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Real-time Expert Seismology & Non-linear Singular Integral Equations for Oil Reserves Exploration

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Abstract

A non-linear 3-D elastic waves real - time expert system is proposed for the exploration of several oil and gas reserves, including off-shore oil reserves, of the seas all over the world, according to the new theory of "*Real-Time Expert Seismology*". Such Generic Technology will work under Real Time Logic for searching marine oil reserves developed on the continental crust and on deeper water ranging from 300 to 3000 m, or even more. The proposed real - time expert system will be therefore the best device for the exploration of the continental margin areas (shelf, slope and rise) and the very deep waters, too. Also, this expert system will be suitable for the exploration of on-shore oil and gas reserves, as well. Beyond the above, for the determination of the properties of the reservoir materials, when oil reserves are moving through porous media, a new mathematical device is proposed. This problem is very much important for petroleum reservoir engineering and the oil industry. Therefore, the above mentioned problem is reduced to the solution of a non-linear singular integral equation, which is numerically evaluated by using the Singular Integral Operators Method (S.I.O.M.). Also, several properties are analyzed and investigated for the porous medium equation, defined as a Helmholtz differential equation. An application is finally given for a well testing to be checked when a heterogeneous oil reservoir is moving in a porous solid. So, by using the S.I.O.M., then the pressure response from the well test conducted in the above heterogeneous oil reservoir, is numerically calculated and investigated.

Key Word and Phrases

Real-time Expert Seismology, Singular Integral Operators Method (S.I.O.M.), Non-linear Singular Integral Equations, Oil and Gas Reserves, Porous Media, Petroleum Reservoir Engineering, Helmholtz Differential Equation, Real - Time Expert System, Real - Time Logic, Generic Technology.

1. Introduction

The study of the movement of oil reserves through porous media is very much important problem on petroleum reservoir engineering. Therefore, by applying a well testing analysis, then a history matching process takes place for the determination of the properties of the reservoir materials. The movement of oil reserves through porous media, produces both single-phase and multiphase flows. Furthermore, if a well test is conducted, then the well is subjected to a change of the flow rate and the pressure response can be further measured. For the determination of several petroleum reservoir parameters, such as permeability, then numerical calculations should be used, as analytical solutions in most cases are not possible to be derived.

Over the past years several variants of the Boundary Element Method were used for the solution of petroleum reservoir engineering problems. At the end of eight's Lafe and Cheng [27] proposed a BEM for the solution of steady flows in heterogeneous solids. During the same period Masukawa and Horne [28] and Numbere and Tiab [29] applied boundary elements for steady state problems of streamline tracking.

Furthermore, Kikani and Horne [13] solved transient problems by using a Laplace space boundary element model, for the analysis of well tests in several arbitrarily shaped reservoirs. Also, Koh and Tiab [14] used boundary elements to describe the flow around tortuous horizontal wells, for homogeneous, or piecewise homogeneous reservoirs.

Sato and Horne [33], [34] applied perturbation boundary elements for the study of heterogeneous reservoirs. Beyond the above, El Harrouni et al. [4] proposed the use of a transformed form of Darcy's law combined with dual reciprocity boundary element method to handle heterogeneity. On the other hand, Onyejekwe [30] applied a Green element method to isothermal flows with second order reactions. The same author [31], [32] used a combined method of boundary elements together with finite elements for the study of heterogeneous reservoirs. Furthermore, Taigbenu and Onyejekwe [37] applied a transient one-dimensional transport equation by using a mixed Green element method.

During the last years several non-linear singular integral equations methods were used successfully by Ladopoulos [15] - [24] for the solution of applied problems of solid mechanics, elastodynamics, structural analysis, fluid mechanics and aerodynamics. Thus, in the present research, the non-linear singular integral equations will be used in order to determine the properties of the reservoir materials, when oil reserves are moving through porous solids.

By using therefore, the Singular Integral Operators Method (S.I.O.M.) then the pressure response from the well test conducted in a heterogeneous reservoir will be computed. Also, some properties of the porous medium equation, which is a Helmholtz differential equation are proposed and investigated. Thus, basic properties of the fundamental solution will be analyzed and investigated.

Finally, an application is given for a well testing to be investigated when a heterogeneous oil reservoir is moving in a porous medium. Then, this problem will be solved by using the Singular Integral Operators Method and thus the pressure response from the well test conducted in this heterogeneous oil reservoir, will be computed.

Consequently, the non-linear singular integral equation methods which were used with big success for the solution of several engineering problems of fluid mechanics, hydraulics, aerodynamics, solid mechanics, elastodynamics, and structural analysis, are further extended by the present study for the solution of oil reservoir engineering problems. In such case the non-linear singular integral equations are used for the solution of one of the most important and interesting problems for petroleum engineers.

2. Real-time Expert Seismology

The research and development aspects of marine oil reserves can be divided into three main areas:

(a) The acquisition and analysis of geophysical, geological and reservoir engineering data to enable an appreciation to be made of the reserves.
(b) The determination of all-necessary standards and data for the safety to offshore operations.
(c) To assist the development of the offshore supplies industry, and to enable it to play a full part in the development of the marine hydrocarbon resources in worldwide markets in the future.

Worldwide geological surveys by oil companies and scientific institutes indicate that such prospects do not necessarily end at the edge of the continental shelf. Normal probability considerations indicate that main resources will be found in areas of thick sedimentary sequences developed on the continental crust. There is therefore an expectation with good possibilities for finding marine oil resources in deep waters, too. These will be on the shelf, slope and rise of the Earth's margin, and the depths of water would not only range up to 300 m, but also in deeper waters from 300 m to 3000 m, or even much more.

The behavior of a reservoir, depends not only on the properties of the liquid and gas, but also on a series of factors that may be termed as the "properties of the environment". Amongst these are such items as capillary - pressure effects, the reaction of rock when subjected to high stress, pressure and temperature gradients at the shallower levels in the Earth's crust and influences of the compressibility as pressure are reduced by fluid withdrawals.

There are four conditions that must be satisfied so that a geological formation, or a part thereof, should form a suitable reservoir, for example for the accumulation of oil. These are porosity, permeability, seal and closure. The first defines the pore space in the rock - space in which the oil may collect. Permeability is the attribute of the rock that permits the passage of fluid through it. Generally, it is a measure of the degree interconnectedness, of the pore space, but some reservoir (e.g. in the massive limestone deposits, or in igneous intrusions) depend for fluid flow on a network of fractures within the rock.

Furthermore, the seal is the "cap" of the reservoir and prevents the oil from leaking away, while closure is a measure of the vertical extent of the sealed trap or, in the case of resources accumulation bounded below by a moving body of water, of the "height" of the sealed trap where that height is measured along a line perpendicular to the oil - water contact.

Almost all resources occur in sedimentary basins, in porous standstones or limestones and that seal or cap rock is often a clay or shale, or massive unfractured limestone having little or no permeability. On the other hand, three general categories of resources can be mentioned for marine reserves: structural traps, stratigraphic traps and combination traps.

Elastic waves are sound waves, generally three - dimensional and they may be transmitted through matter in any phase - solid, liquid, or gas. Any body vibrating in air gives rise to such waves, as it alternately compresses and rarefies the air adjacent to its surfaces. A body vibrating in a liquid, or in contact with a solid, likewise generates similar longitudinal waves. The frequency of the waves is of course the same as the frequency of the vibrating body which produces them.

The distance between two successive maxima (or between any two successive points in the same phase) is the wavelength of the wave and is denoted by l. Since the wave form, travelling with constant velocity u, advances a distance of one wavelength in a time interval of one period, it follows that the velocity of sound waves u as following:

$$u = l\, v \tag{2.1}$$

where v denotes the frequency.

As it is obvious the velocity u differs when the sound waves are travelling through solid, liquid, or gas. In a solid the elastic waves are moving faster than in a liquid and the air, and in a liquid faster than in the air. Therefore, if somebody is searching for example for oil marine resources over the sea, by transmitting sound waves, then there will be a difference in the velocity of the waves in the air, the sea, the solid bottom and in a potential reservoir.

In order to better explain the new method, consider the example of Figure 1. In this case consider that in the bottom of the sea there is a potential oil reservoir. Then, the speed of the elastic waves in the air (u_{air}), will be different from the speed in the water (u_{water}), and different from the speed in the solid bottom (u_{solid}) and different from the speed in the potential reservoir (u_{oil}), while the frequency of the elastic waves remaining the same when transmitted through every different matter.

A real - time non-linear 3-D plane - polarized elastic waves expert system is proposed in order to explore the marine oil resources, for the several closed seas all over the world, according to the new theory of *"Real-time Expert Seismology"* as proposed by Ladopoulos [25], [26], in contrast to the old theory of *"Reflection Seismology"* (Aki and Richards [1], Hale [9], Thomsen [38], [39], Dellinger, Muir and Karrenbach [3], Harrison and Stewart [10], Tsvankin and Thomsen [40], Alkhalifah and Tsvankin [2], Gaiser [7], Schmelzbach, Green and Horstmeyer [35], Schmelzbach, Horstmeyer and Juhlin [36]). Such Generic Sound Waves Technology will work under Real Time Logic for searching marine hydrocarbon reservoir developed on the continental crust and on deeper waters ranging from 300 m to 3000 m, or even much deeper (Figure 2). There are many deeper water prospects around the seas all over the world, but because of the paucity of the available information it is not possible at present to quantify the amounts that may be recoverable from them. For this reason the proposed real - time elastic waves expert system will be the best device for the exploration of the continental margin areas (shelf, slope and rise) and the very deep waters, too.

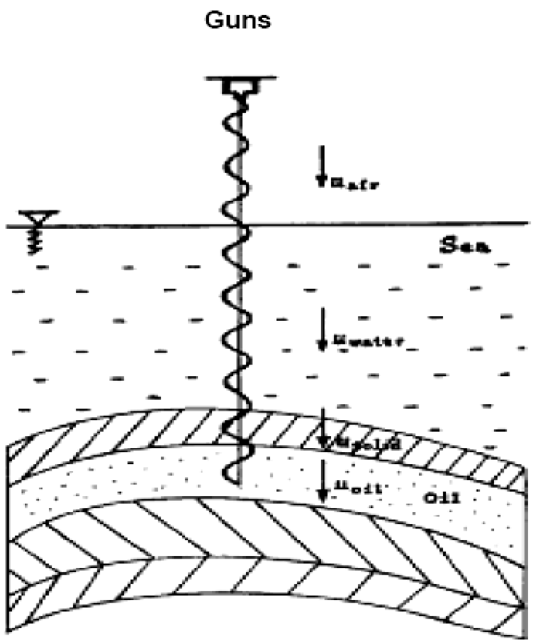

Fig. 1 Elastic Waves Method for the Exploration of Marine Resources.

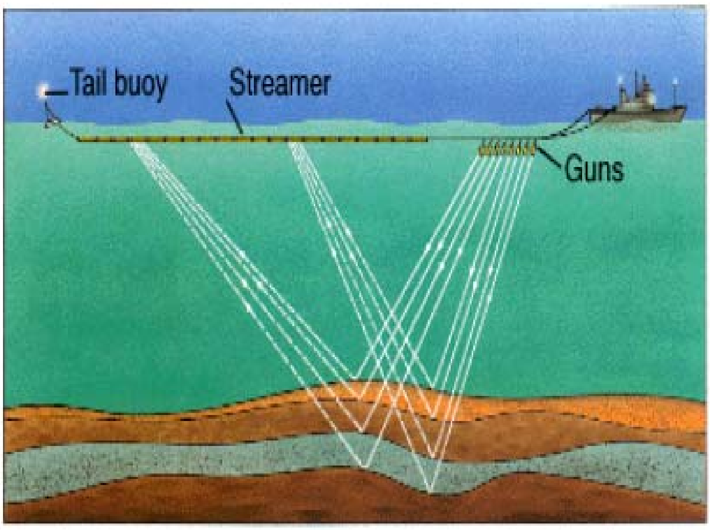

Fig. 2 Real-time Expert Seismology.

By using the proposed modern technology of *"Real-Time Expert Seismology"* the average velocity of the sound waves is calculated by providing important information about the composition of the solids through of which passed the sound waves. For example the velocity of the sound waves through the air is 331 m/sec, through liquid 1500 m/sec and through sedimentary rock 2000 to 5000 m/sec. Furthermore, according to the law of Reflection the angle of reflection equals the angle of incidence (Figure 3). Then according to the new method the arrival times of the seismic waves are analyzed. After the sensor measures the precise arrival time of the wave, then the velocity of the wave can be calculated by using the method which follows.

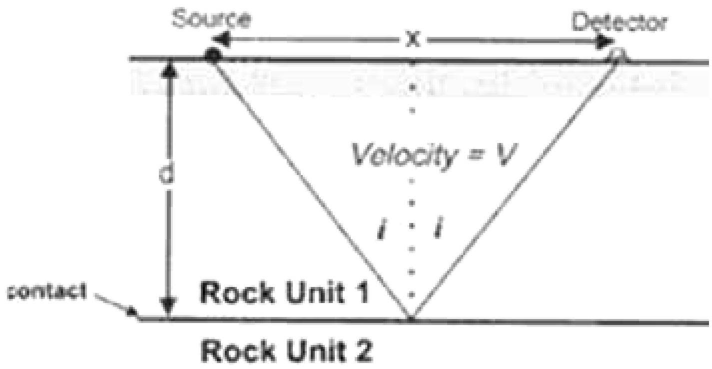

Fig. 3 Law of Reflection.

The travel time *T* of the seismic waves is given by the relation:

$$T = \frac{2\left(d^2 + x^2/4\right)^{1/2}}{v} \tag{2.2}$$

where *d* denotes the depth, *x* the distance between source of wave and the geophone or hydrophone detector and *v* is the average speed.

Beyond the above, from (2.2) follows equation (2.3):

$$T^2 = \frac{4d^2 + x^2}{v^2} \tag{2.3}$$

Also, the normal incident time T_o is given by the formula:

$$T_o = \frac{2d}{v} \tag{2.4}$$

From eqs (2.3) and (2.4) finally follows:

$$T^2 - T_o^2 = \frac{x^2}{v^2} \tag{2.5}$$

Hence, from eqn (2.5) follows that the travel time curve for a constant velocity horizontal layer model is a hyperbola whose apex is at the zero-offset travel time T_o:

$$\frac{T^2}{T_o^2} - \frac{x^2}{(T_o v)^2} = 1 \tag{2.6}$$

Finally from (2.5) the mean velocity is equal to:

$$v = \frac{x}{\sqrt{T^2 - T_o^2}} \tag{2.7}$$

Thus, a real time expert system is used and the apparatus permitted excitation of any combination of elements and reception of any other, visual analysis of the responses, and transfer of the signals to the PC for post processing. The sequencing of transducer excitation, digitiser configuration and subsequent data analysis was performed by a rule based Real-Time Expert System. From the information gathered, the Expert System applies knowledge via a series of software coded rules and provides any one of the following conditions: speed in the air (u_{air}), speed in the water (u_{water}), speed in the solid bottom (u_{solid}) and speed in the potential reservoir (u_{oil}),

Real-time logic (RTL) is a reasoning system for real-time properties of computer based systems. RTL's computational model consists of events, actions, causality relations, and timing constraint (Jahanian and Mok [11], [12], Emnis *et al.* [5], Fritz, Haase and Kalcher [6], Haase [8]). This model is expressed in a first order logic describing the system properties as well as the systems dependency on external events. The RTL system introduces time to the first logic formulas with an event occurrence function, which assign time values to event occurrences. Furthermore, real-time computing in common practice is characterized by two major criteria: deterministic and fast response to external stimulation, and both human and sensor and actor based interaction with the external world. Real-time is an external requirement for a peace of software; it is not a programming technology.

In general, Real-Time Logic uses three types of constraints:

1. Action constants may be primitive or composite. In a composite constant, precedence is imposed by the event-action model using sequential or parallel relations between actions.
2. Event constants are divided into three cases. Start/stop events describe the initiation/termination of an action or subaction. Transition events are those which make a change in state attributes. This means, that a transition event changes an assertion about the state of the real-time system or its environment. The third class, which are the external events, includes those that can be impact the system behavior, but cannot be caused by the system.
3. Integers assigned by the accurance function provide time values, and also denote the number of an event occurance in a sequence.

Moreover, the RTL System introduces time to the first order logic formulas with an event occurance function denoted by e. The mechanism to achieve a timing property of a system is the deduction resolution.

Consider further the following example: Upon pressing button ≠ 20, action TEST is extended within 200 time units. Dusting each execution of this action, the information is sampled and subsequently transmitted to the display panel. Also, the computation time of action TEST is 80 time units.

This example can be further translated into the following two formulas:

$$\forall\ x : e(\ \Omega\ \text{button } 20,\ x) \le e(\uparrow \text{TEST}, x) \wedge$$
$$e(\downarrow \text{TEST}, x) \le e(\ \Omega\ \text{button } 20,\ x) + 200$$
$$\forall\ y : e(\uparrow \text{TEST}, y) + 80 \le e(\downarrow \text{TEST}, y)$$

3. Well Test Analysis for Oil Reservoir

Oil well test analysis is a kind of an important history matching process for the determination of the properties of reservoir materials. Thus, during the movement of oil reservoir through porous media, then both single-phase and multiphase flow occurs. Also, when a petroleum well test is conducted then the well is subjected to a change of its flow rate and the resulting pressure response is possible to be measured. This pressure is further compared to analytical or numerical models in order to estimate reservoir parameters such as permeability.

In general an oil reservoir well test in a single-phase reservoir is calculated by using the porous medium equation:

$$\nabla \bullet \left(\frac{\lambda}{\phi\xi}\nabla p\right) = c_t \frac{\partial p}{\partial t} \tag{3.1}$$

in which λ denotes the permeability, ϕ the porosity, ξ the viscosity, p the pressure of the reservoir, t the time and c_t the compressibility.

By replacing variables as follows:

$$u = \left(\frac{\lambda}{\phi\xi}\right)^{1/2} p \tag{3.2}$$

then (3.1) can be written as:

$$\nabla^2 u + \lambda' u = 0 \tag{3.3}$$

with :

$$\lambda' = -\frac{\nabla^2 (\lambda/\phi\xi)^{1/2}}{(\lambda/\phi\xi)^{1/2}} \tag{3.4}$$

Hence, (3.3) is a Helmholtz differential equation.

Beyond the above, consider by $u^*(\mathbf{x},\mathbf{y})$ the fundamental solution of any point $\mathbf{y}$, because of the source point $\mathbf{x}$. Then, the fundamental solution can be given by the following equation:

$$\nabla^2 u^*(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) + \delta(\mathbf{x},\mathbf{y}) = 0 \tag{3.5a}$$

which may be further written as:

$$u^*_{,ii}(\mathbf{x},\mathbf{y}) + \lambda' u^*(\mathbf{x},\mathbf{y}) + \delta(\mathbf{x},\mathbf{y}) = 0 \tag{3.5b}$$

Thus, (3.5) is the Helmholtz potential equation governing the fundamental solution.

Consider further by u^* the fundamental solution chosen so that to enforce the Helmholtz equation in terms of the function *u,* in a weak form. Then the weak form of Helmholtz equation will be written as following:

$$\int_{\Omega} (\nabla^2 u + \lambda' u) u^* d\Omega = 0 \tag{3.6}$$

in the solution domain *Ω*.

Also, by applying the divergence theorem once in (3.6), one obtains a symmetric weak form:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\Omega} u_{,i} u^*_{,i} d\Omega + \int_{\Omega} \lambda' u u^* d\Omega = 0 \tag{3.7}$$

in which **n** denotes the outward normal vector of the surface *S.*

Therefore, in the symmetric weak form the function *u* and the fundamental solution u^* are only required to be first - order differentiable. By applying further the divergence theorem twice in (3.6) one has:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\partial\Omega} n_i u u^*_{,i} dS + \int_{\Omega} u (u^*_{,ii} + \lambda' u^*) d\Omega = 0 \tag{3.8}$$

Hence, (3.8) is the asymmetric weak form and the fundamental solution u* is required to be second - order differentiable. On the other hand, *u* is not required to be differentiable in the domain *Ω*.

By combining (3.5) and (3.8), then one obtains:

$$u(\mathbf{x}) = \int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} n_i(\mathbf{y}) u(\mathbf{y}) u^*_{,i}(\mathbf{x}.,\mathbf{y}) dS \tag{3.9}$$

which can be further written as:

$$u(\mathbf{x}) = \int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} u(\mathbf{y}) R^*(\mathbf{x},\mathbf{y}) dS \tag{3.10}$$

where $q(\mathbf{y})$ denotes the potential gradient along the outward normal direction of the boundary surface:

$$q(\mathbf{y}) = \frac{\partial u(\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u_{,k}(\mathbf{y}) \quad , \quad \mathbf{y} \in \partial\Omega \tag{3.11}$$

and the kernel function:

$$R^*(\mathbf{x},\mathbf{y}) = \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u^*_{,k}(\mathbf{x},\mathbf{y}) \quad , \quad \mathbf{y} \in \partial\Omega \tag{3.12}$$

By differentiating (3.10) with respect to x_k we obtain the integral equation for potential gradients $u_{,k}(\mathbf{x})$ by the following formula:

$$\frac{\partial u(\mathbf{x})}{\partial x_k}=\int_{\partial\Omega} q(\mathbf{y})\frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial x_k}dS-\int_{\partial\Omega} u(\mathbf{y})\frac{\partial R^*(\mathbf{x},\mathbf{y})}{\partial x_k}dS \qquad (3.13)$$

4. Fundamental Solution's Basic Properties

We rewrite the weak form of (3.5) governing the fundamental solution, as follows:

$$\int_{\Omega}\left[\nabla^2 u^*(\mathbf{x},\mathbf{y})+\lambda' u^*(\mathbf{x},\mathbf{y})\right]c\,d\Omega+c=0, \quad \mathbf{x}\in\Omega \qquad (4.1)$$

where c denotes a constant, considering as the test function.

Also, (4.1) can be written as:

$$\int_{\Omega}\left[u^*_{,ii}(\mathbf{x},\mathbf{y})+\lambda' u^*(\mathbf{x},\mathbf{y})\right]d\Omega+1=0, \quad \mathbf{x}\in\Omega \qquad (4.2)$$

Furthermore, (4.2) takes the form:

$$\int_{\partial\Omega} n_i(\mathbf{y})u^*_{,i}(\mathbf{x},\mathbf{y})dS+\int_{\Omega}\lambda' u^*(\mathbf{x},\mathbf{y})d\Omega+1=0, \quad \mathbf{x}\in\Omega \qquad (4.3)$$

By considering further an arbitrary function *u(x)* in Ω as the test function, then the weak form of (3.5) will be written as:

$$\int_{\Omega}\left[\nabla^2 u^*(\mathbf{x},\mathbf{y})+\lambda' u^*(\mathbf{x},\mathbf{y})+\delta(\mathbf{x},\mathbf{y})\right]u(\mathbf{x})d\Omega=0, \quad \mathbf{x}\in\Omega \qquad (4.4)$$

and also as:

$$\int_{\Omega}\left[u^*_{,ii}(\mathbf{x},\mathbf{y})+\lambda' u^*(\mathbf{x},\mathbf{y})\right]u(\mathbf{x})d\Omega+u(\mathbf{x})=0, \quad \mathbf{x}\in\Omega \qquad (4.5)$$

Finally, (4.5) takes the form:

$$\int_{\partial\Omega}\Phi^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS+\int_{\Omega}\lambda' u^*(\mathbf{x},\mathbf{y})u(\mathbf{x})d\Omega+u(\mathbf{x})=0, \quad \mathbf{x}\in\Omega \qquad (4.6)$$

If $\mathbf{x}$ approaches the smooth boundary $(\mathbf{x}\in\partial\Omega)$, then the first term in (4.6) may be written as:

$$\lim_{x\to\partial\Omega}\int_{\partial\Omega}\Phi^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS=\int_{\partial\Omega}^{CPV}\Phi^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS-\frac{1}{2}u(\mathbf{x}) \qquad (4.7)$$

in the sense of a Cauchy Principal Value (CPV) integral.

For the understanding of the physical meaning of (4.7), we rewrite (4.3) and (4.6) as:

$$\int_{\partial\Omega}^{CPV} \Phi^*(\mathbf{x},\mathbf{y})dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y})d\Omega + \tfrac{1}{2} = 0, \quad x \in \partial\Omega \tag{4.8}$$

and:

$$\int_{\partial\Omega}^{CPV} \Phi^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y})u(\mathbf{x})d\Omega + \tfrac{1}{2}u(\mathbf{x}) = 0, \quad x \in \partial\Omega \tag{4.9}$$

By (4.8) follows that only a half of the source function at point **x** is applied to the domain *Ω*, when the point **x** approaches a smooth boundary, $\mathbf{x} \in \partial\Omega$.

Also, consider another weak form of (3.5) by supposing the vector functions to be the gradients of an arbitrary function *u*(**y**) in *Ω*, chosen in such a way that they have constant values:

$$u_{,k}(\mathbf{y}) = u_{,k}(\mathbf{x}) \;, \quad \text{for } k=1,2,3 \tag{4.10}$$

Then the weak form of (3.5) will be written as:

$$\int_{\Omega} \left[u^*_{,ii}(\mathbf{x},\mathbf{y}) + \lambda'' u^*(\mathbf{x},\mathbf{y}) \right] u_{,k}(\mathbf{y})d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{4.11}$$

By applying further the divergence theorem, then (4.11) takes the form:

$$\int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})dS + \int_{\Omega} \lambda' u^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{4.12}$$

Furthermore, the following property exists:

$$\int_{\partial\Omega} n_i(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,k}(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega} n_k(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,i}(\mathbf{x},\mathbf{y})dS$$

$$= \int_{\Omega} u_i(\mathbf{x})u^*_{,ki}(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega} u_{,i}(\mathbf{x})u^*_{,ik}(\mathbf{x},\mathbf{y})dS = 0 \tag{4.13}$$

By adding (4.12) and (4.13) then one obtains:

$$\int_{\partial\Omega} n_i(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,k}(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega} n_k(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,i}(\mathbf{x},\mathbf{y})dS \tag{4.14}$$

$$+\int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y})u^*_{,k}(\mathbf{x})dS + \int_{\Omega} \lambda'' u^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega + u_{,k}(\mathbf{x}) = 0$$

which takes finally the form:

$$\int_{\partial\Omega} n_i(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,k}(\mathbf{x},\mathbf{y})dS + \int_{\partial\Omega} e_{ikt}R_t u(\mathbf{x})u^*_{,i}(\mathbf{x},\mathbf{y})dS$$

$$+\int_{\partial\Omega} \lambda' u^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega + u_{,k}(\mathbf{x}) = 0 \qquad (4.15)$$

5. Analysis by Non-linear Singular Integral Equations

The porous medium equation (3.1) will be further written in another form, in order a singular integral equations representation to be applicable:

$$\nabla^2 p = -\nabla \ln\left(\frac{\lambda}{\Phi\xi c_t}\right) \bullet \nabla p + \frac{\Phi\xi c_t}{\lambda}\frac{\partial p}{\partial t} \qquad (5.1)$$

By applying the Green Element Method, then (5.1) reduces to the solution of a non-linear singular integral equation:

$$-\frac{\theta}{2\pi}p(r_i) + \int_{\partial\Omega}\left(p\frac{\partial[\ln(r-r_i)]}{\partial n} - \ln(r-r_i)\frac{\partial p}{\partial n}\right)dS +$$

$$+\iint_{\Omega} \ln(r-r_i)\left[-\nabla \ln \Lambda \bullet \nabla p + \frac{1}{\Lambda}\frac{\partial p}{\partial t}\right]d\Omega = 0 \qquad (5.2)$$

in which:

$$\Lambda = \frac{\Phi\xi}{\lambda}c_t \qquad (5.3)$$

In order the non-linear singular integral equation (5.2) to be numerically evaluated, then the Singular Integral Operators Method (S.I.O.M.) will be used. Thus, the non-linear singular integral equation (5.2) is approximated by the formula:

$$-\frac{\theta}{2\pi}p(r_i) + \sum_{e=1}^{M}\left[\int_{\partial\Omega}\left(p\frac{\partial[\ln(r-r_i)]}{\partial n} - \ln(r-r_i)\frac{\partial p}{\partial n}\right)dS + \iint_{\Omega}\left(-\nabla \ln \Lambda \bullet \nabla p + \frac{1}{\Lambda}\frac{\partial p}{\partial t}\right)d\Omega\right] = 0 \qquad (5.4)$$

where M denotes the total number of elements.

Beyond the above, let us introduce the following functions describing the pressure at any point in an element, in terms of the nodal pressures:

$$p(x,y) = N_j(x,y)p_j \qquad (5.5)$$

By replacing (5.5) then (5.4) takes the form:

$$\sum_{e=1}^{M}\left(A_{ij}^{e}p_{j}+B_{ij}^{e}q_{j}-C_{ijl}^{e}\ln\Lambda_{j}p_{l}+D_{ijl}^{e}\frac{1}{\Lambda_{j}}\frac{\partial p_{l}}{\partial t}\right)=0, \qquad i,j,l=1,2,3,4 \qquad (5.6)$$

where:

$$A_{ij}^{e}=\int_{\partial\Omega}\frac{\partial[\ln(r-r_{i})]}{\partial n}\Omega_{j}dS-\delta_{ij}\frac{\theta}{2\pi} \qquad (5.7)$$

$$B_{ij}^{e}=-\int_{\partial\Omega}\ln(r-r_{i})\Omega_{j}dS \qquad (5.8)$$

$$C_{ijl}^{e}=\iint_{\Omega_{j}}\ln(r-r_{i})\left[\frac{\partial N_{j}}{\partial x}\frac{\partial N_{l}}{\partial x}+\frac{\partial N_{j}}{\partial y}\frac{\partial N_{l}}{\partial y}\right]d\Omega \qquad (5.9)$$

$$D_{ijl}^{e}=\iint_{\Omega_{j}}\ln(r-r_{i})N_{j}N_{l}d\Omega \qquad (5.10)$$

6. Well Testings Applications in Heterogeneous Reservoirs

The previous mentioned theory will be applied to the determination of a well testing, which will be checked in an heterogeneous reservoir with a permeability varying from 10 mD to 300 mD (1mDarcy ≈ $10^{-12}\,m^2 = 1(\mu m)^2$).

So, by using the Singular Integral Operators Method (S.I.O.M.) as described by the previous paragraphs, then the computation of the pressure response from the well test conducted in the above heterogeneous reservoir will become possible. Firstly, the pressures were computed in variation with the time. Thus, Table 1 shows the pressure response with respect to the time.

In addition, the pressure derivatives were computed with respect to the time, as shown in Table 2. Such derivatives are very much important of the well testings interpretation as these are some distinct shapes and especially the characteristics of certain reservoir features.

The computational results of the pressures and the pressure derivatives are compared to the analytical solutions of the same well testing problem, if the reservoir was homogeneous with permeability equal to 50 mD. So, the analytical results are shown in Table 1 for the pressures and in Table 2 for the pressure derivatives, correspondingly. From the above Tables it can be seen that there is very small difference between the computational results and the analytical solutions for both the pressures and the pressure derivatives. On the other hand, the above mentioned small difference can be explained because of the diffusive nature of the pressure transport mechanism. Finally same results are shown, correspondingly in Figures 4 and 5, and in three-dimensional form in Figures 4a and 5a.

Table 1

Time (hours)	Pressure (psi) S.I.O.M.	Pressure (psi) Analytical
0.002	7.003	7.022
0.009	10.002	10.013
0.015	12.002	12.031
0.030	12.504	12.523
0.040	13.003	13.014
0.070	13.503	13.502
0.100	14.002	14.033
0.250	14.501	14.521
0.400	15.004	15.032
1.000	15.502	15.514
2.000	16.004	16.023
10.00	17.002	17.022
30.00	17.504	17.524
80.00	18.001	18.042
100.00	19.003	19.032
200.00	20.000	20.030
400.00	20.000	20.020
600.00	20.000	20.010
1000.00	20.000	20.000

Table 2

Time (hours)	Pressure Derivative (psi) S.I.O.M.	Pressure Derivative (psi) Analytical
0.002	1.504	2.002
0.009	2.002	2.003
0.015	2.001	2.003
0.030	2.002	2.002
0.040	2.003	2.002
0.070	2.004	2.003
0.100	2.002	2.004
0.250	2.001	2.002
0.400	2.003	2.003
1.000	2.002	2.002
2.000	2.004	2.003
10.00	2.001	2.002
30.00	2.003	2.003
80.00	2.002	2.003
100.00	1.001	1.301
200.00	0.600	0.800
400.00	0.250	0.260
600.00	0.060	0.060
1000.00	0.030	0.010

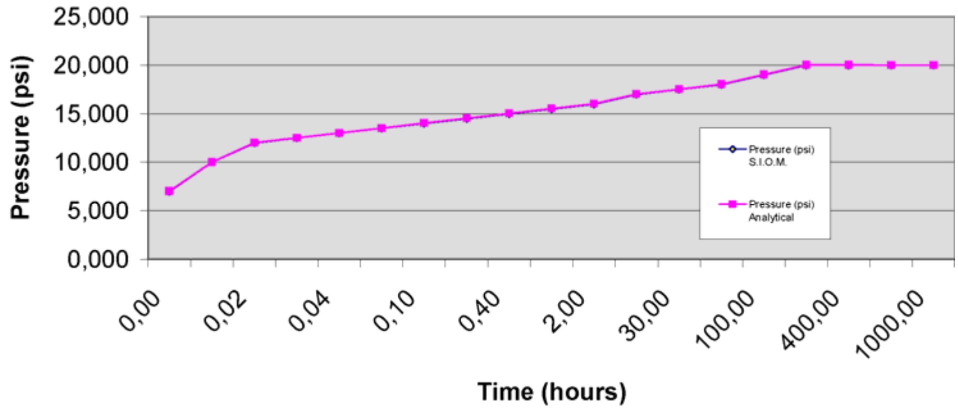

Fig. 4 Pressure Response for Well Test in Heterogeneous Reservoir.

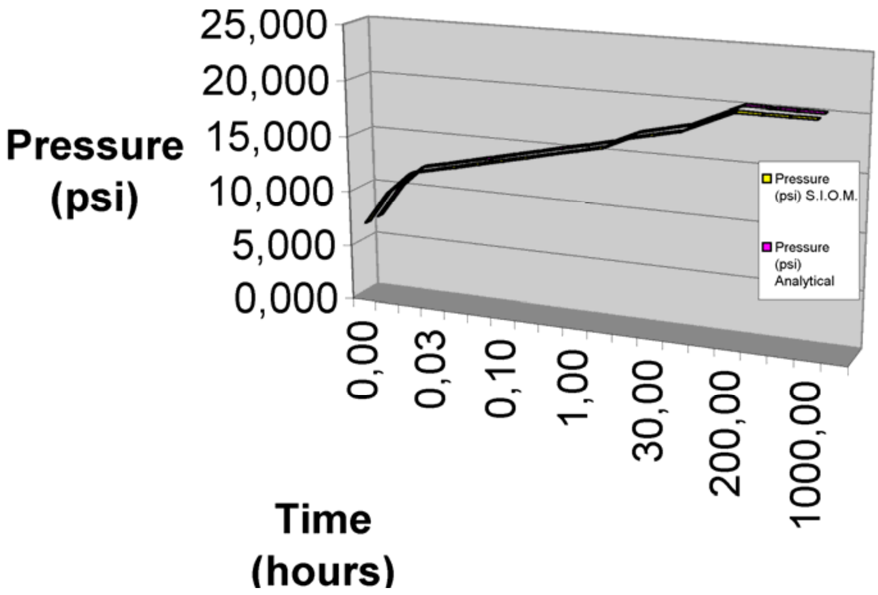

Fig. 4a 3-D Distribution of Pressure Response for Well Test in Heterogeneous Reservoir.

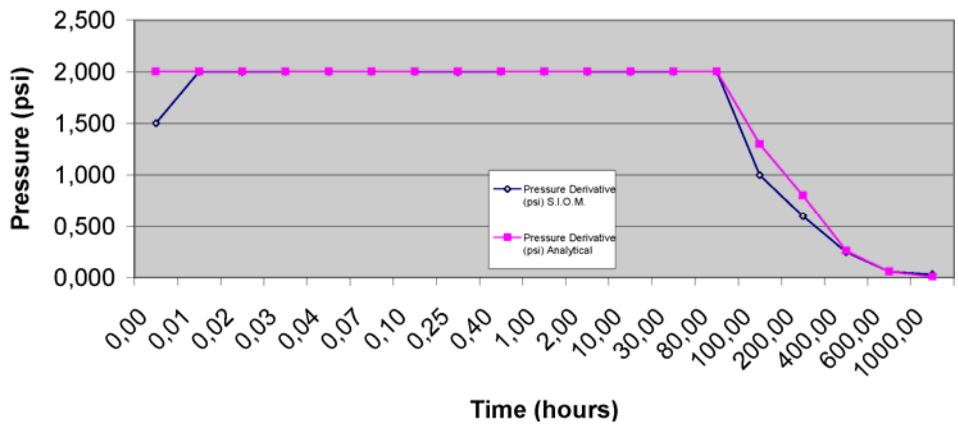

Fig. 5 Pressure Derivative for Well Test in Heterogeneous Reservoir.

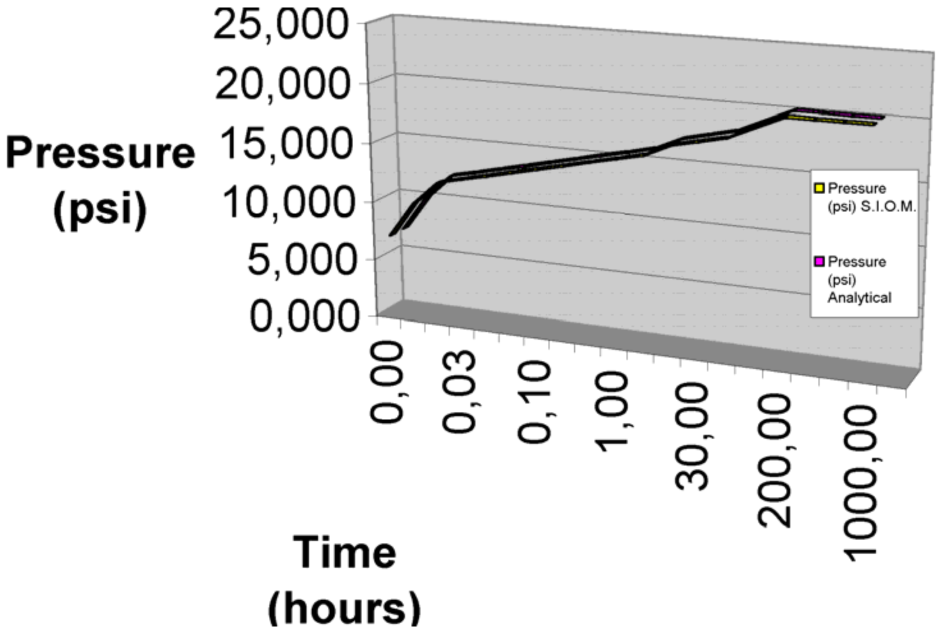

Fig. 5a 3-D Distribution of Pressure Derivative for Well Test in Heterogeneous Reservoir.

7. Conclusions

By the present investigation the new theory of *"Real-time Expert Seismology"* has been introduced and investigated for the exploration of on-shore and off-shore oil reserves. The benefits of the new technology of *"Real-time Expert Seismology"* in comparison to the existing theory of *"Reflection Seismology"* are the following:

1. The new theory is based on the special form of the geological anticlines of the bottom of the sea, in order to decide which areas of the bottom have the most possibilities to include petroleum.
On the other hand, the existing theory is only based to the best chance and do not include any theoretical and sophisticated model.

2. The new theory of elastic (sound) waves is based on the difference of the speed of the sound waves which are traveling through solid, liquid, or gas. In a solid the elastic waves are moving faster than in a liquid and the air, and in a liquid faster than in the air.
Existing theory is based on the application of Snell's law and Zoeppritz equations, which are not giving good results, as these which we are expecting with the new method.

3. The new theory is based on a Real-time Expert System working under Real Time Logic, that gives results in real time, which means every second.
Existing theory do not include real time logic.

From the above three points it can be well understood the evidence of the applicability of the new method of *"Real-time Expert Seismology"*. Also its novelty, as it is based mostly on a theoretical and very sophisticated Real-time Expert model and not to practical tools like the existing method.

Also, a mathematical model has been presented as an attempt to determine the properties of the reservoir materials. So, the study of the movement of oil reserves through porous media is very important for petroleum reservoir engineers. The above mentioned problem was reduced to the solution of a non-linear singular integral equation, which was numerically evaluated by using the Singular Integral Operators Method (S.I.O.M.).

Furthermore, several important properties of the porous medium equation, which is a Helmholtz differential equation, were analyzed and investigated. Thus, the fundamental solution of the porous medium equation was proposed and studied. Also, some basic properties of the fundamental solution were further investigated. These are very important in order the behavior of the non-linear singular integral equation to be well understood.

An application was finally given for a well testing to be checked when a heterogeneous oil reservoir is moving in a porous solid. The above problem was solved by using the Singular Integral Operators Method and thus the pressure response from the well test conducted in the above heterogeneous oil reservoir, was computed. Both the pressures and the pressure derivatives were computed and these values were compared to the analytical solutions of the same well testing problem, if the reservoir was homogeneous with a mean permeability.

Over the last years, non-linear singular integral equation methods have been used with a big success for the solution of several important engineering problems of structural analysis, elastodynamics, hydraulics, fluid mechanics and aerodynamics. For the numerical evaluation of the non-linear singular integral equations of the above problems, were used several aspects of the Singular Integral Operators Method (S.I.O.M.). Thus, by the present research such methods were extended for the solution of oil reserves problems in petroleum reservoir engineering.

The benefits of the new method in comparison to existing methods are the following:

1. The new method is based on the non-linear programming method, by using non-linear singular equations. According to this theory the porous medium equation is reduced to the solution of a non-linear singular integral equation which is then numerically evaluated by using a non-linear programming method.

Existing methods of well test analysis, are using too as a start the porous medium equation, but as this is a complicated differential equation are giving only some analytical solutions for very simple cases or numerical solutions for homogeneous reservoir materials..

2. The new method, as it is a complicated non-linear numerical method can give results for heterogeneous porous media (which of course are the solids in reality) and not only for homogeneous solids as are giving the analytical or numerical existing methods.

So the estimation of the properties and the future petroleum production from a new oil reservoir could be done exactly, and not estimated as by the existing methods.

From the above two points it can be understood the evidence of the applicability of the new method, as it is based on non-linear software. Also its novelty, as it is based on the theory of non-linear singular integral equations.

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