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Non-linear Elastodynamics by 3 & 4-D Real-Time Expert Seismology

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Abstract
"*Non-linear Real-Time Expert Seismology*" is a modern method by using a non-linear 3-D elastic waves real - time expert system, for the exploration of the on-shore and off-shore oil & gas reserves all over the world. The above innovative hydrocarbon exploration method is working by using elastic (seismic) waves moving in an unbounded subsurface medium, for searching the land and marine petroleum reservoir developed on the continental crust and in deeper water ranging from 300 to 3000 m, or even deeper. This new method can be used at any depth of seas and oceans all over the world and for any depth in the subsurface of existing petroleum reserves. By the present paper 3 & 4-D multiphase flows are proposed, which incorporates many 3-D multiphase flows over the same reservoir at specified intervals of time. Hence, by studying multiple time-lapsed 3-D surveys, or three-dimensional subsurface flows, portrays the changes in the reservoir over time. Besides, several mechanical properties of rock regulating the wave propagation phenomenon appear as spatially varying coefficients in a system of time-dependent hyperbolic partial differential equations. The wave equation describes the propagation of the seismic waves through the earth subsurface and this equation is finally reduced to a Helmholz differential equation. Consequently, the Helmholtz differential equation is numerically solved by using the *Singular Integral Operators Method (S.I.O.M.)*. Furthermore, several properties are analyzed and investigated for the wave equation. An application is given to the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium.

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Key Word and Phrases
Singular Integral Operators Method (S.I.O.M.), 3 & 4-D Non-linear Real-time Expert Seismology, Time-Dependent Hyperbolic Partial Differential Equations, Wave Equation, Petroleum Reserves, Helmholtz Differential Equation, Real - Time Expert System.

1. 3 & 4-D Non-linear Real-Time Expert Seismology

The new method *"Non-linear Real-Time Expert Seismology"* is the main and best tool which can be used by the petroleum industry to map oil & gas deposits in the Earth's upper crust. Thus, environmental and civil engineers can also use variants of the above modern technique to locate bedrock, acquifers, and other near-surface features, academic geophysicists can extend it to a tool for imaging the lower crust and mantle. The new petroleum exploration method of *"Non-linear Real-Time Expert Seismology"* was introduced and investigated by E.G.Ladopoulos [25] - [30], as an extension on his methods on non-linear singular integral equations in potential flows, fluid mechanics, structural analysis, solid mechanics, hydraulics and aerodynamics [15] - [24].

Additionally, seismic wave propagation, the physical phenomenon underlying the *"Non-linear Real-Time Expert Seismology"* as well as other types of seismology, is modeled with reasonable accuracy as small-amplitude displacement of a continuum, using various specializations and generalizations of linear elastodynamics. In the above models, there are several mechanical properties of rock regulating the wave propagation phenomenon appear as spatially varying coefficients in a system of time-dependent hyperbolic partial differential equations.

In general, the *"Non-linear Real-Time Expert Seismology"* seeks to extract maps of the Earth's sedimentary crust from transient near-surface recording of echoes, stimulated by explosions or other controlled sound sources positioned near the surface. Hence, reasonably accurate models of seismic energy propagation take the form of hyperbolic systems of partial differential equations, in which the coefficients represent the spatial distribution of various mechanical characteristics of rock, like density, stiffness, etc. Consequently, the exploration geophysics community has developed various methods for estimating Earth structure from seismic data, however the very modern and groundbreaking method *"Non-linear Real-Time Expert Seismology"* seems to be the best tool for on-shore and off-shore petroleum reserves exploration for very deep drillings ranging up to 20,000 or 30,000 m.

Furthermore, 3 & 4-D seismic can help to locate untapped pockets of oil or gas within the reservoir. Typically, 4-D seismic data is processed by subtracting the data from one survey from the data of another. The amount of change in the reservoir is defined by the difference between the two. If no change has occurred over the time period, the result will be zero.

During the last years several variants of integral equations methods were used for the solution of elastodynamics and acoustic problems. Already at the end of sixty's H.A. Shenk [1] stated that the integral equation for potential mathematically failed to yield unique solutions to the exterior acoustic problem and proposed a method in which an over determined system of equations at some characteristic frequencies was formed by combining the surface Helmholtz equation with the corresponding interior Helmholtz equation. Thus, it was analytically proved, that the system of equations provide a unique solution at the same characteristic frequencies, to some extent. However, the above method might fail to produce unique solutions, when the interior points used in the collocation of the Boundary Integral Equations were located on a nodal surface of an interior standing wave.

Besides, at the start of seventy's A.J. Burton and G.F. Miller [2] proposed a combination of the surface Helmholtz integral equation for potential and the integral equation for the normal derivative of potential at the surface, to circumvent the problem of nonuniqueness at characteristic frequencies. Their method was called Composite Helmholtz Integral Equation. Several years later, W.L. Meyer, W.A. Bell, B.T. Zinn and M.P. Stallybrass [3] and T. Terai [4], developed regularization techniques for planar elements for the calculation of sound fields around three dimensional objects by integral equation methods.

On the contrary, Z. Reut [5], investigated further the Composite Helmhholtz Integral Equation Method by introducing the hypersingular integrals. Also, in the above numerical method, the accuracy of the integrations affects the results and the conventional Gauss quadrature can not be used directly.

The basic idea by using the gradients of the fundamental solution to the Helmholtz differential equation for velocity potential, as vector test functions to write the weak form of the original Helmholtz differential equation for potential and so directly to derive a non hypersingular boundary integral equations for velocity potential gradients, was proposed and investigated by H. Okada, H. Rajiyah and S.N. Atluri [6] and H. Okada and S.N. Atluri [9]. The above scientists used the displacement and velocity gradients to directly establish the numerically tractable displacement and displacement gradient boundary integral equations in elasto-plastic solid problems and traction boundary integral equations. Moreover, C.C. Chien, H. Rajiyah and S.N. Atluri [7], employed some known identities of the fundamental solution from the associated interior Laplace problem, to regularize the hypersingular integrals.

In addition, T.W. Wu, A.F. Seybert and G.C. Wan [8], proposed the regularized normal derivative equation, to be converged in the Cauchy principal value sense. The computation of tangential derivatives was required everywhere on the boundary. Also, W.S. Hwang [10], reduced the singularity of the Helmholtz integral equation by using some identities from the associated Laplace equation. On the other hand, the value of the equipotential inside the domain must be computed, because the source distribution for the equipotential surface from the potential theory was used to regularize the weak singularities.

S.A. Yang [11] used further the identities of the fundamental solution of the Laplace problem to efficiently solve the problem of acoustic scattering from a rigid body. Moreover, Z.Y. Yan, K.C. Hung and H. Zheng [12], in order to solve the intensive computation of double surface integral, employed the concept of a discretized operator matrix to replace the evaluation of double surface integral with the evaluation of two discretized operator matrices.

On the other hand, Z.D. Han and S.N. Atluri [13] used traction boundary integral equations for the solution of the Helmholtz equation. Beyond the above, recently was used by S.N. Atluri, Z.D. Han and S. Shen [14] the meshless method, as an alternative numerical method, to eliminate the drawbacks in the Finite Element Method and the Boundary Element Method.

By the present research, the Singular Integral Operators Method (S.I.O.M.) will be used for the solution of elastodynamic problems by using the Helmholtz differential equation. For the above derivation will be used the gradients of the fundamental solution to the Helmholtz differential equation for the velocity potential. Besides, several basic identities governing the fundamental solution to the Helmholtz differential equation for the velocity potential are analyzed and investigated.

Thus, by using the Singular Integral Operators Method (S.I.O.M.), then the acoustic velocity potential will be computed. In addition, some properties of the wave equation, which is a Helmholtz differential equation, are proposed and investigated. Some basic properties of the fundamental solution will be further derived.

An application is finally proposed for the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium. Hence, by using the Singular Integral Operators Method (SIOM), then the acoustic pressure radiated from the above pulsating sphere will be computed. This is very important in petroleum reservoir engineering in order the size of the reservoir to be evaluated.

Consequently, the S.I.O.M. which was used very successfully for the solution of several engineering problems of fluid mechanics, hydraulics, aerodynamics, solid mechanics, potential flows and structural analysis, are further extended in the present investigation for the solution of petroleum reservoir engineering problems in elastodynamics.

2. 3 & 4-D Non-linear Seismic Wave Motion for Elastodynamics

In general, seismic wavelengths run in the tens of meters, so it is reasonable to presume that the mechanical properties of rocks responsible for seismic wave motion might be locally homogeneous on length scales of millimeters or less, which means that the Earth can be modeled as a mechanical continuum. Hence, except possible for a few meters around the source location, the wavefield produced in seismic reflection experiments does not appear to result in extended damage or deformation, so the waves are entirely transient. These considerations suggest a non-linear wave motion as a mechanical model in elastodynamics.

In an homogeneous medium the equations of elastodynamics are given by the following formulas:

$$\rho \frac{\partial v}{\partial t} = \nabla \bullet \boldsymbol{\sigma} + b \tag{2.1}$$

$$\frac{\partial \sigma}{\partial t} = \frac{1}{2} C(\nabla v + \nabla v^T) + \gamma \tag{2.2}$$

in which v denotes the particle velocity field, $\boldsymbol{\sigma}$ the stress tensor, b a body force density, γ a defect in the elastic constitutive law, ρ the mass density, t the time and C the Hooke's tensor.

Furthermore, the right hand sides b and γ provide a variety of representations for external energy input to the system.

Consequently, the new method for on-shore and off-shore petroleum reserves exploration *"Non-linear Real-Time Expert Seismology"* uses transient energy sources and produce transient wave fields. Thus, the appropriate initial conditions for the system of eqs (2.1) and (2.2) are as following:

$$v = 0, \ \sigma = 0, \qquad \text{for: } t << 0 \tag{2.3}$$

The Hooke's tensor, for isotropic elasticity, has only two independent parameters, the compressional and shear wave speeds c_p and c_s. It is instructive to examine direct measurements of these quantities, made in a borehole. Hence, there are two types of elastic waves produced: I) P-waves, which are primary or “compressional” waves, and II) S-waves, or shear waves.

By the present research, the seismic problem will not be developed in the generalized context of the elastodynamic system (2.1) and (2.2). Instead, our research will be limited to a special case of seismology. Hence, by the current model, it is supposed that the material does not support shear stress. Then the stress tensor becomes scalar, $\sigma = -pI$, p being the pressure, and only one significant component, the bulk modulus κ, is left in the Hooke tensor.

Thus, the system (2.1) and (2.2) reduces to:

$$\rho \frac{\partial v}{\partial t} = -\nabla p + b \tag{2.4}$$

$$\frac{1}{\kappa}\frac{\partial p}{\partial t} = -\nabla \bullet \mathbf{v} + h \tag{2.5}$$

where the energy source is represented as a constitutive law defect h.

On the other hand, the proposed model predicts wave motion c with spatially varying wave speed:

$$c = \sqrt{\frac{\kappa}{\rho}} \tag{2.6}$$

in which ρ is the mass density and κ the bulk modulus.

In addition, it is very convenient to represent the elastodynamics in terms of the acoustic velocity potential : $u(x,t) = \int_{-\infty}^{t} p(x,s)ds$, with the following results:

$$p = \frac{\partial u}{\partial t}$$

and:

$$\mathbf{v} = \frac{1}{\rho}\nabla u \tag{2.7}$$

By using (2.6) and (2.7), then the acoustic system (2.4) and (2.5) reduces to the wave equation, because of the propagation of seismic waves through an unbounded homegeneous solid:

$$\frac{1}{\rho c^2}\frac{\partial^2 u}{\partial t^2} - \nabla \bullet \frac{1}{\rho}\nabla u = h \tag{2.8}$$

Furthermore, by assuming that density ρ to be constant and that the source (transient constitutive law defect h) is an isotropic point radiator located at the source point, then the wave equation (2.8) reduces to the following Helmholtz differential equation:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0 \tag{2.9}$$

Besides, for time harmonic waves with a time factor $e^{-i\omega t}$, then the wave equation (2.9) reduces to:

$$\nabla^2 u + k^2 u = 0 \tag{2.10}$$

in which the wave number k is equal to:

$$k = \frac{\omega}{c} \tag{2.11}$$

with ω the angular frequency and c the speed of sound in the medium at the equilibrium state.

The fundamental solution of the wave equation (2.1) at any field point $\mathbf{y}$ due to a point sound source $\mathbf{x}$, for the two dimensions is valid as following:

$$u^*(\mathbf{x},\mathbf{y}) = \frac{i}{4} H_0^{(1)}(kr) \tag{2.12}$$

and:

$$\frac{\partial u^*}{\partial r}(\mathbf{x},\mathbf{y}) = -\frac{i}{4} k H_1^{(1)}(kr) \tag{2.13}$$

where $i = \sqrt{-1}$, $H_0^{(1)}(kr)$ denotes the Hankel function of the first kind and r is the distance between the field point $\mathbf{y}$ and the source point $\mathbf{x}$ ($r = |\mathbf{x} - \mathbf{y}|$).

Beyond the above, the fundamental solution of the wave equation (2.1) for the three dimensions is equal to:

$$u^*(\mathbf{x},\mathbf{y}) = \frac{1}{4\pi r} e^{-ikr} \tag{2.14}$$

and:

$$\frac{\partial u^*}{\partial r}(\mathbf{x},\mathbf{y}) = \frac{e^{-ikr}}{4\pi r^2}(-ikr - 1) \tag{2.15}$$

The fundamental solution $u^*(\mathbf{x},\mathbf{y})$ is further governed by the wave equation:

$$\nabla^2 u^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y}) + \Delta(\mathbf{x},\mathbf{y}) = 0 \tag{2.16}$$

So, eqn (2.16) is referred as the Helmholtz potential equation governing the fundamental solution.

Also, consider the weak form of the Helmholtz equation to be given by the relation:

$$\int_{\Omega} (\nabla^2 u + k^2 u) u^* d\Omega = 0 \tag{2.17}$$

in the solution domain *Ω*.

Then, by applying the divergence theorem once in (2.17), we obtain a symmetric weak form:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\Omega} u_{,i} u_{,i}^{\ *} d\Omega - \int_{\Omega} k^2 u u^* d\Omega = 0 \tag{2.18}$$

in which **n** denotes the outward normal vector of the surface *S*.

Thus, in the symmetric weak form the function u and the fundamental solution u^* are only required to be first - order differentiable. Also, by applying the divergence theorem twice in (2.17) then one has:

$$\int_{\partial\Omega} n_i u_{,i} u^* dS - \int_{\partial\Omega} n_i u u_{,i}^{\ *} dS + \int_{\Omega} u(u_{,ii}^* + k^2 u^*) d\Omega = 0 \tag{2.19}$$

Thus, (2.19) is the asymmetric weak form and the fundamental solution u^* is required to be second - order differentiable. Furthermore, *u* is not required to be differentiable in the domain *Ω*.

By combining further eqs (2.16) and (2.19), then we obtain:

$$u(\mathbf{x}) = \int_{\partial\Omega} n_i(\mathbf{y}) u_{,i}(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} n_i(\mathbf{y}) u(\mathbf{y}) u_{,i}^*(\mathbf{x}.,\mathbf{y}) dS \tag{2.20}$$

which can be further written as:

$$u(\mathbf{x}) = \int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} u(\mathbf{y}) R^*(\mathbf{x},\mathbf{y}) dS \tag{2.21}$$

where $q(\mathbf{y})$ denotes the potential gradient along the outward normal direction of the boundary surface:

$$q(\mathbf{y}) = \frac{\partial u(\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u_{,k}(\mathbf{y}) \quad , \quad \mathbf{y} \in \partial\Omega \tag{2.22}$$

and the kernel function:

$$R^*(\mathbf{x},\mathbf{y}) = \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial n_y} = n_k(\mathbf{y}) u_{,k}^*(\mathbf{x},\mathbf{y}) \quad , \quad \mathbf{y} \in \partial\Omega \tag{2.23}$$

By differentiating (2.21) with respect to x_k , we obtain the integral equation for potential gradients $u_{,k}(\mathbf{x})$ by the following relation:

$$\frac{\partial u(\mathbf{x})}{\partial x_k} = \int_{\partial\Omega} q(\mathbf{y}) \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial x_k} dS - \int_{\partial\Omega} u(\mathbf{y}) \frac{\partial R^*(\mathbf{x},\mathbf{y})}{\partial x_k} dS \tag{2.24}$$

3. Some Properties of the Fundamental Solution

The weak form of (2.6) governing the fundamental solution, takes the following form:

$$\int_{\Omega} \left[\nabla^2 u^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y})\right] c d\Omega + c = 0, \quad \mathbf{x} \in \Omega \tag{3.1}$$

where c denotes a constant, considering as the test function.

Additionally, (3.1) can be written as:

$$\int_{\Omega} \left[u^*_{,ii}(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y})\right] d\Omega + 1 = 0, \quad \mathbf{x} \in \Omega \tag{3.2}$$

Thus, (3.2) takes the following form:

$$\int_{\partial\Omega} n_i(\mathbf{y}) u^*_{,i}(\mathbf{x},\mathbf{y}) dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y}) d\Omega + 1 = 0, \quad \mathbf{x} \in \Omega \tag{3.3}$$

Moreover, by considering an arbitrary function *u(x)* in *Ω* as the test function, then the weak form of (2.6) can be written as:

$$\int_{\Omega} \left[\nabla^2 u^*(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y}) + \Delta(\mathbf{x},\mathbf{y})\right] u(\mathbf{x}) d\Omega = 0, \quad \mathbf{x} \in \Omega \tag{3.4}$$

and further as:

$$\int_{\Omega} \left[u^*_{,ii}(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y})\right] u(\mathbf{x}) d\Omega + u(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega \tag{3.5}$$

Finally, (3.5) takes the form:

$$\int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) d\Omega + u(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega \tag{3.6}$$

On the other hand, if $\mathbf{x}$ approaches the smooth boundary $(\mathbf{x} \in \partial\Omega)$, then the first term in (3.6) will be written as:

$$\lim_{x \to \partial\Omega} \int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS = \overset{CPV}{\int_{\partial\Omega}} \Phi^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS - \frac{1}{2} u(\mathbf{x}) \tag{3.7}$$

in the sense of a Cauchy Principal Value (CPV) integral.

For the understanding of the physical meaning of (3.7), then eqs (3.3) and (3.6) may be written as following:

$$\overset{CPV}{\int_{\partial\Omega}} \Phi^*(\mathbf{x},\mathbf{y})dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y})d\Omega + 1/2 = 0, \quad x \in \partial\Omega \tag{3.8}$$

and:

$$\overset{CPV}{\int_{\partial\Omega}} \Phi^*(\mathbf{x},\mathbf{y})u(\mathbf{x})dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y})u(\mathbf{x})d\Omega + 1/2\, u(\mathbf{x}) = 0, \quad x \in \partial\Omega \tag{3.9}$$

Consequently, from (3.8) follows that only a half of the source function at point **x** is applied to the domain Ω, when the point **x** approaches a smooth boundary, $\mathbf{x} \in \partial\Omega$.

Let us further consider another weak form of eqn (3.6) by supposing the vector functions to be the gradients of an arbitrary function $u(\mathbf{y})$ in Ω, chosen in such a way that they have constant values:

$$u_{,k}(\mathbf{y}) = u_{,k}(\mathbf{x}) \ , \quad \text{for } k=1,2,3 \tag{3.10}$$

Hence, the weak form of eqn (3.6) takes the following form:

$$\int_{\Omega} \left[u^*_{,ii}(\mathbf{x},\mathbf{y}) + k^2 u^*(\mathbf{x},\mathbf{y})\right] u_{,k}(\mathbf{y})d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{3.11}$$

By applying further the divergence theorem, then eqn (3.11) may be written as:

$$\int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{3.12}$$

Besides, the following property exists:

$$\begin{aligned} &\int_{\partial\Omega} n_i(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,k}(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega} n_k(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,i}(\mathbf{x},\mathbf{y})dS \\ &= \int_{\Omega} u_i(\mathbf{x})u^*_{,ki}(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega} u_{,i}(\mathbf{x})u^*_{,ik}(\mathbf{x},\mathbf{y})dS = 0 \end{aligned} \tag{3.13}$$

By adding eqs (3.12) and (3.13) then one obtains:

$$\begin{aligned} &\int_{\partial\Omega} n_i(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,k}(\mathbf{x},\mathbf{y})dS - \int_{\partial\Omega} n_k(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,i}(\mathbf{x},\mathbf{y})dS \\ &+ \int_{\partial\Omega} \Phi^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y})u_{,k}(\mathbf{x})d\Omega + u_{,k}(\mathbf{x}) = 0 \end{aligned} \tag{3.14}$$

which takes finally the form:

$$\int_{\partial\Omega} n_i(\mathbf{y})u_{,i}(\mathbf{x})u^*_{,k}(\mathbf{x},\mathbf{y})dS + \int_{\partial\Omega} e_{ikt}R_t u(\mathbf{x})u^*_{,i}(\mathbf{x},\mathbf{y})dS$$

$$+\int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y}) u_{,k}(\mathbf{x}) d\Omega + u_{,k}(\mathbf{x}) = 0 \tag{3.15}$$

4. Application of Singular Integral Operators Method (SIOM)

By the current investigation the regularization of the Singular Integral Operators Method will be considered together with the possibility of satisfying the SIOM in a weak form at $\partial\Omega$, through a generalized Petrov - Galerkin formula.

Consequently, by subtracting (3.6) from (2.21), we obtain:

$$\int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} [u(\mathbf{y}) - u(\mathbf{x})] R^*(\mathbf{x},\mathbf{y}) dS + \int_{\Omega} k^2 u^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) d\Omega = 0 \tag{4.1}$$

By using further (3.9), then (4.1) can be applied at point $\mathbf{x}$ on the boundary $\partial\Omega$, as follows:

$$\int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS - \int_{\partial\Omega} [u(\mathbf{y}) - u(\mathbf{x})] R^*(\mathbf{x},\mathbf{y}) dS = \int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS + \frac{1}{2} u(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega \tag{4.2}$$

Additionally, the Petrov-Galerkin scheme can be used in order the weak form of (4.2) to be written as:

$$\int_{\partial\Omega} f(\mathbf{x}) dS_x \int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS_y - \int_{\partial\Omega} f(\mathbf{x}) dS_x \int_{\partial\Omega} [u(\mathbf{y}) - u(\mathbf{x})] R^*(\mathbf{x},\mathbf{y}) dS_y$$
$$= \int_{\partial\Omega} f(\mathbf{x}) dS_x \int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y}) u(\mathbf{x}) dS_y + \frac{1}{2} \int_{\partial\Omega} f(\mathbf{x}) u(\mathbf{x}) dS_x \tag{4.3}$$

where $u(\mathbf{x})$ denotes a test function on the boundary $\partial\Omega$.

By using further (3.9), then from (4.3) follows:

$$\frac{1}{2} \int_{\partial\Omega} f(\mathbf{x}) u(\mathbf{x}) dS_x = \int_{\partial\Omega} f(\mathbf{x}) dS_x \int_{\partial\Omega} q(\mathbf{y}) u^*(\mathbf{x},\mathbf{y}) dS_y$$
$$- \int_{\partial\Omega} f(\mathbf{x}) dS_x \int_{\partial\Omega}^{CPV} R^*(\mathbf{x},\mathbf{y}) u(\mathbf{y}) dS_y \tag{4.4}$$

Finally, by choosing the test function $f(\mathbf{x})$ in such way to be identical to a function which is energy-conjugate to $u(\mathbf{x})$, then the following Galerkin SIOM is obtained:

$$\frac{1}{2}\int_{\partial\Omega}\bar{q}(\mathbf{x})u(\mathbf{x})dS_x = \int_{\partial\Omega}\bar{q}(\mathbf{x})dS_x\int_{\partial\Omega}q(\mathbf{y})u^*(\mathbf{x},\mathbf{y})dS_y$$
$$-\int_{\partial\Omega}\bar{q}(\mathbf{x})dS_x\overset{CPV}{\int_{\partial\Omega}}R^*(\mathbf{x},\mathbf{y})u(\mathbf{y})dS_y \tag{4.5}$$

Hence, (4.5) is referred to a symmetric Galerkin SIOM.

5. Non-linear Application by Seismic Wave Motion

An application of the previous theory is proposed, to the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium (Figure 1).

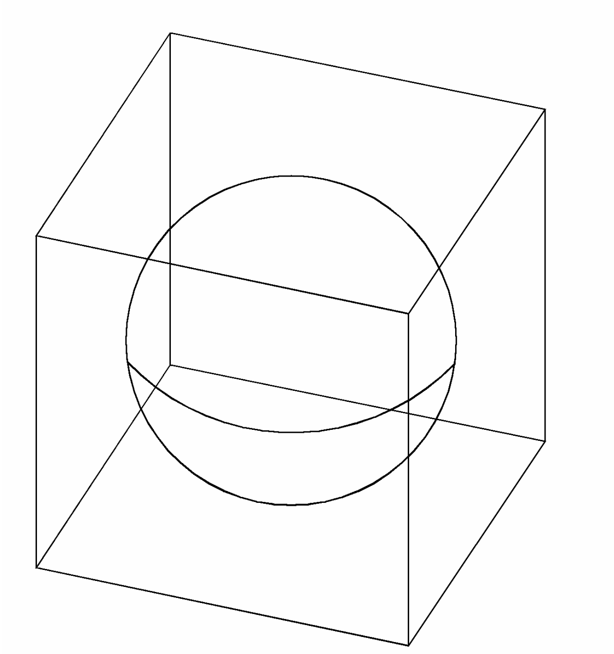

Fig. 1 Pulsating Sphere Radiating Field into an Infinite Homogeneous Medium.

Consequently, by using the Singular Integral Operators Method (S.I.O.M.) as described in the previous paragraphs, then the computation of the acoustic pressure radiated from the above pulsating sphere is determined.

Additionally, the analytical solution of the acoustic pressure for a sphere of radius a, pulsating with uniform radial velocity v_a, is given in [7]:

$$\frac{p(r)}{z_0 v_a} = \frac{a}{r}\frac{ika}{(1+ika)}e^{-ik(r-a)} \tag{5.1}$$

in which $p(r)$ denotes the acoustic pressure at distance r, z_0 is the characteristic impedance and k the wave number.

In Table 1 and Table 2, the real and imaginary parts of dimensionless surface acoustic pressures are shown with respect to the reduced frequency ka. Thus, the computational results by using the

S.I.O.M. were compared to the analytical solutions of the same problem. From the above Tables it can be seen that there is very small difference between the computational results and the analytical solutions. Finally same results are plotted, in Figures 2 and 3, and in three-dimensional form in Figures 2a and 3a.

Table 1

ka	**Re($p(a)/z_0v_a$) Analytical**	**Re($p(a)/z_0v_a$) S.I.O.M.**
0.00	0.00	0,00
0.40	0.10	0.11
0.60	0.30	0.32
0.75	0.40	0.41
1.00	0.50	0.52
1.25	0.60	0.60
1.50	0.70	0.71
2.00	0.80	0.80
2.50	0.86	0.87
3.00	0.90	0.91
3.50	0.92	0.93
4.00	0.94	0.94
4.50	0.96	0.96
5.00	0.97	0.97
6.00	0.98	0.98
7.00	0.99	0.99
7.50	0.99	0.99

Table 2

ka	**Im($p(a)/z_0v_a$) Analytical**	**Im($p(a)/z_0v_a$) S.I.O.M.**
0.00	0.00	0,00
0.20	0.22	0.23
0.40	0.30	0.31
0.50	0.40	0.41
0.60	0.45	0.45
0.80	0.48	0.48
1.00	0.50	0.49
1.50	0.45	0.45
2.00	0.40	0.40
2.50	0.35	0.36
3.00	0.30	0.31
3.50	0.26	0.26
4.00	0.24	0.24
4.50	0.21	0.21
5.00	0.19	0.20
5.50	0.17	0.17
6.00	0.15	0.15
6.50	0.14	0.14
7.00	0.12	0.12
7.50	0.11	0.11

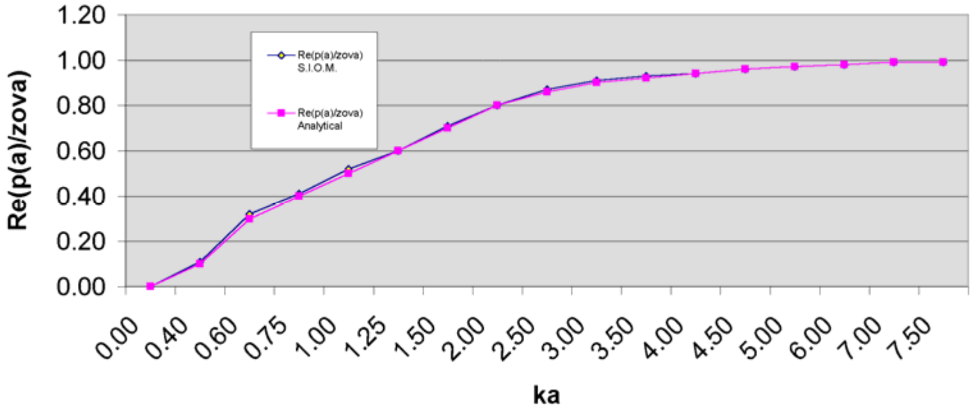

Fig. 2 Real Part of Dimensionless Surface Acoustic Pressure of a Pulsating .

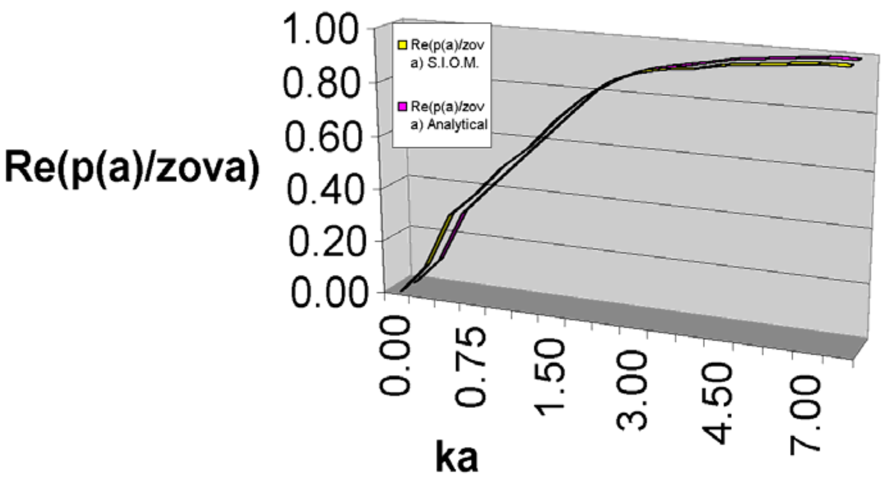

Fig. 2a 3-D Distribution of Real Part of Dimensionless Surface Acoustic Pressure of a Pulsating .

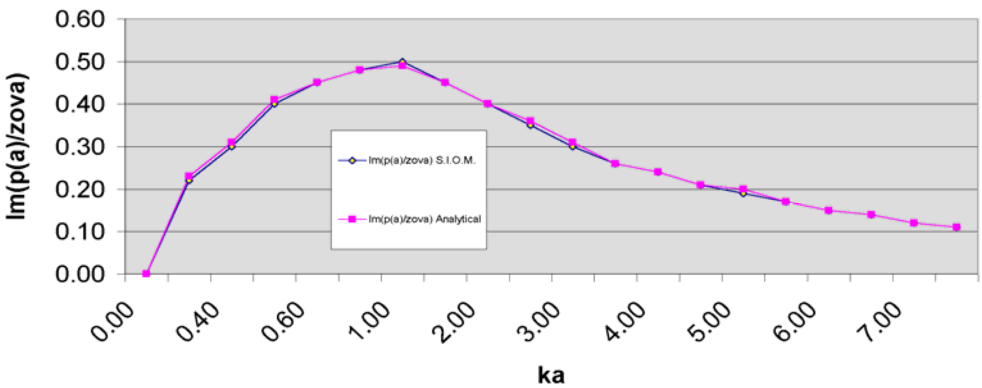

Fig. 3 Imaginary Part of Dimensionless Surface Acoustic Pressure of a Pulsating .

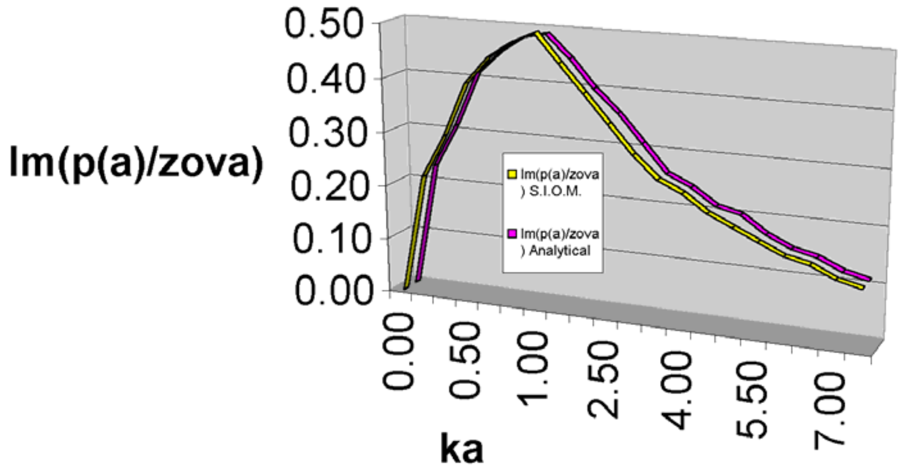

Fig. 3a 3-D Distribution of Imaginary Part of Dimensionless Surface Acoustic Pressure of a Pulsating .

6. Conclusions

The modern technology *"Real-time Expert Seismology"* as was introduced and investigated by E.G.Ladopoulos [25]-[30] is further improved for the exploration of on-shore and off-shore petroleum reservoir. Such a groundbreaking method can be used at any depth of seas and oceans all over the world ranging from 300 to 3000 m, or even deeper and for any depth like 20,000 m or 30,000 m in the subsurface of existing petroleum and gas reserves.

Hence, the benefits of the new theory of *"Real-time Expert Seismology"* in comparison to the old theory of *"Reflection Seismology"* are the following:

1. The new method uses the special form of the crests of the geological anticlines of the bottom of the sea, in order to decide which areas of the bottom have the most possibilities to include petroleum.

On the contrary, the existing theory is only based to the best chance and do not include any theoretical and sophisticated model.

2. The new method of elastic (sound) waves is based on the difference of the speed of the sound waves which are traveling through solid, liquid, or gas. In a solid the elastic waves are moving faster than in a liquid and the air, and in a liquid faster than in the air.

Existing theory is based on the application of Snell's law and Zoeppritz equations, which are not giving good results, as these which we are expecting with the new method.

3. The new method is based on a Real-time Expert System working under Real Time Logic, that gives results in real time, which means every second.

Existing theory does not include real time logic.

Consequently, from the above three points it can be well understood the evidence of the applicability of the modern technology of *"Non-linear Real-time Expert Seismology".* Moreover its novelty, as it is based mostly on a theoretical and very sophisticated Real-time Expert model and not to practical tools like the existing method.

By the current paper, the Singular Integral Operators Method (S.I.O.M.) has been used for the solution of the elastodynamic problems used in *"Non-linear Real-time Expert Seismology"* by applying the Helmholtz differential equation. In this derivation the gradients of the fundamental solution to the Helmholtz differential equation for the velocity potential, has been used. Furthermore, several basic identities governing the fundamental solution to the Helmholtz differential equation for the velocity potential were analyzed and investigated.

Thus, by using the S.I.O.M., then the acoustic velocity potential has be computed. Also, several properties of the wave equation, which is a Helmholtz differential equation, were proposed and investigated. Some basic properties of the fundamental solution have been further derived.

Finally, an application was given to the determination of the seismic field radiated from a pulsating sphere into an infinite homogeneous medium. Consequently, by using the S.I.O.M., then the acoustic pressure radiated from the above pulsating sphere has be computed. This is very important in hydrocarbon reservoir engineering in order the size of the reservoir to be evaluated.

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