_i and u_k are the components of the velocity $\mathbf{u}$ of the solid at the place and time considered.

Proof
The proof of the present Corollary follows directly from (2.14).

Corollary 2.2
The relative stress tensor σ_{ik} is not symmetrical.

Proof
The law of conservation of angular momentum requires the absolute stress tensor σ_{ik}^o to be symmetrical:

$$\sigma_{ik}^o=\sigma_{ki}^o \tag{2.26}$$

Thus, from (2.3) , (2.14) and (2.26) follows:

$$\sigma_{ik}-\sigma_{ki}=-g_iu_k+g_ku_i=[-(\mathbf{u},\boldsymbol{\sigma})_iu_k+(\mathbf{u},\boldsymbol{\sigma})_ku_i]/c^2\neq 0 \tag{2.27}$$

and therefore the relative stress tensor σ_{ik} is not symmetrical.

Corollary 2.3
In the stationary system S^O the following formulas are valid:

$$\sigma_{ik}^0 = \sigma_{ik} = \sigma_{ki} = \sigma_{ki}^0 \ , \ D_i^0 = g_i^0 = \sigma_{i4} = 0, \ \sigma_{44} = -E^0 \tag{2.28}$$

Proof
As in the stationary system S^O (Fig. 1) we have $\mathbf{u}^0 = 0$, then directly from (2.3), (2.14) and (2.24) follows the required (2.28).

Theorem 2.3
The mechanical energy-momentum tensor T_{ik} is the sum of a kinetik part K_{ik} and a potential part P_{ik}:

$$T_{ik} = K_{ik} + P_{ik} \tag{2.29}$$

in which:

$$K_{ik} = E^0 U_i U_k / c^2 = m^0 U_i U_k \tag{2.30}$$

where U_i denotes the four-velocity of the matter and the potential part P_{ik} is determined completely by the relative stress tensor σ_{ik}, and will be called the stress four-tensor.

Proof
The mechanical energy-momentum tensor T_{ik} satisfies the following relation:

$$T_{ik} U_k = -E^0 U_i \tag{2.31}$$

in which U_i is the four-velocity of the solid and E^0 the energy density in the stationary system. Eqn (2.31) is easily proved in the stationary system by using (2.28) and putting $U_i^0 = (0,0,0,ic)$.

Beyond the above, the following relations are valid:

$$U_i T_{ik} U_k \big/ c^2 = U_i^0 T_{ik}^0 U_k^0 \big/ c^2 = -T_{44}^0 = E^0(x_1) \tag{2.32}$$

and by introducing the tensor:

$$\Delta_{ik} = \delta_{ik} + U_i U_k \big/ c^2 \tag{2.33}$$

which satisfies the relations:

$$U_i \Delta_{ik} = \Delta_{ik} U_k = 0 \tag{2.34}$$

then, we can form the following symmetrical tensor:

$$P_{ik} = \Delta_{in} T_{nm} \Delta_{mk} = P_{ki} \tag{2.35}$$

which is orthogonal to U_i:

$$U_i P_{ik} = P_{ik} U_k = 0 \tag{2.36}$$

Hence, by combining (2.31), (2.32), (2.33) and (2.35) we obtain:

$$P_{ik} = T_{ik} - E^0 U_i U_k / c^2 \tag{2.37}$$

Furthermore, in the stationary system the following relations are satisfied:

$$P_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik}, \quad P_{i4}^0 = P_{4i}^0 = 0 \tag{2.38}$$

and (2.37) can be also written as (2.29).

Theorem 2.4
The connection between the relative $\boldsymbol{\sigma}$ and the absolute $\boldsymbol{\sigma}^0$ stress tensors is given by the formula:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u} \bullet (\boldsymbol{\sigma}^0, \mathbf{u}) \frac{(\gamma - 1)}{u^2} - (\boldsymbol{\sigma}^0, \mathbf{u}) \bullet \mathbf{u} \frac{(\gamma - 1)}{\gamma u^2} - (\mathbf{u} \bullet \mathbf{u})(\mathbf{u}, \boldsymbol{\sigma}^0, \mathbf{u}) \frac{(\gamma - 1)^2}{\gamma u^4} \tag{2.39}$$

where: $$\gamma = 1/(1 - u^2 / c^2)^{1/2} \tag{2.40}$$

and $\mathbf{u}$ is the velocity of the solid at the place and time considered, c the speed of light and the notation $\mathbf{a} \bullet \mathbf{b}$ denotes the direct product of the spatial vectors **a** and **b**.

Proof
In the stationary system the following tensor denotes a real symmetrical matrix:

$$P_{ik}^0 = \sigma_{ik}^0 = \sigma_{ik} \tag{2.41}$$

So, the normalized eigenvectors $\mathbf{d}^{(j)0}$, j = 1,2,3 satisfy the orthonormality and completeness relations:

$$(\mathbf{d}^{(j)0}, \mathbf{d}^{(l)0}) = \delta_{jl} \tag{2.42a}$$

and:

$$\mathbf{d}_i^{(j)0}\mathbf{d}_k^{(j)0} = \delta_{ik} \quad (j,l = 1,2,3) \tag{2.42b}$$

Beyond the above, consider the principal stresses $p_{(j)}^0$ which are the three roots of the following equation:

$$\left|P_{ik}^0 - \xi\delta_{ik}\right| = \left|\sigma_{ik}^0 - \xi\delta_{ik}\right| = 0 \tag{2.43}$$

in which ξ is the unknown.

The matrix P_{ik}^0 may be further written in terms of the eigenvectors $\mathbf{d}^{(j)0}$ and eigenvalues $p_{(j)}^0$ as follows:

$$P_{ik}^0 = p_{(j)}^0 d_i^{(j)0} d_k^{(j)0} \tag{2.44}$$

Thus, in any system S we have:

$$P_{ik} = p_{(j)}^0 d_i^{(j)} d_k^{(j)} \tag{2.45}$$

and:

$$\sigma_{ik} = P_{ik} - P_{i4}U_k / U_4 \tag{2.46}$$

By using (2.29), (2.30), (2.45) and (2.46), then we obtain:

$$P_{ik} = m^0 U_i U_k + p_{(j)}^0 d_i^{(j)} d_k^{(j)} \tag{2.47}$$

and:

$$\sigma_{ik} = P_{ik} - P_{i4}U_k / U_4 = p_{(j)}^0 d_i^{(j)} \left(d_k^{(j)} + i d_4^{(j)} u_k / c\right) \tag{2.48}$$

Consequently, by using the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, then (2.48) can be further written as:

$$\boldsymbol{\sigma} = p_{(j)}^0 \left[(\mathbf{d}^{(j)} \bullet \mathbf{d}^{(j)}) + \frac{i}{c} d_4^{(j)} (\mathbf{d}^{(j)} \bullet \mathbf{u}) \right], \quad j = 1,2,3 \tag{2.49}$$

where the vectors $\mathbf{d}^{(j)}$ satisfy the tensor relations:

$$d_i^{(j)} d_i^{(l)} = \delta_{jl} \tag{2.50}$$

and:

$$d_i^{(j)} d_k^{(j)} = \Delta_{ik} \tag{2.51}$$

where Δ_{ik} is given by (2.33).

In addition, if the spatial axes of the stationary and the moving systems have the same orientation, then one has:

$$\begin{aligned} \mathbf{d}^{(j)} &= \mathbf{d}^{(j)0} + \mathbf{u}(\mathbf{u}, \mathbf{d}^{(j)0})(\gamma - 1)/u^2 \\ d_4^{(j)} &= i(\mathbf{u}, \mathbf{d}^{(j)0})\gamma/c \end{aligned} \tag{2.52}$$

For the special case $i = k = 4$, we obtain from (2.47) and (2.52):

$$d = -P_{44} = -m^0 U_4^2 - p_{(j)}^0 (\mathbf{u}, \mathbf{d}^{(j)0})^2 \gamma^2 / c^2 \tag{2.53}$$

Furthermore, in the stationary system, (2.49) reduces to:

$$\boldsymbol{\sigma}^0 = p_{(j)}^0 \left(\mathbf{d}^{(j)0} \bullet \mathbf{d}^{(j)0} \right) \tag{2.54}$$

So, from eqn. (2.53) we obtain the following transformation law for the energy density:

$$d = \frac{d^0 + (\mathbf{u}, \boldsymbol{\sigma}^0, \mathbf{u}/c^2)}{1 - u^2/c^2} \tag{2.55}$$

$$(\mathbf{u}, \boldsymbol{\sigma}^0, \mathbf{u}) = u_i \sigma_{ik}^0 u_k \tag{2.56}$$

and for the mass density:

$$m = \frac{m^0 + (\mathbf{u}, \boldsymbol{\sigma}^0, \mathbf{u}/c^2)}{1 - u^2/c^2} \tag{2.57}$$

Thus, from (2.47) and (2.52) for $k = 4$ we obtain the following relation for the momentum density **g:**

$$\begin{aligned} \mathbf{g} &= \mathbf{u}\left[d^0 + (\mathbf{u}, \boldsymbol{\sigma}^0, \mathbf{u})(1 - \gamma^{-1})/u^2\right]\gamma^2/c^2 + (\boldsymbol{\sigma}^0, \mathbf{u})\gamma/c^2 \\ (\boldsymbol{\sigma}^0, \mathbf{u})_i &= \sigma_{ik}^0 u_k \end{aligned} \tag{2.58}$$

Finally, by substituting (2.52) in (2.48) we obtain the required relation (2.39).

Corollary 2.4
Consider the special case, where the motion of the matter at the point considered is parallel to the x_1-axis, i.e. $\mathbf{u} = (u,0,0)$ (Figure 2). Then the connection between the absolute $\boldsymbol{\sigma}$ and the relative $\boldsymbol{\sigma}^0$ stress tensor is given by the ***Universal Equation of Elasticity***:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^0 & \gamma\sigma_{12}^0 & \gamma\sigma_{13}^0 \\ \frac{1}{\gamma}\sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \frac{1}{\gamma}\sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.59}$$

where γ is given by (2.40).

Proof
The required formula (2.59) follows immediately from (2.39) by putting $\mathbf{u} = (u,0,0)$.

3. Relativistic Thermo-Elastic Stress Analysis for Non-linear Frames

In the previous paragraph it has been regarded our system, the elastic body, as a purely mechanical system. However, all macroscopic systems are in reality thermo-dynamical systems with properties depending on non-mechanical variables such as the proper temperature T^o, and the question which arises is to what kind of thermodynamical processes may be described by an energy-momentum tensor.

Consequently, it is clear that all properties in which heat energy is transferred from one part of the system to another are excluded, for heat flow in the manner would give rise to a non-vanishing energy current in the rest system.

Theorem 3.1
Consider a general system of continuously distributed ponderable or visible matter, inside which invisible heat conduction can take place, while the motion of the visible matter is described by the four-velocity U_i. Then the energy-momentum tensor of the general system is given by the relation:

$$T_{ik} = M_{ik} + H_{ik} \tag{3.1}$$

in which M_{ik} is the mechanical part of the energy-momentum tensor and H_{ik} the heat part.

Furthermore, the mechanical part M_{ik} is valid as following:

$$M_{ik} = d^0 U_i U_k / c^2 + P_{ik} \tag{3.2}$$

and the heat part:

$$H_{ik} = \left(U_i V_k + V_i U_k\right) / c^2 \tag{3.3}$$

where the four-vector V_i satisfies the relation:

$$V_i = -\Delta_{ik} T_{kj} U_j = -T_{ik} U_k - d^0 U_i \tag{3.4}$$

in which d^0 denote the normalized eigenvectors, Δ_{ik} is the tensor given by (2.33) and P_{ik} the potential part of the energy momentum tensor.

Proof. The four-vector V_i is orthogonal to U_i:

$$U_i V_i = 0 \tag{3.5}$$

and so, one obtains:

$$V_i = \left(\mathbf{V}, i\left(\mathbf{V}, \mathbf{u}\right)/c\right) \tag{3.6}$$

where $\mathbf{u}$ denotes the velocity of the matter.

Thus, in the stationary system, (3.6) reduces to:

$$V_i^0 = \left(\mathbf{V}^0, 0\right) \tag{3.7}$$

Moreover, by replacing (2.33) into (2.35) and using (2.32) and (3.4), then we obtain instead of (2.37):

$$P_{ik} = T_{ik} - d^0 U_i U_k / c^2 - \left(U_i V_k + V_i U_k\right) / c^2 \tag{3.8}$$

Consequently, from (3.8) follows the required relation (3.1), instead of (2.29).

Theorem 3.2
Consider the general system of continuously matter described in the previous theorem, inside which invisible heat conduction can take place, while the motion of the matter is described by the four-velocity U_i or by the velocity u_i.

Then, the connection between the energy-momentum tensor T_{ik} and the relative stress tensor σ_{ik} of the general system is given by the relation:

$$T_{ik} = g_i u_k + \sigma_{ik} + u_i \, \xi_k / c^2 \tag{3.9}$$

with:

$$\xi_k = U_4 \left(V_k - V_4 U_k / U_4\right) / ic \tag{3.10}$$

in which V_k denotes the four-vector given by (3.4), g_i the momentum density and c the speed of light.

Proof. The quantity ξ_k seems to be the most important part of ξ_{ik} :

$$\xi_{ik} = H_{ik} - H_{i4} U_k / U_4 = U_i \left(V_k - V_4 U_k / U_4\right) / c^2 \tag{3.11}$$

Moreover, ξ_k can be written by the following form by using (2.40) and (3.6):

$$\xi_k = (\boldsymbol{\xi}, 0) \tag{3.12}$$

with:

$$\boldsymbol{\xi} = \gamma\left[\mathbf{V} - \mathbf{u}(\mathbf{V}, \mathbf{u}) / c^2\right] \tag{3.13}$$

Beyond the above, in the stationary system, $\boldsymbol{\xi}^0$ is equal to the heat current density $\mathbf{V}^0$:

$$\boldsymbol{\xi}^0 = \mathbf{V}^0 \tag{3.14}$$

By combining (3.10) and (3.11), then one obtains:

$$\xi_{ik} = U_i \, \xi_k / \gamma c^2 \tag{3.15}$$

So, by using (2.48), (3.1), (3.2), (3.11) and (3.15), one has:

$$T_{ik} - T_{i4} U_k / U_4 = \sigma_{ik} + \xi_{ik} = \sigma_{ik} + U_i \, \xi_k / \gamma c^2 \tag{3.16}$$

which finally reduces to the required formula (3.9).

Lemma 3.1
Consider the general system of continuously matter, inside which invisible heat conduction can take place. Then the momentum density $\mathbf{g}$ of this system is given by the ***Universal Equation of***

Thermo-Elasticity:

$$\mathbf{g} = m\mathbf{u} + \frac{(\mathbf{u}, \boldsymbol{\sigma})}{c^2} + \frac{\boldsymbol{\xi}}{c^2} \tag{3.17}$$

where $\mathbf{u}$ denotes the velocity of the matter at the place and time considered, $\boldsymbol{\sigma}$ the relative stress tensor, $\boldsymbol{\xi}$ is given by (3.13) and m is the total mass density given by (2.15).

Proof. From (3.9), we obtain for the energy current density:

$$D_k = Eu_k + u_i \sigma_{ik} + \xi_k \tag{3.18}$$

which can be further written as:

$$\mathbf{D} = E\mathbf{u} + (\mathbf{u}, \boldsymbol{\sigma}) + \boldsymbol{\xi} \tag{3.19}$$

Consequently, from (3.19) by using the formula of the momentum density $\mathbf{g}$:

$$\mathbf{g} = \mathbf{D}/c^2 \tag{3.20}$$

one obtains the required relation (3.17) which is a generalization of (2.14), for a general system with heat conduction.

4. Conclusions

A combination of the theories of thermo-elasticity and special relativity has been introduced, in order to investigate the thermo-elastic stress behavior for non-linear (moving) structures. So, the theory of relativity mainly known for its theoretical aspects, was directed to more applicable forms. The following Table 1 shows some characteristic values of the velocity for the moving structure in connection with the corresponding values of γ.

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	c	∞

Consequently, from the above Table follows that for small velocities 50,000 km/h to 200,000 km/h, $\gamma = 1$ and therefore the absolute and the relative stress tensors are nearly the same. On the other hand, for bigger velocities 50,000 km/h to $10^8\ km/h$ these two kinds of stress tensors begin to become different, while for velocities $c/3$, $c/2$ or $3c/4$ (c = speed of light), the variable γ takes bigger values and thus, the relative stress tensor is different than the absolute one. Finally, for values of the velocity for the non-linear (moving) structure near the speed of light, the variable γ takes much bigger values, and at the end when the velocity becomes equal to the speed of light, then γ tends to infinity.

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