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Dynamic Modeling and Digital Optimal Control of an Overhead Crane

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Abstract
This paper presents digital optimal control of an overhead crane. Hence, the nonlinear dynamic model of the system is derived via Lagrange's principle, the linearized equations of the system are expressed. The motor voltage of and displacement of the trolley are presumed as input and output of the system respectively, and the state-space equations of the system are presented. Then, digital optimal feedback control law is obtained for tracking of trolley, and some simulations are performed which verify the proposed method.

Key Word and Phrases
Digital Controller, Optimal Control, Overhead Crane, Tracking, Dynamic Modeling.

1. Introduction

Cranes are used in various industries such as transportation and construction tasks. A crane often includes a hoisting mechanism which is suspended from a point on the support mechanism. Based on the support mechanism, cranes can be classified as: overhead (overhead) cranes, tower (rotary) crane, and boom carne. Overhead cranes are commonly composed of a cart moving in a fixed support, while a cable is suspended from a point on the cart to transport the payload. The overhead cranes have some advantages other the other models of the crane such as low cost, easy assembly and less maintenance, and have attracted a great deal of interests [1, 2]. Thus, the dynamic analysis of such a system is an important task, and treated by researchers, recently. For the modeling of the crane, two approaches are usually used including a lumped-mass or a distributed-mass modeling. For the lumped-mass modeling of the crane, the system is modeled by a massless cable as a hoisting line and a lumped mass as a payload [3-5].

On the other hand, in the distributed-mass modeling, the hoisting line is modeled as a continuous string, and the payload assumed a lumped mass as a boundary condition of the system [6, 7]. Hubbel et al. [8] used an open-loop continuous control named input-shaping to control the motion of an overhead crane. In this method, the input control profile is determined as unwanted oscillation during travel and residual pendulations are avoided [9]. Also, a hybrid input-shaping strategy and a continuous PD-type fuzzy logic control scheme are implemented in [10] to control an overhead crane system. however this method if effective, but the input-shaping method leaks from being an open loop control scheme, and is not robust to disturbances and parameter uncertainties [4]. Moustafa and Ebied [11] used a nonlinear modeling and anti-swing control method for the overhead cranes. Also, in [12] a fuzzy logic feedback controller is proposed to control an intelligent crane system.

The current research is concerned with the digital optimal control of an overhead crane. In order this to be effected, the nonlinear and linear dynamic equations of the system are derived. Then, the digital model of the system is presented in state-space form. As the problem is the tracking of the cart position, the optimal control law is formulated, and the optimal control feedback gains are obtained. In addition, some simulation results are presented to verify the method.

2. Dynamic modeling

In this section, the dynamic model of the overhead carne is presented. The dynamic equation of the system is derived using Lagrange principle. Figure 1 shows an overhead crane moves in two-dimensional space. The crane consists of a cart (trolley) transverses in horizontal direction, while a massless pendulum connects on the cart and hoists the payload.

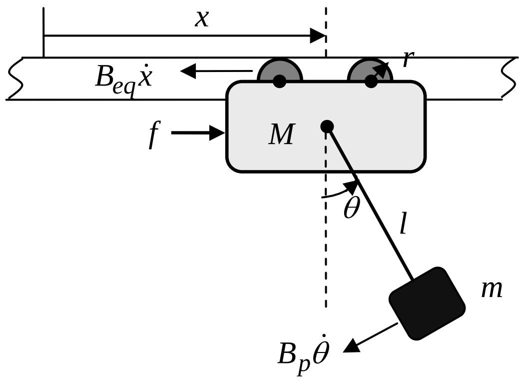

Fig. 1 The Overhead Crane

As, the Lagrange principle is used, the lagrangian function can be stated as:

$$L = \frac{1}{2}m\left[\dot{x}^2 + \dot{l}^2 + (l\dot{\theta})^2 + 2\dot{x}\dot{l}\sin\theta + 2\dot{x}l\dot{\theta}\cos\theta\right] + \frac{1}{2}M\dot{x}^2 + mgl\cos\theta \quad (2.1)$$

In addition, the parameters of the overhead system are presented in Table 1:

Table 1. Parameters of the overhead crane

Parameters	Nomenclatures
Cart position	x
Cart velocity	$\dot{x}$
Pendulum angular displacement	θ
Pendulum angular velocity	$\dot{\theta}$
Pendulum length	l
Mass of the cart system	M
Payload mass	m
Gravitational constant of earth	g
Radius of wheels of cart	r
DC motor voltage of cart	e
Force exerted to cart	f
Motor armature resistance	R
Motor torque constant	k
Viscous damping coefficient of pendulum axis	B_p
Equivalent viscous damping coefficient	B_{eq}

The principle of Lagrange for dynamic systems is expressed as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j - Q_{j,lost} \quad (2.2)$$

where q_j represents the generalized coordinates, Q_j is the generalized external force, and $Q_{j,lost}$ is defined as the generalized force related to the viscous damping of the system. The generalized coordinates are treated as $q_1 = x, q_2 = \theta$.

According to Fig. 1, the generalized damping force can be written as:

$$Q_{x,lost} = B_{eq}\dot{x}$$
$$Q_{\theta,lost} = B_p l\dot{\theta} \tag{2.3}$$

Using Eq. (2.2) for $q_1 = x$ results in:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = f - B_{eq}\dot{x}$$
$$\Rightarrow \frac{d}{dt}\left[(M+m)\dot{x} + m\dot{l}\sin\theta + ml\dot{\theta}\cos\theta\right] - 0 = f - B_{eq}\dot{x} \tag{2.4}$$

and Eq. (2.2) is used for $q_2 = \theta$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = -B_p l\dot{\theta}$$
$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2}m\left(2l^2\dot{\theta} + 2\dot{x}l\cos\theta\right)\right] - \left[\frac{1}{2}m\left(2\dot{x}\dot{l}\cos\theta - 2\dot{x}l\dot{\theta}\sin\theta\cos\theta\right) - mgl\sin\theta\right] = -B_p l\dot{\theta} \tag{2.5}$$

Thus, the nonlinear equations of the system are expressed as:

$$(M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + 2m\dot{l}\dot{\theta}\cos\theta + m\ddot{l}\sin\theta = f - B_{eq}\dot{x}$$
$$l\ddot{\theta} + 2\dot{l}\dot{\theta} + \ddot{x}\cos\theta + g\sin\theta = -B_p\dot{\theta} \tag{2.6}$$

Besides, the linear force f is originated from the torque of motor of trolley T [13]:

$$T = rf$$
$$T = \frac{k}{R}e - \frac{k^2}{R}\omega \tag{2.7}$$
$$\dot{x} = r\omega$$

Thus, from Eq. (2.6) and Eq. (2.7), the nonlinear model of the system is:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = \frac{1}{r}\left(\frac{k}{R}e - \frac{k^2}{Rr}\dot{x}\right)$$
$$\ddot{x}\cos\theta + l\ddot{\theta} + g\sin\theta = 0 \tag{2.8}$$

Also, if the state vector is defined as $X = \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}$, and the continuous dynamic model of the system is linearized, then the continuous linear system can be expressed as:

$$\dot{X} = AX + Bu$$
$$y = CX + Du \tag{2.9}$$

where the matrices of the continuous model are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{k^2}{Rr^2M} & \frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{k^2}{Rr^2Ml} & -\frac{(M+m)g}{Ml} & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ \frac{k}{RrM} \\ 0 \\ -\frac{k}{RrMl} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \quad D = 0 \tag{2.10}$$

To express the digital model of the system, one can assume the sampling time as T, and use the function d2c in MATLAB [14] to descretize the system model given by Eq. (2.10). Consequently, the digital model of the system is:

$$\begin{aligned} X\left((k+1)T\right) &= G\,X\left(kT\right) + H\,u(kT) \\ y\left(kT\right) &= E\,X\left(kT\right) + F\,u(kT) \end{aligned} \tag{2.11}$$

3. Optimal control of discrete system

As the digital model of the system is described by Eq. (2.11), a digital optimal controller must be developed. For the digital system, if the state-space equation of the system is given by Eq. (2.11), then the cost function of the digital system is defined as [14]:

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left[X(k)\,Q\,X(k) + u(k)\,R\,u(k)\right] \tag{3.1}$$

where *Q* is the state weighting matrix related to state vector *X(k)*, and *R* is the input weighting function related to input control effort *u(k)*.

In order to solve the optimal control problem, some mathematical effort has to be be done, which finally result in the optimal feedback law as $u = -K\,X(k) = -\left[(R + H^T PH)^{-1} H^T PG\right]X(k)$, and the following equation must be satisfied [14]:

$$-P + G^T P\left(I + HR^{-1}H^T P\right)^{-1} G + Q = 0 \tag{3.2}$$

where K is the feedback gain matrix of the digital system, and P is assumed as an positive definite matrix. To solve the foregoing equation, one can use the dlqr function in MATLAB software, and the gain matrix K is achieved.

4. Simulation results

In this section, the optimal control results of the digital system are considered. The digital system is considered for the sampling time T=0.1, 1 second. the parameter values of the system is given as:

Table 2. Parameter values of the overhead crane [16]

Parameter	Parameter values
Pendulum length	$l = 0.3302\,m$
Mass of the cart system	$M = 1.073\ kg$
Payload mass	$m = 0.23\,kg$
Gravitational constant of earth	$g = 9.81\,m/s^2$
Radius of wheels of cart	$r = 0.006\,m$
Motor armature resistance	$R = 2.6\Omega$
Motor maximum voltage	$e_{\max} = 12V$
Motor torque constant	$k = 0.00767\ Vs/rad$

As the weighting matrices are assumed as $Q = diag(1), R = 0.1$, the optimal feedback law of the system is obtained either from Eq. (3.1) or Eq. (3.2).

The simulation results for the cart position, pendulum angle, and the input voltage are shown by next Figures:

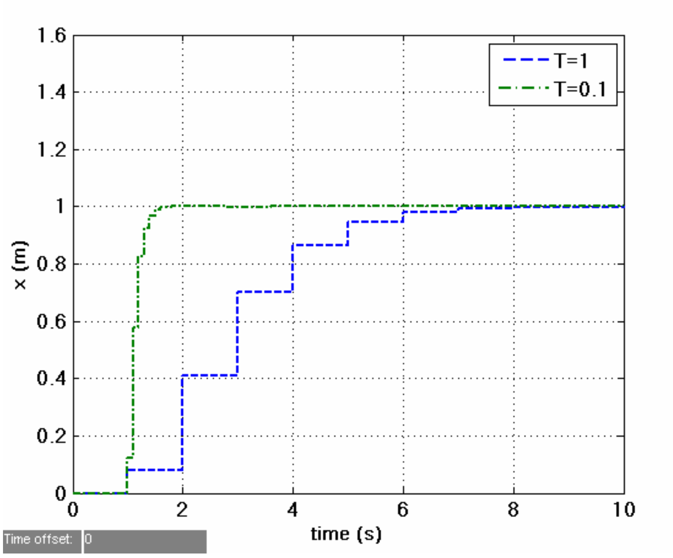

Fig. 2 The displacement of the cart

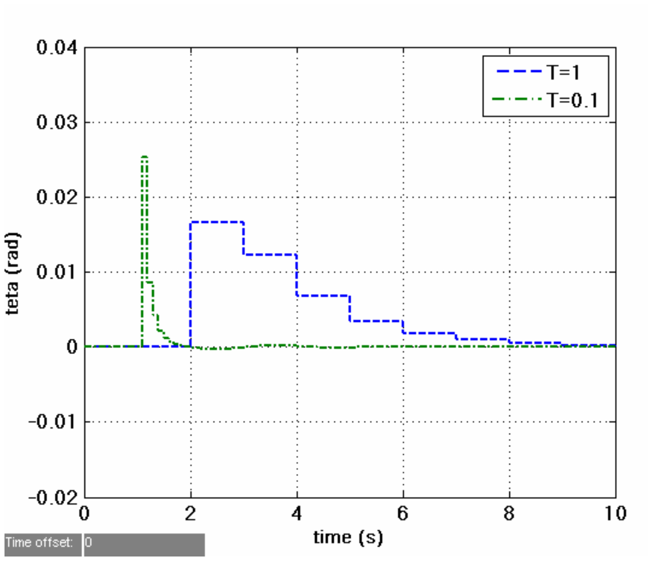

Fig. 3 The angular displacement of the trolley

As it is seen in Fig. 2 and Fig. 3, decreasing the sampling time cause the better response of the system.

4. Conclusions

The present research has presented a digital optimal control strategy for tracking of an overhead crane system. The dynamic of the continuous system have been derived, and the equivalent linear digital system has been presented.

Hence, by using, the optimal control formulation for tracking problems, the corresponding optimal feedback laws have been obtained, and the simulation results have been presented. The results have shown that decreasing the sampling time of the digital model can be resulted in better response.

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