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Mathematical Modeling and Nonlinear Dynamic Analysis of Flexible Manipulators using Finite Element Method

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Abstract
By the current research, mathematical modeling and dynamic analysis of flexible manipulators are presented based on finite element method. In order this to be effected, each link of flexible manipulator is modeled by finite number of elements and the displacement of element is formulated based on nodal coordinates and shape functions of beam element. Then, the kinetic and potential energy of the system is developed using the displacement in the reference coordinate systems. In addition, by employing the Lagrange principle, the nonlinear dynamic model of the system is derived. Besides, simulation results are presented to validate the proposed method.

Key Word and Phrases
Flexible Manipulator, Mathematical Modeling, Nonlinear Dynamic Analysis, Finite Element Method.

1. Introduction

Flexible manipulators exhibit many advantages over their traditional rigid ones: they have light weight, their motors are smaller, they consume less energy, and their production is frugal. Because of these important features, the application of flexible manipulators are exceedingly developed during last decades, and they have been achieved an important role in many fields of science such as surgical operation [1, 2], nuclear application [3, 4], and aero space structures [5, 6]. Thus, the mathematical modeling and dynamic analysis of such system is important and treated by some authors: Book [7] analyzed the dynamic behavior of flexible manipulators based on recursive lagrangian method.

Moreover, a Newton-Euler approach is presented in [8] to model the dynamic of a flexible robot. Meghdari and Fahimi [9] used an analytical method to decouple the dynamic equations of elastic manipulators. Furthermore, a lumped model of a planer flexible manipulator is presented in [10]. Singh [10] used an extended Hamilton’s principle to derive the equation of motion of the flexible manipulator. Besides, Korayem et al. [11, 12] presented the dynamic modeling of flexible manipulator systems, based on assumed mode method. In their method, the flexible behavior of the system is modeled via eigenvalue functions multiplied by modal coordinate of the system.

In this paper, the mathematical analysis and dynamic modeling of flexible manipulator is presented based on finite element method. Each link of the system is modeled by finite number of elements, and the displacement vector of each point of the robot is formulated in the reference coordinate by means of finite formulation of beam element. Then, the kinetic and potential energies of the system are presented, and the dynamic model of the system is derived using Lagrange principle. Finally, simulation results are presented.

2. Finite element formulation for mathematical model of the system

To present the mathematical and dynamic model of the flexible manipulators, the system with m number of links, each link is divided to n_i elements with length of l_{ij}. As the total displacement of each point of the flexible manipulator can be presented as $\vec{r}_{ij}$. According to Figure 1, the reference coordinate system is shown by OXY, and the local coordinate system attached to ith link is assumed as $O_i X_i Y_i$.

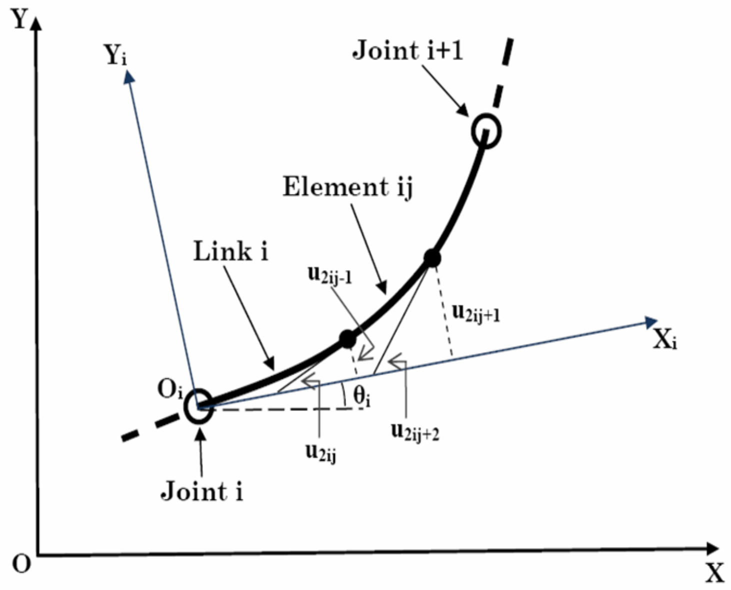

Fig. 1 The flexible manipulator

The parameters of the flexible manipulator are shown in Table 1.

Table 1. Parameters of flexible manipulator

Parameters	Nomenclatures
Jth element of ith link	ij
Displacement vector of element	$\vec{r}_{ij}$
Displacement vector of ith joint	$\vec{r}_{o_i}$
Angular displacement of ith joint	θ_i
Number of elements of ith link	n_i
Length of ith link	L_i
Mass per length of ith link	m_i
Gravitational constant of earth	g
Length of jth element of ith link	l_{ij}
Elasticity modulus of ith link	E_i
Moment of inertia of ith link	I_i
Rotation matrix between local and global coordinate system	T_0^i

To present the total displacement vector of *ijth* element of the system in the global coordinate system, this vector is assumed as a summation of displacement of O_i, and the deflection of the link in the local coordinate $O_i X_i Y_i$:

$$\vec{r}_{ij} = \vec{r}_{ij,r} + \vec{r}_{ij,f} = \vec{r}_{O_i} + T_0^i \begin{bmatrix} (j-1)l_i + x_{ij} \\ y_{ij} \end{bmatrix} \tag{2.1}$$

where y_{ij} is the deflection of element due to flexibility of system in the local coordinates. By implementation of finite element method, this displacement is presumed a summation of Hermitian shape function multiplied to nodal coordinate of the element [13]:

$$y_{ij}(x_{ij},t)=\sum_{k=1}^{4}\phi_k(\,x_{ij}\,)\;u_{2ij-2+k} \tag{2.2}$$

where ϕ_k shows the shape function and $u_{2ij-2+k}$ are the nodal coordinate of the systems, and are given as [13]:

$$\phi_1(x_{ij})=1-3\frac{x_{ij}^2}{l_{ij}^2}+2\frac{x_{ij}^3}{l_{ij}^3} \tag{2.3}$$

$$\phi_2(x_{ij})=x_{ij}-2\frac{x_{ij}^2}{l_{ij}}+\frac{x_{ij}^3}{l_{ij}^2} \tag{2.4}$$

$$\phi_3(x_{ij})=3\frac{x_{ij}^2}{l_{ij}^2}-2\frac{x_{ij}^3}{l_{ij}^3} \tag{2.5}$$

$$\phi_4(x_{ij})=-\frac{x_{ij}^2}{l_{ij}}+\frac{x_{ij}^3}{l_{ij}^2} \tag{2.6}$$

As the displacement vector of element is formulated, the kinetic energy of the element is stated as follows:

$$T_{ij}=\frac{1}{2}\int_0^{l_i} m_i\left[\frac{\partial \vec{r}_{ij}}{\partial t}^T.\frac{\partial \vec{r}_{ij}}{\partial t}\right]dx_{ij}\qquad 0<x_{ij}<l_{ij} \tag{2.7}$$

If, the vectors $\vec{z}_{ij}=\left[\theta_i\quad u_{2ij-1}\quad u_{2ij}\quad u_{2ij+1}\quad u_{2ij+2}\right]^T$ and $\vec{\psi}_{ij}=\left[u_{2ij-1}\quad u_{2ij}\quad u_{2ij+1}\quad u_{2ij+2}\right]^T$ are defined, then Eq. (2.7) can be rewritten as:

$$T_{ij}=\frac{1}{2}\dot{\vec{z}}_{ij}^{\,T}\,M_{ij}\,\dot{\vec{z}}_{ij}$$
$$M_{ij}(l,p)=\int_0^{l_i} m_i\left[\frac{\partial \vec{r}_{ij}}{\partial z_{ij_l}}\right]^T.\left[\frac{\partial \vec{r}_{ij}}{\partial z_{ij_p}}\right]dx_{ij} \tag{2.8}$$

Besides, the potential energy of the element is shown as V_{ij}, and is a summation of gravitational potential energy V_{ij_g}, and elastic potential energy V_{ij_e}:

$$V_{ij}=V_{gij}+V_{eij} \tag{2.8a}$$

The gravitational potential energy is given as:

$$V_{gij}=\int_0^{l_i} m_i g\left[0\quad 1\right]\vec{r}_{ij}\,dx_{ij} \tag{2.9}$$

And the elastic energy of the system is:

$$V_{eij}=\frac{1}{2}\int_0^{l_i} EI_i\left(\frac{\partial^2 y_{ij}}{\partial x_{ij}^2}\right)dx_{ij}=\frac{1}{2}\vec{\psi}_{ij}^{\,T}\,K_{ij}\,\vec{\psi}_{ij} \tag{2.10}$$

where the stiffness matrix K_{ij} is presented as:

$$K_{ij}=\frac{EI_i}{l_i^3}\begin{bmatrix}12 & 6l_i & -12 & 6l_i\\ 6l_i & 4l_i^2 & -6l_i & 2l_i^2\\ -12 & -6l_i & 12 & -6l_i\\ 6l_i & 2l_i^2 & -6l_i & 4l_i^2\end{bmatrix} \tag{2.11}$$

Beyond the above, the generalized coordinate vector is defined as $\vec{q}$, and the total kinetic and potential energy of the system can be written as:

$$T(\vec{q},\dot{\vec{q}})=\sum_{i=1}^{m}\sum_{j=1}^{n_i}T_{ij}\qquad V(\vec{q})=\sum_{i=1}^{m}\sum_{j=1}^{n_i}V_{ij} \tag{2.12}$$

Then, the Lagrange function is introduced as $L(\vec{q},\dot{\vec{q}})=T-V$, and the Lagrange' principle is developed. The principle of Lagrange for dynamic systems is expressed as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=Q_j \tag{2.13}$$

where q_j represents the generalized coordinates, Q_j is the generalized external force. Thus, by implementation of Lagrange principle, the nonlinear dynamic equations of the system are summarized as follows:

$$M\,\ddot{\vec{q}}+f\left(\vec{q},\dot{\vec{q}}\right)=B\vec{\tau} \tag{2.14}$$

As in Eq. (2.14) is presented, the nonlinear dynamic model of the system is developed, and no linearization is done. Consequently, the nonlinear terms affect the dynamic of the system.
It must be further noticed that for each link of the flexible manipulator, the first node is coincided on the joint of the link. Thus, these nodal coordinates are zero:

$$u_{i1}(t)=0\ ,\ u_{i2}(t)=0 \tag{2.15}$$

3. Dynamic model of a single link manipulator

For a single-link flexible manipulator, as the link modeled by one element, the generalized coordinate vector of the system is $\vec{q}=[\theta_1 \quad u_3 \quad u_4]$, where θ_1 is angular displacement of the robot joint, u_3 and u_4 are the elastic deflection and slope of the end point of the flexible manipulator.

Moreover, the rotation matrix of the system is:

$$T_0^1=\begin{bmatrix}\cos(\theta_1) & -\sin(\theta_1)\\ \sin(\theta_1) & \cos(\theta_1)\end{bmatrix} \tag{3.1}$$

and the total displacement of any point of the robot is:

$$\vec{r}_1=\begin{bmatrix} x_{11}\cos(\theta_1)-\sin(\theta_1)\left((\frac{3x_{11}^2}{L1^2}-\frac{2x_{11}^3}{L1^3})u_3+(\frac{-x_{11}^2}{L1}+\frac{x_{11}^3}{L1^2})u_4\right)\\ x_{11}\sin(\theta_1)+\cos(\theta_1)\left((\frac{3x_{11}^2}{L1^2}-\frac{2x_{11}^3}{L1^3})u_3+(\frac{-x_{11}^2}{L1}+\frac{x_{11}^3}{L1^2})u_4\right)\end{bmatrix} \tag{3.2}$$

So, the kinetic and potential energies of the single-link flexible manipulator are stated as:

$$T=\frac{1}{420}m_1L_1\left(2L_1^2\dot{\theta}_1^2u_4^2+147L_1\dot{\theta}_1\dot{u}_3-21L_1^2\dot{\theta}_1\dot{u}_4+78\dot{\theta}_1^2u_3^2+2L_1^2\dot{u}_4^2-22L_1\dot{\theta}_1^2u_3u_4+78\dot{u}_3^2+70L_1^2\dot{\theta}_1^2-22L_1\dot{u}_3\dot{u}_4\right) \tag{3.3}$$

$$V_g=\frac{1}{12}m_1gL_1\left(6u_3\cos(\theta_1)-L_1u_4\cos(\theta_1)+6L_1\sin(\theta_1)\right) \tag{3.4}$$

$$V_e=\frac{2EI_1}{L_1^3}\left(3u_3^2-3L_1u_3u_4+L_1^2u_4^2\right) \tag{3.5}$$

Thus, the dynamic equation of the system can be derived, using Lagrange principle.
To simulate the dynamic behavior of the system, the parameters are given as: *m1=5 kg, L1=1m, I1=5e-9, E=20e9 pa, g=9.81*. The simulation results are as follows:

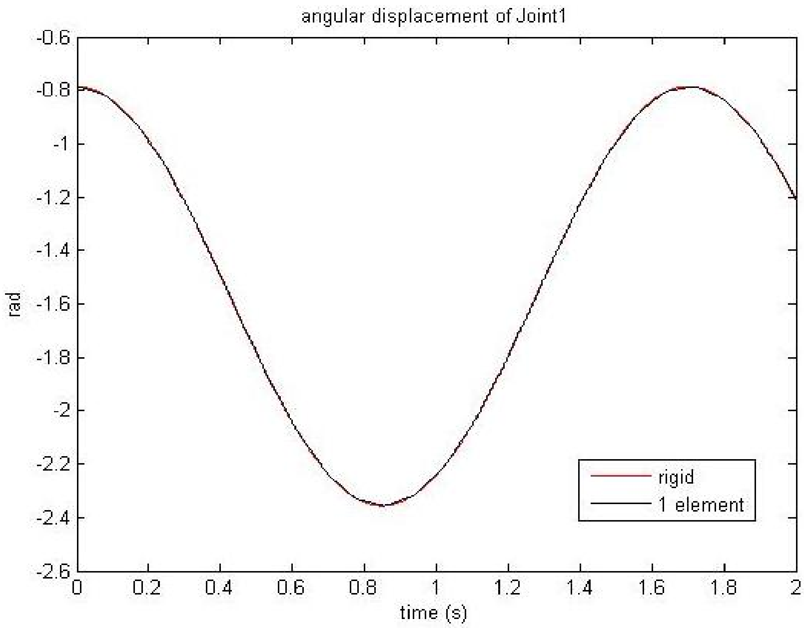

Fig. 2 Angular displacement of flexible manipulator

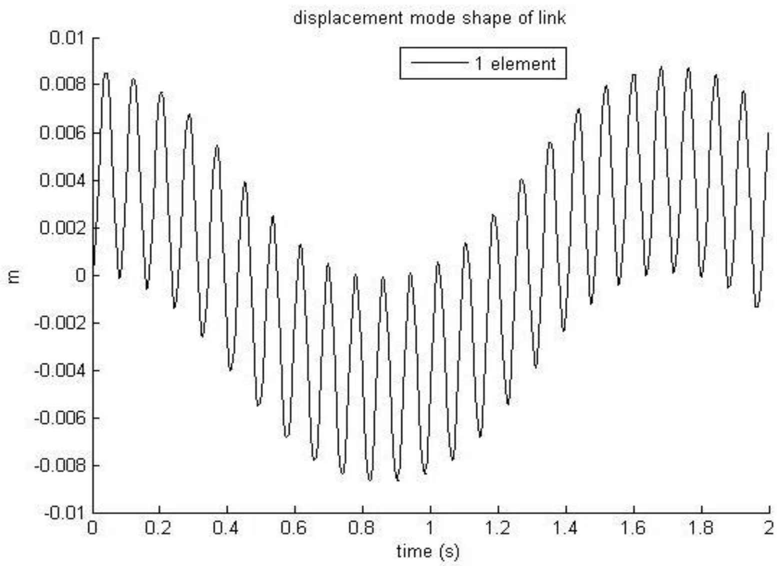

Fig. 3 Displacement mode shape of the flexible manipulator

4. Conclusions

In this paper, the nonlinear dynamic analysis of the flexible manipulators has been studied using finite element method. The total displacement vector of the system has been formulated using Hermitian shape function.

Hence, the total displacement of the elastic arm in reference coordinate system has been presented, the Lagrange principle has been used to derive the nonlinear dynamic motion of the elastic manipulator. Finally, the proposed method has been employed to derive the dynamic equations of a single-link manipulator, and some simulations are done.

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