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Non-linear Structural Analysis for Future Spacecraft with no Speed Limits

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Abstract
For the design of the future spacecraft with no speed limits, as its speed could approach the speed of light, the new theory of ***"Universal Mechanics"*** is introduced and investigated. The proposed theory of *"Universal Mechanics"* consists of the combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*. Such theory is applied for non-linear airframes. Hence, according to the above theories there is a considerable difference between the absolute stress tensor and the stress tensor of the non-linear airframe. For very big speeds of the future spacecraft, like c/3, c/2 or 3c/4 (c=speed of light), then the difference between the two stress tensors is very much increased. Consequently, for the future spacecraft with very high speeds, the relative stress tensor will be therefore very much different than the absolute stress tensor. In addition, for velocities near the speed of light, then the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. The theory of *"Relativistic Elasticity"* is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results in the ***"Universal Equation of Elasticity"***. Moreover, the theory of *"Relativistic Thermo-Elasticity"* is a combination between the theories of "Classical Thermo-Elasticity" and "Special Relativity" and results in the ***"Universal Equation of Thermo-Elasticity"***. So, the *"Universal Equation of Elasticity"*, and the *"Universal Equation of Thermo-Elasticity"* are parts of the general theory of *"Universal Mechanics"*.

Key Word and Phrases
Future Spacecraft, Non-linear Airframe, Relativistic Elasticity, Relativistic Thermo-Elasticity, Relative Stress Tensor, Absolute Stress Tensor, Energy-Momentum Tensor, Universal Mechanics, Universal Equation of Elasticity, Universal Equation of Thermo-Elasticity.

1. Universal Mechanics for Non-linear Airframes

The scope by the International Space Agencies is to achieve in the future, an absolute spacecraft moving with very high speeds, even approaching the speed of light. Such future spacecraft would behave like a non-linear airframe. Hence, how far could be this future ? According to the current investigation and research such future could be much closer than everybody believes. For the future spacecraft the relative stress tensor will be much different than the absolute stress tensor and so special solid should be used for the construction of the next generation spacecraft.

Beyond the above, the suitable choice of the solid which should be used for the construction of the future spacecraft is under investigation, but such solid will be very much different than the usual composite materials.

Consequently, it will be shown that there is a significant difference between the absolute stress tensor and the stress tensor of the airframe even for lower speeds. In addition, for bigger speeds the difference of the two stress tensors will be very much increased. So, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is very much different than the absolute one and for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the non-linear airframe is included in the theory proposed by E.G.Ladopoulos [30] - [32] under the term *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* and the final formula which results from the above theories is called the ***"Universal Equation of Elasticity"*** and the ***"Universal Equation of Thermo-Elasticity"***, correspondingly.

Both theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* are included in a more general theory under the term ***"Universal Mechanics"***.

In addition, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et al. [17]-[22] proposed singular integral equation methods applied to elasticity, plasticity and fracture mechanics theories. In the above mentioned publications the ***Singular Integral Operators Method (S.I.O.M.)*** is proposed for the numerical solution of the multidimensional singular integral equations in which the stress tensor analysis of the linear elastic theory is reduced. Also, the theory of linear singular integral equations was extended to non-linear singular integral equations, too. [23]-[29]. Thus, the theory of *"Universal Mechanics"* and correspondingly the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"* will be applied for the design of the elastic stress analysis of the airframes.

Moreover, the classical theory of elastic stress analysis and thermo-elastic stress analysis began to be analyzed in the early nineteenth century and was further developed during the twentieth century. In the past, several important monographs were published on the classical theory of elasticity and thermo-elasticity. [33]-[52].

Over the past years special attention has been given, by many scientists worldwide, on the theoretical aspects of the special theory of relativity. Consequently, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [53]–[60]. In addition, by the current research and investigation will be shown that the **"*relative stress tensor is not symmetrical*"**, while, as it is well known, the **"*absolute stress tensor is symmetrical*"**. Such a difference is very important for the design of the future spacecraft of very high speeds.

2. Relativistic Elasticity - Universal Equation of Elasticity for Non-linear Airframes

Consider the state of stress at a point in the stationary frame S^0, defined by the following symmetrical stress tensor: (Fig.1)

$$\sigma^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix} \tag{2.1}$$

where: $$\sigma_{21}^0 = \sigma_{12}^0,\, \sigma_{31}^0 = \sigma_{13}^0,\, \sigma_{32}^0 = \sigma_{23}^0 \tag{2.2}$$

In addition, consider an infinitesimal face element df with a directed normal, defined by a unit vector $\mathbf{n}$, at definite point p in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to df.

Hence, the force is valid as:

$$\mathrm{d}\,\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}\,f \tag{2.3}$$

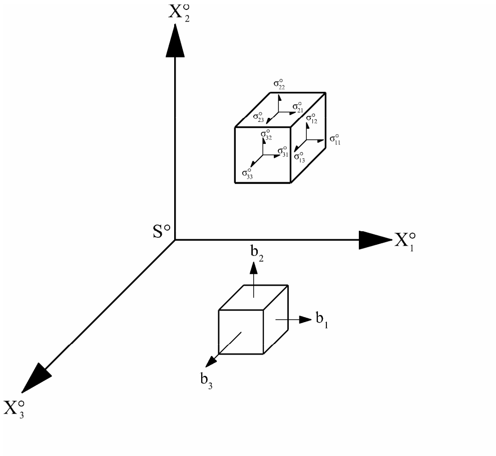

Fig. 1 The state of stress σ_{ik}^0 in the stationary system S^0 .

The components $\sigma_i(\mathbf{n})$ of $\boldsymbol{\sigma}(\mathbf{n})$ are linear functions of the components n_k of $\mathbf{n}$:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k \,, \; i, k = 1,2,3 \tag{2.4}$$

in which σ_{ik} is the elastic stress tensor, also called as the relative stress tensor, in contrast to the space part σ_{ik}^0 of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor. [53], [54} (Fig. 2).

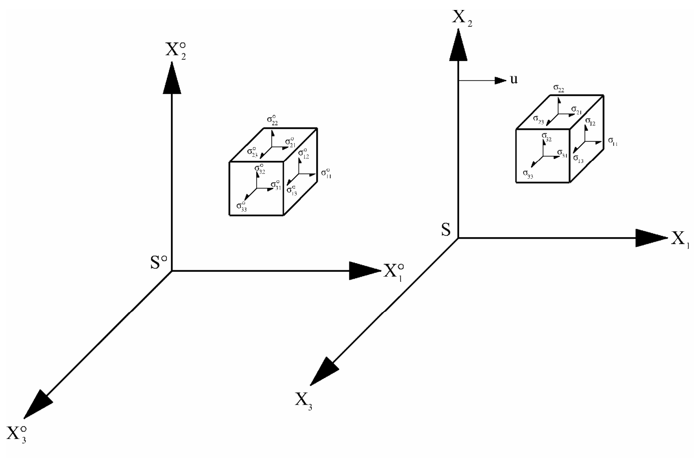

Fig. 2 The state of stress σ_{ik}^0 in the stationary system S^0 and σ_{ik} in the airframe system with velocity u parallel to the x_1 - axis.

Moreover, the connection between the absolute and relative stress tensors is defined as:

$$\sigma_{ik}^0 = \sigma_{ik} + g_i u_k \,, \; i, k = 1,2,3 \tag{2.5}$$

where g_i are the components of the momentum density $\mathbf{g}$ and u_k the components of the velocity $\mathbf{u}$ of the matter.

The connection between $\mathbf{g}$ and the energy flux $\mathbf{s}$, is equal to:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

in which c denotes the speed of light (= 300.000 *km/sec*).

Beyond the above, the total work done per unit time by elastic forces on the matter inside the closed surface f can be given by the formula:

$$W = \int_f (\boldsymbol{\sigma}(\mathbf{n}) \cdot \mathbf{u}) \mathrm{d} f = \int_f \sigma_{ik} n_k u_i \, \mathrm{d} f = -\int_\upsilon \frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \mathrm{d} \upsilon \,, \; i, k = 1,2,3 \tag{2.7}$$

where the integration in the last integral is extended over the interior υ of the surface f.

So, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is valid as:

$$\delta W = -\frac{\vartheta(u_i \sigma_{ik})}{\vartheta x_k} \delta\upsilon \tag{2.8}$$

Furthermore, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d} t}(h\delta\upsilon) = \delta W \tag{2.9}$$

in which h denotes the total energy density, including the elastic energy and $\mathrm{d}/\mathrm{d} t$ is the substantial time derivative.

Eq. (2.9) is valid as:

$$\frac{\mathrm{d}}{\mathrm{d} t}(h\delta\upsilon) = \left(\frac{\vartheta h}{\vartheta t} + \frac{\vartheta h}{\vartheta x_k} u_k \right) \delta\upsilon + h\delta\upsilon \frac{\vartheta u_k}{\vartheta x_k} = \left[\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k} (h u_k) \right] \delta\upsilon \tag{2.10}$$

which finally leads to the relation:

$$\frac{\vartheta h}{\vartheta t} + \frac{\vartheta}{\vartheta x_k} (h u_k + u_i \sigma_{ik}) = 0 \tag{2.11}$$

Hence, the total energy flow is valid as:

$$\mathbf{s} = \mathbf{hu} + (\mathbf{u} \cdot \boldsymbol{\sigma}) \tag{2.12}$$

where $(\mathbf{u} \cdot \boldsymbol{\sigma})$ is a space vector with components $(\mathbf{u} \cdot \boldsymbol{\sigma})_k = u_i \sigma_{ik}$.

So, the total momentum density can be written as:

$$\mathbf{g} = \frac{\mathbf{s}}{c^2} = \mu \mathbf{u} + \frac{(\mathbf{u} \cdot \boldsymbol{\sigma})}{c^2} \tag{2.13}$$

in which $\mu = h/c^2$ denotes the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) one has:

$$\sigma_{ik} - \sigma_{ki} = -g_i u_k + g_k u_i = [-(\mathbf{u} \cdot \boldsymbol{\sigma})_i u_k + (\mathbf{u} \cdot \boldsymbol{\sigma})_k u_i] / c^2 \neq 0 \qquad (2.14)$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame S^0 the velocity $u^0 = 0$ and thus, from (2.5), (2.12) and (2.13) the following expressions are obtained:

$$\sigma_{ik}^0 = \sigma_{ik} = \sigma_{ki} = \sigma_{ki}^0 \ (i, k = 1,2,3) \qquad (2.15)$$

Moreover, the mechanical energy-momentum tensor satisfies the following relation:

$$T_{ik} U_k = -h^0 U_i \qquad (2.16)$$

where U_i is the four-velocity of the matter, in the Lorentz system and $U_i^0 = (0,0,0,ic)$.

Consequently, the following scalar can be formed:

$$U_i T_{ik} U_k / c^2 = U_i^0 T_{ik}^0 U_k^0 / c^2 = -T_{44}^0 = h^0 (x_1) \qquad (2.17)$$

with $h^0 (x_1)$ the invariant rest energy density considered as a scalar function of the coordinates (x_i) (i = 1,2,3) in S. (Fig. 2)

Furthermore, by applying the tensor:

$$\Delta_{ik} = \delta_{ik} + U_i U_k / c^2 \qquad (2.18)$$

which satisfies the relations:

$$U_i \Delta_{ik} = \Delta_{ik} U_k = 0 \qquad (2.19)$$

then, the following symmetrical tensor can be formed:

$$S_{ik} = \Delta_{i1} T_{1m} \Delta_{mk} = S_{ki} \qquad (2.20)$$

which is orthogonal to U_i:

$$U_i S_{ik} = S_{ik} U_k = 0 \qquad (2.21)$$

By combining eqs. (2.16), (2.17) and (2.20) we obtain:

$$S_{ik} = T_{ik} - h^0 U_i U_k / c^2 \qquad (2.22)$$

Also, in the stationary system S_0 we have:

$$S_{ik}^{0} = \sigma_{ik}^{0} = \sigma_{ik}\,,\ S_{i4}^{0} = S_{4i}^{0} = 0 \tag{2.23}$$

Eq. (2.22) may also be written as:

$$T_{ik} = \xi_{ik} + S_{ik} \tag{2.24}$$

in which:

$$\xi_{ik} = h^{0} U_i U_k \big/ c^{2} = \mu^{0} U_i U_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^{0} = h^{0} \big/ c^{2} \tag{2.26}$$

is the proper mass density.

We introduce further in every system S the quantity:

$$\sigma_{ik} = S_{ik} - S_{i4} U_k \big/ U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is valid as:

$$\sigma_{ik} = T_{ik} - T_{i4} U_k \big/ U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S_{ik}^{0} = \sigma_{ik}^{0} = \sigma_{ik}$$

in the stationary system is a real symmetrical matrix. The corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p_{(j)}^{0}$, the principal stresses, are the three roots of the following algebraic equation, where λ is the unknown:

$$\left| S_{ik}^{0} - \lambda \delta_{ik} \right| = \left| \sigma_{ik}^{0} - \lambda \delta_{ik} \right| = 0 \tag{2.30}$$

The matrix S_{ik}^{0} can be further written in terms of the eigenvalues and eigenvectors as:

$$S_{ik}^{0} = \sigma_{ik}^{0} = p_{(j)}^{0} h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

Then, from eqs. (2.23) and (2.31) we obtain the following form of the stress four-tensor in S^{o}:

$$S_{ik}^{0} = p_{(j)}^{0} h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

So, in any system S we have:

$$S_{ik} = p_{(j)}^{0} h_i^{(j)} h_k^{(j)} \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) follow the expressions:

$$T_{ik} = \mu^0 U_i U_k + p_{(j)}^{0} h_i^{(j)} h_k^{(j)} \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p_{(j)}^{0} h_k^{(j)} \left(h_k^{(j)} + i h_4^{(j)} u_k / c \right) \tag{2.35}$$

By putting:

$$h_i^{(j)} = (\mathbf{h}^{(j)}, h_4^{(j)}) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors $\mathbf{a}$ and $\mathbf{b}$, then eqn (2.35) can be written for the relative stress tensor $\boldsymbol{\sigma}$ as following:

$$\boldsymbol{\sigma} = p_{(j)}^{0} \left[\mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h_4^{(j)} (\mathbf{h}^{(j)} \bullet \mathbf{u}) \right], \; j = 1,2,3 \tag{2.37}$$

Beyond the above, the triad vectors $h_i^{(j)}$ satisfy the tensor relations:

$$h_i^{(j)} h_i^{(\rho)} = \delta^{j\rho} \tag{2.38}$$

$$h_i^{(j)} h_k^{(j)} = \Delta_{ik} \tag{2.39}$$

with Δ_{ik} given by (2.18).

If the stationary system S^0 for every event point is chosen in such a way that the spatial axes in S^0 and in S have the same orientation, we have:

$$\begin{aligned} \mathbf{h}^{(j)} &= \mathbf{h}^{(j)0} + \left\{ \mathbf{u}(\mathbf{u} \cdot \mathbf{h}^{(j)0})(\gamma - 1) \right\} / u^2 \\ h_4^{(j)} &= i \mathbf{u} \cdot \mathbf{h}^{(j)0} \, \gamma / c \end{aligned} \tag{2.40}$$

with:

$$\gamma = 1 / (1 - u^2 / c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ follows:

$$h = -T_{44} = -\mu^0 U_4^2 - p_{(j)}^{0} (\mathbf{u} \cdot \mathbf{h}^{(j)0})^2 \cdot \gamma^2 / c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces to:

$$\boldsymbol{\sigma}^0 = p_{(j)}^{0} \left(\mathbf{h}^{(j)0} \bullet \mathbf{h}^{(j)0} \right) \tag{2.43}$$

Hence, from (2.42) we have the following transformation law for the energy density:

$$h = \frac{h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^2}{1-u^2/c^2}$$
$$\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u} = u_i\sigma^0_{ik}u_k \tag{2.44}$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}/c^4}{1-u^2/c^2} \tag{2.45}$$

From (2.40) and (2.34) with $k = 4$, one obtains the momentum density $\mathbf{g}$ with the components $g_i = T_{i4}/ic$:

$$\mathbf{g} = \mathbf{u}\left[h^0 + \mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u}(1-\gamma^{-1})/u^2\right]\gamma^2/c^2 + (\boldsymbol{\sigma}^0\cdot\mathbf{u})\,\gamma/c^2$$
$$(\boldsymbol{\sigma}^0\cdot\mathbf{u})_i = \sigma^0_{ik}u_k \tag{2.46}$$

In addition, from (2.40) and (2.35) one has the relative stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u}\bullet(\boldsymbol{\sigma}^0\cdot\mathbf{u})(\gamma-1)/u^2 - (\boldsymbol{\sigma}^0\cdot\mathbf{u})\bullet\mathbf{u}(\gamma-1)/\gamma u^2$$
$$-(\mathbf{u}\bullet\mathbf{u})(\mathbf{u}\cdot\boldsymbol{\sigma}^0\cdot\mathbf{u})\,(\gamma-1)^2/\gamma u^4 \tag{2.47}$$

In the special case $\mathbf{u} = (u,0,0)$, where the notation of the matter at the point considered is parallel to the x_1-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$h = \left(h^0 + \frac{u^2}{c^2}\sigma^0_{11}\right)\gamma^2$$
$$g_{x_1} = \gamma^2\left(\mu^0 + \frac{\sigma^0_{11}}{c^2}\right)u$$
$$g_{x_2} = \frac{\gamma\sigma^0_{21}}{c^2}u \tag{2.48}$$
$$g_{x_3} = \frac{\gamma\sigma^0_{31}}{c^2}u$$

and the relative stress tensor gives the ***Universal Equation of Elasticity***:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma^0_{11} & \gamma\sigma^0_{12} & \gamma\sigma^0_{13} \\ \frac{1}{\gamma}\sigma^0_{21} & \sigma^0_{22} & \sigma^0_{23} \\ \frac{1}{\gamma}\sigma^0_{31} & \sigma^0_{32} & \sigma^0_{33} \end{bmatrix} \tag{2.49}$$

where γ is given by (2.41).

Finally, as it could be easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

3. Theory of Relativistic Thermo-Elasticity - Universal Equation of Thermo-Elasticity for Non-linear Airframes

In the previous section the system under investigation, which is the elastic body, was regarded as a purely mechanical system. However, all macroscopic systems are in reality thermo-dynamical systems with properties depending on non-mechanical variables such as the proper temperature T^o, and so the question which arises is to what kind of thermodynamical processes may be described by an energy-momentum tensor.

Consequently, it is clear that all properties in which heat energy is transferred from one part of the system to another are excluded, for heat flow in the manner would give rise to a non-vanishing energy current in the rest system.

Consider further a general system of continuously distributed ponderable or visible matter, inside which invisible heat conduction can take place, while the motion of the visible matter is described by the four-velocity U_i. Then the energy-momentum tensor of the general system can be given by the following relation:

$$T_{ik} = M_{ik} + H_{ik} \tag{3.1}$$

where M_{ik} denotes the mechanical part of the energy-momentum tensor and H_{ik} the heat part.

Furthermore, the mechanical part M_{ik} is valid by the following formula:

$$M_{ik} = d^0 U_i U_k / c^2 + S_{ik} \tag{3.2}$$

and the heat part:

$$H_{ik} = \left(U_i V_k + V_i U_k\right) / c^2 \tag{3.3}$$

where the four-vector V_i satisfies the relation:

$$V_i = -\Delta_{ik} T_{kj} U_j = -T_{ik} U_k - d^0 U_i \tag{3.4}$$

in which d^0 denote the normalized eigenvectors, Δ_{ik} is the tensor given by (2.18) and P_{ik} the potential part of the energy momentum tensor.

The four-vector V_i is orthogonal to U_i:

$$U_i V_i = 0 \tag{3.5}$$

and so one obtains:

$$V_i = \left(\mathbf{V}, i\left(\mathbf{V}, \mathbf{u}\right)/c\right) \tag{3.6}$$

where $\mathbf{u}$ denotes the velocity of the matter.

So, in the stationary system, (3.6) reduces to:

$$V_i^0 = \left(\mathbf{V}^0, 0\right) \tag{3.7}$$

In addition, by replacing (2.18) into (2.20) and using (2.17) and (3.4), then one has instead of (2.22):

$$S_{ik} = T_{ik} - d^0 U_i U_k / c^2 - \left(U_i V_k + V_i U_k\right) / c^2 \tag{3.8}$$

Consequently, from (3.8) follows the required relation (3.1), instead of (2.24).

Consider further the general system of continuously matter described previously inside which invisible heat conduction can take place, while the motion of the matter is described by the four-velocity U_i or by the velocity u_i .

Then, for the connection between the energy-momentum tensor T_{ik} and the relative stress tensor σ_{ik} of the general system, the following relation is valid:

$$T_{ik} = g_i u_k + \sigma_{ik} + u_i \, \xi_k / c^2 \tag{3.9}$$

with:

$$\xi_k = U_4 \left(V_k - V_4 U_k / U_4\right) / ic \tag{3.10}$$

in which V_k denotes the four-vector given by (3.4), g_i the momentum density and c the speed of light.

The quantity ξ_k seems to be the most important part of ξ_{ik} :

$$\xi_{ik} = H_{ik} - H_{i4} U_k / U_4 = U_i \left(V_k - V_4 U_k / U_4\right) / c^2 \tag{3.11}$$

Beyond the above, ξ_k can be written by the following form by using (2.41) and (3.6):

$$\xi_k = \left(\boldsymbol{\xi}, 0\right) \tag{3.12}$$

with:

$$\boldsymbol{\xi} = \gamma \left[\mathbf{V} - \mathbf{u}\left(\mathbf{V}, \mathbf{u}\right) / c^2\right] \tag{3.13}$$

In the stationary system, $\boldsymbol{\xi}^0$ is equal to the heat current density $\mathbf{V}^0$:

$$\boldsymbol{\xi}^0 = \mathbf{V}^0 \tag{3.14}$$

By combining (3.10) and (3.11), then one has:

$$\xi_{ik} = U_i \, \xi_k / \gamma c^2 \tag{3.15}$$

So, by using (2.35), (3.1), (3.2), (3.11) and (3.15), we obtain:

$$T_{ik} - T_{i4} U_k / U_4 = \sigma_{ik} + \xi_{ik} = \sigma_{ik} + U_i \, \xi_k / \gamma c^2 \qquad (3.16)$$

which finally reduces to the required formula (3.9).

Let us consider the general system of continuously matter, inside which invisible heat conduction can take place. Then the momentum density **g** of this system is given by the ***Universal Equation of Thermo-Elasticity:***

$$\mathbf{g} = m\mathbf{u} + \frac{(\mathbf{u}, \boldsymbol{\sigma})}{c^2} + \frac{\boldsymbol{\xi}}{c^2} \qquad (3.17)$$

where **u** denotes the velocity of the matter at the place and time considered, **σ** the relative stress tensor, **ξ** is given by (3.13) and $m = E / c^2$ is the total mass density.

From (3.9), we obtain for the energy current density:

$$D_k = E u_k + u_i \sigma_{ik} + \xi_k \qquad (3.18)$$

which can be further written as:

$$\mathbf{D} = E\mathbf{u} + (\mathbf{u}, \boldsymbol{\sigma}) + \boldsymbol{\xi} \qquad (3.19)$$

So, from (3.19) by using the formula of the momentum density **g**:

$$\mathbf{g} = \mathbf{D} / c^2 \qquad (3.20)$$

we obtain the required relation (3.17) which is a generalization, for a general system with heat conduction.

4. Universal Mechanics by Elastic Stress Analysis for Non-linear Airframes

Consider the stationary frame of Fig. 1 with Γ_1 the portion of the boundary of the body on which displacements are presented, Γ_2 the surface of the body on which the force tractions are employed and Γ the total surface of the body equal to $\Gamma_1 + \Gamma_2$.

Also, for the principal of virtual displacements, for linear elastic problems then the following formula is valid:

$$\int_{\Omega} (\sigma^0_{jk,j} + b_k) u_k \, \mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \overline{p}_k) u_k \, \mathrm{d}\Gamma \qquad (4.1)$$

in which u_k are the virtual displacements, satisfying the homogeneous boundary conditions $\overline{u}_k \equiv 0$ on Γ_1, b_k the body forces (Fig. 1) and p_k the surface tractions at the point k of the body. (Fig. 3)

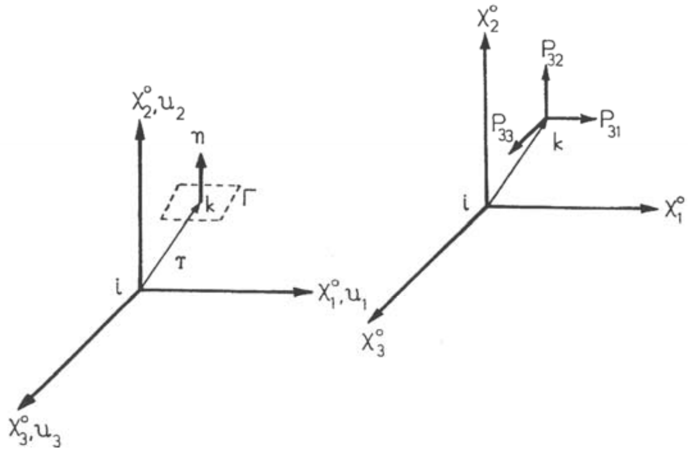

Fig. 3 The stationary system S^0.

Eqn (4.1) can be further written as following if u_k do not satisfy the previous conditions on Γ_1:

$$\int_{\Omega}(\sigma^0_{jk,j}+b_k)u_k\,\mathrm{d}\Omega=\int_{\Gamma_2}(p_k-\overline{p}_k)u_k\,\mathrm{d}\Gamma+\int_{\Gamma_1}(\overline{u}_k-u_k)p_k\,\mathrm{d}\Gamma \tag{4.2}$$

where $p_k = n_j\sigma^0_{jk}$ are the surface tractions corresponding to the u_k system.

Then, by integrating (4.2) follows:

$$\int_{\Omega}b_k u_k\,\mathrm{d}\Omega-\int_{\Omega}\sigma^0_{jk}\varepsilon_{jk}\,\mathrm{d}\Omega=-\int_{\Gamma_2}\overline{p}_k u_k\,\mathrm{d}\Gamma-\int_{\Gamma_1}p_k u_k\,\mathrm{d}\Gamma+\int_{\Gamma_1}(\overline{u}_k-u_k)p_k\,\mathrm{d}\Gamma \tag{4.3}$$

in which ε_{jk} are the strains.

By a second integration then (4.3) reduces to:

$$\begin{gathered}\int_{\Omega}b_k u_k\,\mathrm{d}\Omega+\int_{\Omega}\sigma^0_{jk,j}u_k\,\mathrm{d}\Omega=\\-\int_{\Gamma_2}\overline{p}_k u_k\,\mathrm{d}\Gamma-\int_{\Gamma_1}p_k u_k\,\mathrm{d}\Gamma+\int_{\Gamma_1}\overline{u}_k p_k\,\mathrm{d}\Gamma+\int_{\Gamma_2}u_k p_k\,\mathrm{d}\Gamma\end{gathered} \tag{4.4}$$

In addition, a fundamental solution should be found, satisfying the equilibrium equations, of the following type:

$$\sigma^0_{jk,j}+\Delta^i_l=0 \tag{4.5}$$

where Δ^i_l denotes the Dirac delta function which represents a unit load at i in the l direction.

The fundamental solution for a three-dimensional isotropic body is: [31]

$$u_{lk}^{*} = \frac{1}{16\pi G(1-v)r}\left[(3-4v)\Delta_{lk} + \frac{\vartheta r}{\vartheta x_l}\frac{\vartheta r}{\vartheta x_k}\right]$$

$$p_{lk}^{*} = -\frac{1}{8\pi(1-v)r^2}\left[\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{lk} + 3\frac{\vartheta r}{\vartheta x_l}\frac{\vartheta r}{\vartheta x_k}\right] - (1-2v)\left[\frac{\vartheta r}{\vartheta x_l}n_k - \frac{\vartheta r}{\vartheta x_k}n_l\right]\right] \tag{4.6}$$

in which G is the shear modulus, v Poisson's ratio, n the normal to the surface of the body, Δ_{lk} Kronecker's delta, r the distance from the point of application of the load to the point under consideration and n_j the direction cosines (Fig.3).

The displacements at a point are given as following:

$$u^i = \int_\Gamma up \,\mathrm{d}\Gamma - \int_\Gamma pu \,\mathrm{d}\Gamma + \int_\Omega bu \,\mathrm{d}\Omega \tag{4.7}$$

Consequently, (4.7) takes the following form for the "l" component:

$$u_l^i = \int_\Gamma u_{lk} p_k \,\mathrm{d}\Gamma - \int_\Gamma p_{lk} u_k \,\mathrm{d}\Gamma + \int_\Omega b_k u_{lk} \,\mathrm{d}\Omega \tag{4.8}$$

By differentiating u at the internal points, one obtains the stress-tensor for an isotropic medium:

$$\sigma_{ij}^0 = \frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_l}{\vartheta x_l} + G\left(\frac{\vartheta u_i}{\vartheta x_j} + \frac{\vartheta u_j}{\vartheta x_i}\right) \tag{4.9}$$

Beyond the above, after carrying out the differentiation we have:

$$\begin{aligned}\sigma_{ij}^0 = &\int_\Gamma \left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta u_{ik}}{\vartheta x_j} + \frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right] p_k \,\mathrm{d}\Gamma + \\ &+ \int_\Omega \left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta u_{ik}}{\vartheta x_j} + \frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right] b_k \,\mathrm{d}\Omega - \\ &- \int_\Gamma \left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta p_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta p_{ik}}{\vartheta x_j} + \frac{\vartheta p_{jk}}{\vartheta x_i}\right)\right] u_k \,\mathrm{d}\Gamma\end{aligned} \tag{4.10}$$

Eq. (4.10) can be further written as follows:

$$\sigma_{ij}^0 = \int_\Gamma D_{kij} p_k \,\mathrm{d}\Gamma - \int_\Gamma S_{kij} u_k \,\mathrm{d}\Gamma + \int_\Omega D_{kij} b_k \,\mathrm{d}\Omega \tag{4.11}$$

in which the third order tensor components D_{kij} and S_{kij} are:

$$D_{kij} = \frac{1}{8\pi(1-v)r^2}\left[(1-2v)\left[\Delta_{ki} r_{,j} + \Delta_{kj} r_{,i} - \Delta_{ij} r_{,k}\right] + 3r_{,i} r_{,j} r_{,k}\right] \tag{4.12}$$

$$S_{kij} = \frac{G}{4\pi(1-v)r^3}\left[3\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{ij} r_{,k} + v(\Delta_{ik} r_{,j} + \Delta_{jk} r_{,i}) - 5r_{,i} r_{,j} r_{,k}\right]\right.$$
$$\left. + 3v(n_i r_{,j} r_{,k} + n_j r_{,i} r_{,k}) + (1-2v)(3n_k r_{,i} r_{,j} + n_j \Delta_{ik} + n_i \Delta_{jk}) - (1-4v)n_k \Delta_{ij}\right] \tag{4.13}$$

with: $r_{,i} = \dfrac{\vartheta r}{\vartheta x_i}$

Finally, because of eqs (2.49) and (4.11) by considering the moving system S of Fig. 2, then the stress-tensor reduces to the following form:

$$\begin{aligned}
\sigma_{11} &= \sigma_{11}^0 \\
\sigma_{12} &= \gamma\sigma_{12}^0 \\
\sigma_{13} &= \gamma\sigma_{13}^0 \\
\sigma_{21} &= \frac{1}{\gamma}\sigma_{21}^0 \\
\sigma_{22} &= \sigma_{22}^0 \\
\sigma_{23} &= \sigma_{23}^0 \\
\sigma_{31} &= \frac{1}{\gamma}\sigma_{31}^0 \\
\sigma_{32} &= \sigma_{32}^0 \\
\sigma_{33} &= \sigma_{33}^0
\end{aligned} \tag{4.14}$$

where σ_{ij}^0 are given by. (4.11) to (4.13).

So, Table 1 shows the values of γ as given by (2.41) for some arbitrary values of the velocity u of the moving aerospace structure:

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	C	∞

From Table 1 follows that for small velocities 50,000 *km*/*h* to 200,000 *km*/*h*, the absolute and the relative stress tensor are nearly the same. On the other hand, for bigger velocities like $c/3$, $c/2$ or $3c/4$ (c = speed of light), the variable γ takes values more than the unit and thus, relative stress tensor is very different from the absolute one. In addition, for values of the velocity for the moving structure near the speed of light, the variable γ takes bigger values, while when the velocity is equal to the speed of light, then γ tends to the infinity.

Consequently, the Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et al. [22] will be used for the numerical evaluation of the stress tensor (3.11), for every specific case.

5. Conclusions

By the current research in the area of aerospace and aeronautical technologies the theory of *"Universal Mechanics"* has been investigated and applied for the design the future spacecraft moving with very high speeds, even approaching the speed of light, as the plan of the International Space Agencies is to achieve such spacecraft in the future. Such new generation spacecraft was studied like a non-linear airframe. The future investigation concerns to the determination of the proper composite materials or any other kind of materials for the construction of the next generation spacecraft, as usual composite solids are not suitable for such constructions.

The theory of *"Universal Mechanics"* and correspondingly the *"Universal Equation of Elasticity"* and the *"Universal Equation of Thermo-Elasticity"* show that there is a considerable difference between the absolute stress tensor of the airframe even for lower speeds. For bigger speeds the difference between the two stress tensors is very much increased. *"Universal Mechanics"* results as a combination of the theories of *"Relativistic Elasticity"* and *"Relativistic Thermo-Elasticity"*.

Consequently, for the structural design of the next generation spacecraft will be used the stress tensor of the airframe in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.).

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