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4-D Porous Medium Analysis by Non-linear Singular Integral Equations Method in Petroleum Hydraulic Fracturing more Convenient than the Corresponding 3-D Analysis

E.G. Ladopoulos
Interpaper Research Organization
8, Dimaki Str.
Athens, GR - 106 72, Greece
eladopoulos@interpaper.org

Abstract
For the determination of the properties of the reservoir materials, when petroleum reserves are moving through porous media, a modern mathematical model is further improved. This problem is very much important for petroleum reservoir engineering and the major oil industry and especially in hydraulic fracturing. Hence, hydraulic fracturing is used to increase the rate at which fluids, such as petroleum, water, or natural gas can be recovered from subterranean natural reservoirs. Reserves are typically porous sandstones, limestones or dolomite rocks, but also include "unconventional reservoirs" such as shale rock or coal beds. Consequently, the above mentioned problem is reduced to the solution of a non-linear singular integral equation, which is numerically evaluated by using the Non-linear Singular Integral Operators Method (N.S.I.O.M.). Furthermore, several properties are analyzed and investigated for the porous medium equation, defined as a Helmholtz differential equation. An application is finally given for a well testing to be checked when a heterogeneous oil reservoir is moving in a porous solid. Thus, by using the S.I.O.M., then the pressure response from the well test conducted in the above heterogeneous petroleum reserves, is numerically evaluated and investigated.

Key Word and Phrases
Petroleum Reserves, Multiphase Flows, 4 Porous Medium Analysis, Non-linear Singular Integral Operators Method (N.S.I.O.M.), Non-linear Singular Integral Equation, Hydraulic Fracturing, Helmholtz Differential Equation.

1. Introduction

Hydraulic fracturing is a well-stimulation method in which rock is fractured by a hydraulically pressurized liquid made of water, sand, and chemicals. A high-pressure fluid (usually chemicals and sand suspended in water) is injected into a wellbore to create cracks in the deep-rock formations through which natural gas and petroleum will flow more freely. When the hydraulic pressure is removed from the well, small grains of hydraulic fracturing proppants (either sand or aluminium oxide) hold the fractures open. Hence, the hydraulic fracture is formed by pumping fracturing fluid into a wellbore at a rate sufficient to increase pressure at the target depth in order to exceed that of the fracture pressure gradient of the rock.

Consequently, hydraulic fracturing is used to increase the rate at which fluids, such as petroleum, water, or natural gas can be recovered from subterranean natural reservoirs. Reserves are typically porous sandstones, limestones or dolomite rocks, but also include "unconventional reservoirs" such as shale rock or coal beds. Hydraulic fracturing enables the extraction of natural gas and petroleum from rock formations deep below the earth's surface (generally 2,000–6,000 m), which is greatly below typical groundwater reservoir levels. At such depth, there may be insufficient permeability or reservoir pressure to allow natural gas and petroleum to flow from the rock into the wellbore at high economic return.

Typically, 90% of the hydraulically pressurized fluid is water and 9.5% is sand with chemical additives accounting to about 0.5%. Furthermore, proppants are solid material, typically treated sand or man-made ceramic materials, designed to keep an induced hydraulic fracture open, during or following a fracturing treatment.

It is added to the fracturing fluid which may vary in composition depending on the type of fracturing used, and can be gel, foam or slickwater-based. Additionally, there may be unconventional fracking fluids. Fluids make tradeoffs in such material properties as viscosity, where more viscous fluids can carry more concentrated proppant; the energy or pressure demands to maintain a certain flux pump rate (flow velocity) that will conduct the proppant appropriately; pH, various rheological factors, among others. Hence, sand is used to “prop open” fractures in shale rock and allow oil and gas to flow freely. The goal of the current investigation is to propose an “innovative” technology to reduce the amount of sand required to drill and maintain productive wells. Thus, by the current research a new method will be further improved in order to minimize the quantity of sand which is used for the hydraulic fracturing solution when injected downhole. The proposed modern technology will have the potential to improve the environment for affected communities. Drilling new wells requires thousands of truck trips through these communities, which snarls traffic and creates noise, dust and exhaust fumes. Trucks carrying sand account for a large percentage of the truck traffic to drilling fields.

Weight limits on community roadways prevent sand delivery trucks from hauling maximum capacity loads. So, in order to improve the health and safety of these communities, a groundbreaking technology will be proposed in order to reduce the amount of sand required for hydraulic fracturing of petroleum and gas wells. By using sand more efficiently will eliminate thousands of truck trips every year, as sand is combined with hydraulic fracturing fluid and injected as a slurry into horizontal wells. Thus, the fluid fractures the shale rock and the sand fills the fractures with a strong but porous material that allows oil and gas to flow freely from the fractured rock. The proposed new method will keep sand suspended in solution so that it can efficiently fill each fracture would reduce the amount of sand and trucks required to maximize petroleum and gas recovery from wells.

Hence, in order to minimize the requested sand for the hydraulic fracturing model, the Non-linear Singular Integral Operators Method (N.S.I.O.M.) for porous medium analysis will be used. By using the above method then the viscosity of the fracturing fluid will be modified. So, the study of the movement of oil reserves through porous media is very much important problem on petroleum reservoir engineering. By applying therefore a well testing analysis, then a history matching process takes place for the determination of the properties of the reservoir materials. The movement of petroleum & gas reserves through porous media, produces both single-phase and multiphase flows. Furthermore, if a well test is conducted, then the well is subjected to a change of the flow rate and the pressure response can be further measured. For the determination of several oil reservoir parameters, such as permeability, then numerical calculations should be used, as analytical solutions in most cases are not possible to be derived.

During the last years several non-linear singular integral equations methods were used successfully by E.G. Ladopoulos [1] - [36] for the solution of many applied problems of petroleum engineering. Consequently, by the present research, the non-linear singular integral equations will be proposed in order to determine the properties of the reservoir materials, when petroleum reserves are moving through porous solids.

By using therefore, the Non-linear Singular Integral Operators Method (N.S.I.O.M.), then the pressure response from the well test conducted in a heterogeneous reservoir will be computed. Furthermore, some properties of the porous medium equation, which is a Helmholtz differential equation are proposed and investigated. Hence, basic properties of the fundamental solution will be analyzed and investigated. Consequently, the non-linear singular integral equation methods which were used with big success for the solution of several oil engineering problems, are further extended by the present study for the solution of hydraulic fracturing problems. In such case the non-linear singular integral equations are used for the solution of one of the most important and interesting problems for petroleum engineers.

2. 4-D Well Test Analysis with Hydraulic Fracturing

Petroleum well test analysis is a kind of an important history matching process for the determination of the properties of reservoir materials. Thus, during the movement of oil reservoir through porous media, then both single-phase and multiphase flow occurs. Furthermore, when a petroleum well test is conducted then the well is subjected to a change of its flow rate and the resulting pressure response is possible to be measured. This pressure is further compared to analytical or numerical models in order to estimate reservoir parameters such as permeability.

In general, a petroleum reservoir well test in a single-phase reservoir is calculated by using the porous medium equation:

$$\nabla \bullet (\frac{\lambda}{\phi\xi} \nabla p) = c_t \frac{\partial p}{\partial t} \tag{2.1}$$

in which λ denotes the permeability, ϕ the porosity, ξ the viscosity, p the pressure of the reservoir, t the time and c_t the compressibility.

By replacing variables as follows:

$$u = (\frac{\lambda}{\phi\xi})^{1/2} p \tag{2.2}$$

then (2.1) may be written as:

$$\nabla^2 u + \lambda' u = 0 \tag{2.3}$$

with :

$$\lambda' = -\frac{\nabla^2 (\lambda / \phi\xi)^{1/2}}{(\lambda / \phi\xi)^{1/2}} \tag{2.4}$$

Hence, (2.3) is a Helmholtz differential equation.

Moreover, consider by $u^*(\mathbf{x},\mathbf{y})$ the fundamental solution of any point **y**, because of the source point **x**. Then, the fundamental solution can be given by the following equation:

$$\nabla^2 u^*(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) + \delta(\mathbf{x},\mathbf{y}) = 0 \tag{2.5a}$$

which may be further written as:

$$u^*_{,ii}(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) + \delta(\mathbf{x},\mathbf{y}) = 0 \tag{2.5b}$$

Thus, (2.5) is the Helmholtz potential equation governing the fundamental solution.

Beyond the above, consider by u^* the fundamental solution chosen so that to enforce the Helmholtz equation in terms of the function u, in a weak form. Then the weak form of Helmholtz equation will be written as following:

$$\int_{\Omega} (\nabla^2 u + \lambda' u) u^* d\Omega = 0 \tag{2.6}$$

in the solution domain Ω.

Also, by applying the divergence theorem once in (3.6), one obtains a symmetric weak form:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\Omega} u_{,i} u^*_{,i} d\Omega + \int_{\Omega} \lambda' u u^* d\Omega = 0 \tag{2.7}$$

in which **n** denotes the outward normal vector of the surface *S*.

So, in the symmetric weak form the function u and the fundamental solution u^* are only required to be first - order differentiable. By applying further the divergence theorem twice in (2.6) we have:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\partial\Omega} n_i u u^*_{,i} dS + \int_{\Omega} u(u^*_{,ii} + \lambda' u^*) d\Omega = 0 \tag{2.8}$$

Thus, (2.8) is the asymmetric weak form and the fundamental solution u^* is required to be second - order differentiable. On the contrary, u is not required to be differentiable in the domain Ω.

By combining (2.5) and (2.8), then one has:

$$u(\mathbf{x}) = \int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} n_i(\mathbf{y}) u(\mathbf{y}) u^*_{,i}(\mathbf{x}.,\mathbf{y}) dS \tag{2.9}$$

which can be further written as:

$$u(\mathbf{x}) = \int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} u(\mathbf{y}) R^*(\mathbf{x},\mathbf{y}) dS \tag{2.10}$$

where $q(\mathbf{y})$ denotes the potential gradient along the outward normal direction of the boundary surface:

$$q(\mathbf{y}) = \frac{\partial u(\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u_{,k}(\mathbf{y}) \quad , \quad \mathbf{y} \in \partial\Omega \tag{2.11}$$

and the kernel function:

$$R^*(\mathbf{x},\mathbf{y}) = \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u^*_{,k}(\mathbf{x},\mathbf{y}) \quad , \quad \mathbf{y} \in \partial\Omega \tag{2.12}$$

By differentiating (2.10) with respect to x_k we obtain the integral equation for potential gradients $u_{,k}(\mathbf{x})$ by the following formula:

$$\frac{\partial u(\mathbf{x})}{\partial x_k} = \int_{\partial\Omega} q(\mathbf{y}) \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial x_k} dS - \int_{\partial\Omega} u(\mathbf{y}) \frac{\partial R^*(\mathbf{x},\mathbf{y})}{\partial x_k} dS \tag{2.13}$$

3. Fundamental Solution's Basic Properties

We rewrite the weak form of (2.5) governing the fundamental solution, as follows:

$$\int_{\Omega} \left[\nabla^2 u^*(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) \right] c d\Omega + c = 0, \quad \mathbf{x} \in \Omega \tag{3.1}$$

where c denotes a constant, considering as the test function.

Beyond the above, (3.1) can be written as:

$$\int_{\Omega} \left[u^*_{,ii}(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) \right] d\Omega + 1 = 0, \quad \mathbf{x} \in \Omega \tag{3.2}$$

Moreover, (3.2) takes the form:

$$\int_{\partial\Omega} n_i(\mathbf{y}) u^*_{,i}(\mathbf{x},\mathbf{y}) dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y}) d\Omega + 1 = 0, \quad \mathbf{x} \in \Omega \tag{3.3}$$

By considering further an arbitrary function *u(x)* in Ω as the test function, then the weak form of (2.5) will be written as:

$$\int_{\Omega} \left[\nabla^2 u^*(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) + \delta(\mathbf{x},\mathbf{y}) \right] u(\mathbf{x}) d\Omega = 0, \quad \mathbf{x} \in \Omega \tag{3.4}$$

and also as:

$$\int_{\Omega} \left[u^*_{,ii}(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) \right] u(\mathbf{x}) d\Omega + u(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega \tag{3.5}$$

Finally, (3.5) takes the form:

$$\int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) d\Omega + u(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega \tag{3.6}$$

If $\mathbf{x}$ approaches the smooth boundary $(\mathbf{x} \in \partial\Omega)$, then the first term in (3.6) may be written as:

$$\lim_{x \to \partial\Omega} \int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS = \int_{\partial\Omega}^{CPV} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS - \frac{1}{2} u(\mathbf{x}) \tag{3.7}$$

in the sense of a Cauchy Principal Value (CPV) integral.

For the understanding of the physical meaning of (3.7), we rewrite (3.3) and (3.6) as:

$$\int_{\partial\Omega}^{CPV} \Phi^*(\mathbf{x},\mathbf{y}) dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y}) d\Omega + 1/2 = 0, \quad x \in \partial\Omega \tag{3.8}$$

and:

$$\int_{\partial\Omega}^{CPV} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) d\Omega + 1/2\, u(\mathbf{x}) = 0, \quad x \in \partial\Omega \tag{3.9}$$

By (3.8) follows that only a half of the source function at point **x** is applied to the domain *Ω*, when the point **x** approaches a smooth boundary, $\mathbf{x} \in \partial\Omega$.

Furthermore, consider another weak form of (2.5) by supposing the vector functions to be the gradients of an arbitrary function *u*(**y**) in *Ω*, chosen in such a way that they have constant values:

$$u_{,k}(\mathbf{y}) = u_{,k}(\mathbf{x}) \ , \quad \text{for } k=1,2,3 \tag{3.10}$$

Then the weak form of (2.5) will be written as:

$$\int_{\Omega} \left[u^{*}_{,ii}(\mathbf{x},\mathbf{y}) + \lambda'' u^{*}(\mathbf{x},\mathbf{y}) \right] u_{,k}(\mathbf{y}) d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{3.11}$$

By applying further the divergence theorem, then (3.11) takes the form:

$$\int_{\partial\Omega} \Phi^{*}(\mathbf{x},\mathbf{y}) u_{,k}(\mathbf{x}) dS + \int_{\Omega} \lambda' u^{*}(\mathbf{x},\mathbf{y}) u_{,k}(\mathbf{x}) d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{3.12}$$

In addition, the following property exists:

$$\begin{aligned} &\int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{x}) u^{*}_{,k}(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} n_k(\mathbf{y}) u_{,i}(\mathbf{x}) u^{*}_{,i}(\mathbf{x},\mathbf{y}) dS \\ &= \int_{\Omega} u_i(\mathbf{x}) u^{*}_{,ki}(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} u_{,i}(\mathbf{x}) u^{*}_{,ik}(\mathbf{x},\mathbf{y}) dS = 0 \end{aligned} \tag{3.13}$$

By adding (3.12) and (3.13) then we have:

$$\begin{aligned} &\int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{x}) u^{*}_{,k}(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} n_k(\mathbf{y}) u_{,i}(\mathbf{x}) u^{*}_{,i}(\mathbf{x},\mathbf{y}) dS \\ &+ \int_{\partial\Omega} \Phi^{*}(\mathbf{x},\mathbf{y}) u^{*}_{,k}(\mathbf{x}) dS + \int_{\Omega} \lambda'' u^{*}(\mathbf{x},\mathbf{y}) u_{,k}(\mathbf{x}) d\Omega + u_{,k}(\mathbf{x}) = 0 \end{aligned} \tag{3.14}$$

which takes finally the form:

$$\begin{aligned} &\int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{x}) u^{*}_{,k}(\mathbf{x},\mathbf{y}) dS + \int_{\partial\Omega} e_{ikt} R_t u(\mathbf{x}) u^{*}_{,i}(\mathbf{x},\mathbf{y}) dS \\ &+ \int_{\partial\Omega} \lambda' u^{*}(\mathbf{x},\mathbf{y}) u_{,k}(\mathbf{x}) d\Omega + u_{,k}(\mathbf{x}) = 0 \end{aligned} \tag{3.15}$$

4. New Aspects of Hydraulic Fracturing by Non-linear Singular Integral Equations

The porous medium equation (2.1) will be also written in another form, in order a singular integral equations representation to be applicable:

$$\nabla^2 p = -\nabla \ln\left(\frac{\lambda}{\Phi \xi c_t}\right) \bullet \nabla p + \frac{\Phi \xi c_t}{\lambda} \frac{\partial p}{\partial t} \tag{4.1}$$

By applying the Green Element Method, then (4.1) reduces to the solution of a non-linear singular integral equation:

$$-\frac{\theta}{2\pi} p(r_i) + \int_{\partial\Omega} \left(p \frac{\partial [\ln(r - r_i)]}{\partial n} - \ln(r - r_i) \frac{\partial p}{\partial n} \right) dS + \iint_{\Omega} \ln(r - r_i) \left[-\nabla \ln \Lambda \bullet \nabla p + \frac{1}{\Lambda} \frac{\partial p}{\partial t} \right] d\Omega = 0 \tag{4.2}$$

where:

$$\Lambda = \frac{\Phi \xi}{\lambda} c_t \tag{4.3}$$

In order the non-linear singular integral equation (4.2) to be numerically calculated, then the Non-linear Singular Integral Operators Method (N.S.I.O.M.) will be used. Hence, the non-linear singular integral equation (4.2) is approximated by the formula:

$$-\frac{\theta}{2\pi} p(r_i) + \sum_{e=1}^{M} \left[\int_{\partial\Omega} \left(p \frac{\partial [\ln(r - r_i)]}{\partial n} - \ln(r - r_i) \frac{\partial p}{\partial n} \right) dS + \iint_{\Omega} \left(-\nabla \ln \Lambda \bullet \nabla p + \frac{1}{\Lambda} \frac{\partial p}{\partial t} \right) d\Omega \right] = 0 \tag{4.4}$$

where M denotes the total number of elements.

Beyond the above, we introduce the following functions describing the pressure at any point in an element, in terms of the nodal pressures:

$$p(x, y) = N_j(x, y) p_j \tag{4.5}$$

By replacing (4.5) then (4.4) takes the form:

$$\sum_{e=1}^{M} \left(A_{ij}^e p_j + B_{ij}^e q_j - C_{ijl}^e \ln \Lambda_j p_l + D_{ijl}^e \frac{1}{\Lambda_j} \frac{\partial p_l}{\partial t} \right) = 0, \qquad i, j, l = 1,2,3,4 \tag{4.6}$$

where:

$$A_{ij}^e = \int_{\partial\Omega} \frac{\partial [\ln(r - r_i)]}{\partial n} \Omega_j dS - \delta_{ij} \frac{\theta}{2\pi} \tag{4.7}$$

$$B_{ij}^{e} = -\int_{\partial\Omega} \ln(r - r_i)\Omega_j dS \tag{4.8}$$

$$C_{ijl}^{e} = \iint_{\Omega_j} \ln(r - r_i)\left[\frac{\partial N_j}{\partial x}\frac{\partial N_l}{\partial x} + \frac{\partial N_j}{\partial y}\frac{\partial N_l}{\partial y}\right] d\Omega \tag{4.9}$$

$$D_{ijl}^{e} = \iint_{\Omega_j} \ln(r - r_i) N_j N_l d\Omega \tag{4.10}$$

5. Applications of Heterogeneous Reservoirs Well Testings

The previous mentioned theory will be applied to the determination of a well testing, which will be checked in an heterogeneous reservoir with a permeability varying from 10 mD to 300 mD (1mDarcy $\approx 10^{-12}$ $m^2 = 1(\mu m)^2$).

Thus, by using the Non-linear Singular Integral Operators Method (N.S.I.O.M.) as described by the previous paragraphs, then the computation of the pressure response from the well test conducted in the above heterogeneous reservoir will become possible. Firstly, the pressures were computed in variation with the time. Consequently, Table 1 shows the pressure response with respect to the time.

Furthermore, the pressure derivatives were computed with respect to the time, as shown in Table 2. Such derivatives are very much important of the well testings interpretation as these are some distinct shapes and especially the characteristics of certain reservoir features.

The computational results of the pressures and the pressure derivatives are compared to the analytical solutions of the same well testing problem, if the reservoir was homogeneous with permeability equal to 50 mD. Thus, the analytical results are shown in Table 1 for the pressures and in Table 2 for the pressure derivatives, correspondingly. From the above Tables it can be seen that there is very small difference between the computational results and the analytical solutions for both the pressures and the pressure derivatives. On the contrary, the above mentioned small difference can be explained because of the diffusive nature of the pressure transport mechanism. Finally same results are shown, correspondingly in Figures 4 and 5, and in three-dimensional form in Figures 4a and 5a.

Table 1

Time (hours)	Pressure (psi) S.I.O.M.	Pressure (psi) Analytical
0.002	7.003	7.022
0.009	10.002	10.013
0.015	12.002	12.031
0.030	12.504	12.523
0.040	13.003	13.014
0.070	13.503	13.502
0.100	14.002	14.033
0.250	14.501	14.521
0.400	15.004	15.032
1.000	15.502	15.514
2.000	16.004	16.023
10.00	17.002	17.022

30.00	17.504	17.524
80.00	18.001	18.042
100.00	19.003	19.032
200.00	20.000	20.030
400.00	20.000	20.020
600.00	20.000	20.010
1000.00	20.000	20.000

Table 2

Time (hours)	Pressure Derivative (psi) S.I.O.M.	Pressure Derivative (psi) Analytical
0.002	1.504	2.002
0.009	2.002	2.003
0.015	2.001	2.003
0.030	2.002	2.002
0.040	2.003	2.002
0.070	2.004	2.003
0.100	2.002	2.004
0.250	2.001	2.002
0.400	2.003	2.003
1.000	2.002	2.002
2.000	2.004	2.003
10.00	2.001	2.002
30.00	2.003	2.003
80.00	2.002	2.003
100.00	1.001	1.301
200.00	0.600	0.800
400.00	0.250	0.260
600.00	0.060	0.060
1000.00	0.030	0.010

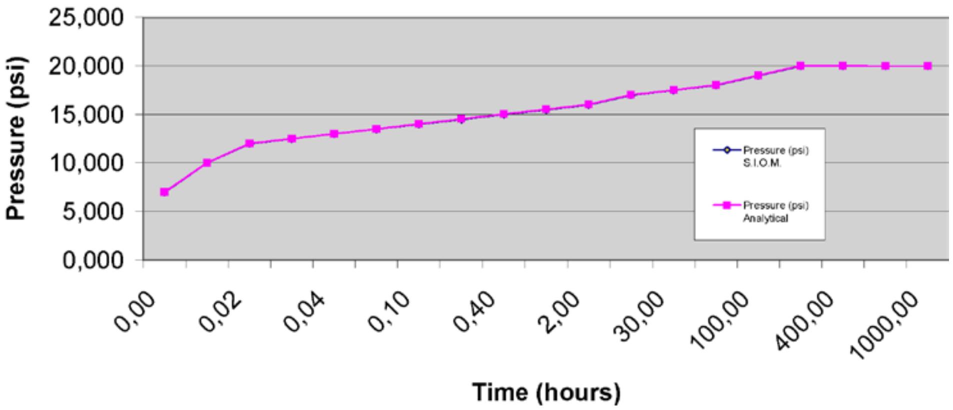

Fig. 4 Pressure Response for Well Test in Heterogeneous Reservoir.

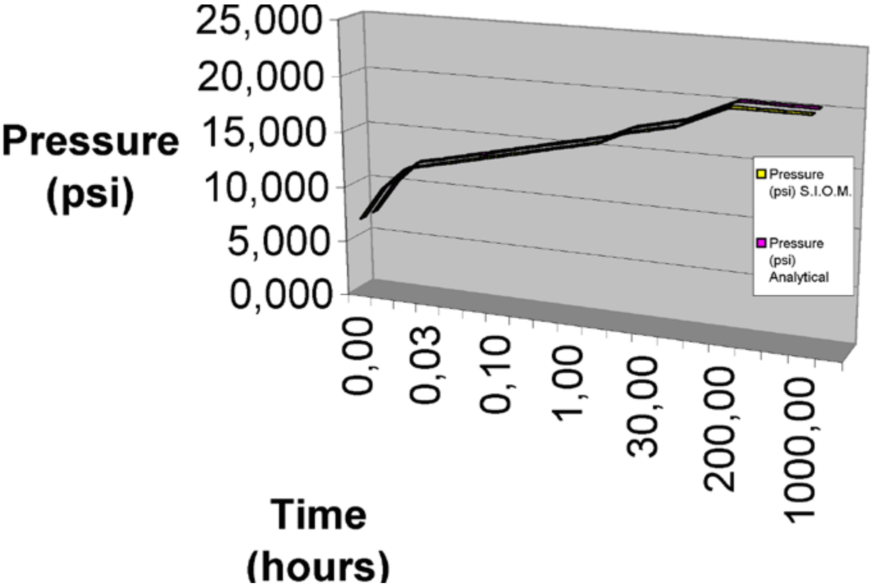

Fig. 4a 3-D Distribution of Pressure Response for Well Test in Heterogeneous Reservoir.

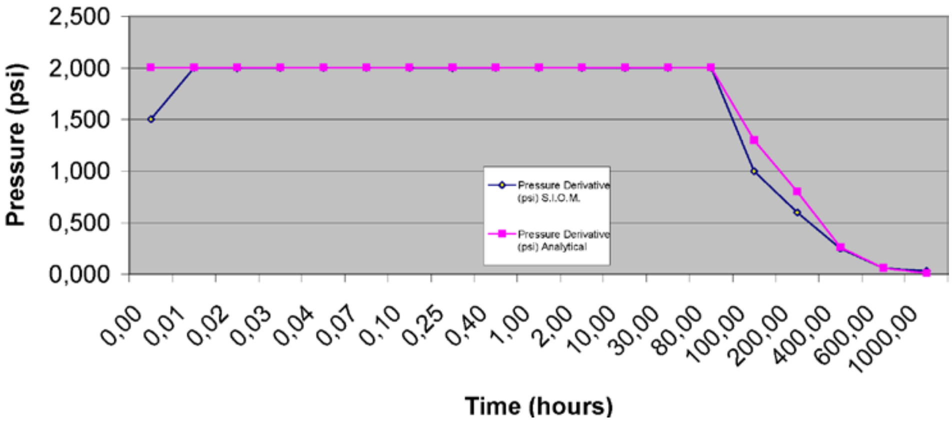

Fig. 5 Pressure Derivative for Well Test in Heterogeneous Reservoir.

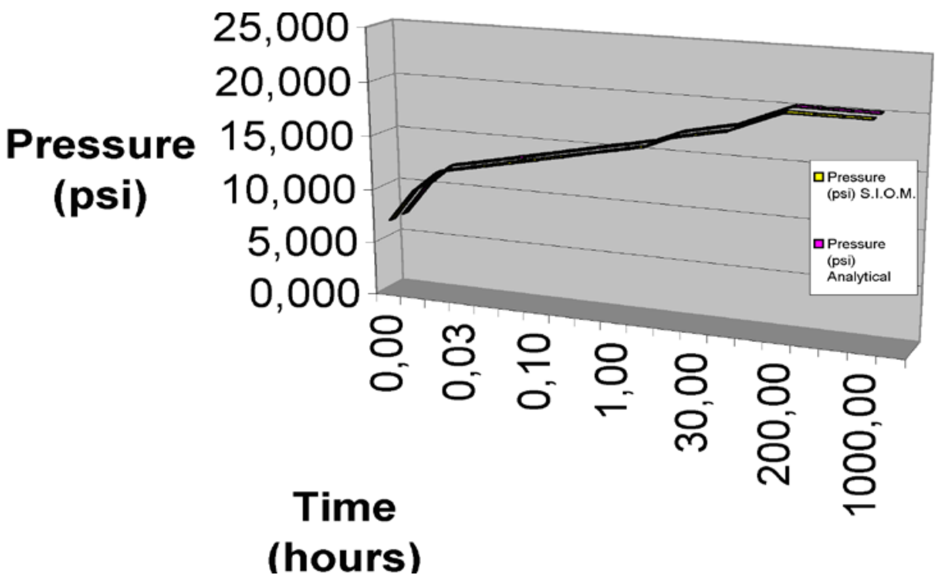

Fig. 5a 3-D Distribution of Pressure Derivative for Well Test in Heterogeneous Reservoir.

6. Conclusions

By the present research a new mathematical method has been improved as an attempt to determine the properties of the reservoir materials. Thus, the study of the movement of oil & gas reserves through porous media is very important for petroleum reservoir engineers. The above mentioned problem was reduced to the solution of a non-linear singular integral equation, which was numerically calculated by using the Non-linear Singular Integral Operators Method (S.I.O.M.).

Beyond the above, several important properties of the porous medium equation, which is a Helmholtz differential equation, were analyzed and investigated. Thus, the fundamental solution of the porous medium equation was proposed and studied. Moreover, some basic properties of the fundamental solution were further investigated. These are very important in order the behavior of the non-linear singular integral equation to be well understood.

An application was also given for a well testing to be checked when a heterogeneous petroleum reservoir is moving in a porous solid. The above problem was solved by using the Non-linear Singular Integral Operators Method and thus the pressure response from the well test conducted in the above heterogeneous oil reservoir, was computed. Both the pressures and the pressure derivatives were computed and these values were compared to the analytical solutions of the same well testing problem, if the reservoir was homogeneous with a mean permeability.

Over the past years, non-linear singular integral equation methods have been used with a big success for the solution of several important engineering problems of structural analysis, elastodynamics, hydraulics, fluid mechanics and aerodynamics. For the numerical solution of the non-linear singular integral equations of the above problems, were used several aspects of the Singular Integral Operators Method (S.I.O.M.). So, by the current investigation such methods were extended for the solution of oil reserves problems in petroleum reservoir engineering.

The benefits of the new technology in comparison to existing methods are the following:

1. The new method is based on the non-linear programming method, by using non-linear singular equations. According to this theory the porous medium equation is reduced to the solution of a non-linear singular integral equation which is then numerically evaluated by using a non-linear programming method. Existing methods of well test analysis, are using too as a start the porous medium equation, but as this is a complicated differential equation are giving only some analytical solutions for very simple cases or numerical solutions for homogeneous reservoir materials..

2. The new method, as it is a complicated non-linear numerical method can give results for heterogeneous porous media (which of course are the solids in reality) and not only for homogeneous solids as are giving the analytical or numerical existing methods. Hence, the estimation of the properties and the future petroleum production from a new oil reservoir could be done exactly, and not estimated as by the existing methods.

From the above two points it can be understood the evidence of the applicability of the new method, as it is based on non-linear software. Furthermore its novelty, as it is based on the theory of non-linear singular integral equations.

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