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Application of the Peak - Load Method to CT and WS Tests for Determining Nonlinear Fracture Parameters of Concrete

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Abstract
To analyze a concrete structure using fracture mechanics, its fracture parameters need to be determined. Many nonlinear fracture models have been proposed in design codes and by investigators for use in determining fracture parameters of concrete. The two-parameter fracture model (TPM) requires two fracture parameters to describe fracture-dominated failure of concrete structures: the critical stress intensity factor, K_{Ic}^{s}, and the critical crack tip opening displacement, $CTOD_c$. In this model, the fracture parameters are obtained from one of two experimental methods: the compliance method and the peak-load method of which the theoretical basis is to solve four simultaneous nonlinear equations. However, eccentric compression prisms, beams and notched split-tension specimens, namely, cubes, cylinders, diagonal cubes, holed cylinders have been used in the peak-load method based on the two-parameter fracture model. In this study, the peak-load method was initially applied to compact-tension (CT) and wedge-splitting (WS) data from six series of experimental studies described in the literature. The results of the peak-load method appear to be viable and very promising.

Key Word and Phrases
Concrete, Compact-tension test, Wedge-splitting test, Two-parameter model, Peak-load method, Double-K model.

1. Introduction

Applications of linear elastic fracture mechanics (LEFM) to concrete were initiated by Kaplan [1] in 1961 and continued by Kesler et al. [2] in 1972, who concluded that LEFM was not valid for cement based composite materials. This inapplicability of LEFM is due to the existence of an inelastic zone in front of the crack tip in concrete with large-scale and full cracks. This so-called fracture process zone (FPZ) is ignored by LEFM. For this reason, several investigators have developed nonlinear fracture mechanics approaches to characterize the FPZ.

These approaches primarily involve the fictitious crack model (FCM) of Hillerborg et al. [3], the crack band model of Bazant and Oh [4], the two-parameter model (TPM) of Jenq and Shah [5], the effective crack model of Nallathambi and Karihaloo [6], the size effect model of Bazant and Kazemi [7], the double-K model of Xu and Reinhardt [8], and the double-G model of Xu and Zhang [9]. In contrast to LEFM, in which a single fracture parameter such as the critical stress intensity factor is used, these models need at least two experimentally determined fracture parameters to characterize the failure of concrete structures. Accordingly, they require either multiple tests (at least three) or a closed-loop testing system. Analysis of an existing structure using fracture mechanics is impossible using many of the approaches mentioned above, and even for those approaches for which it is possible, specimens cored from structures must be tested after being processed to a specific geometry. However, in the peak-load method based on nonlinear analysis, the use of the cylindrical specimens, which can also be taken from existing structures by core drilling, lead to great advantage for estimating fracture properties of existing structures based on nonlinear fracture mechanics.

Concrete splitting specimens have been commonly used in concrete fracture testing because they have certain advantages, such as compactness and lightness, compared to beams. Additionally, cubical and cylindrical test specimens have the following advantages [10, 11].

1) These specimens are easy to handle, and there is no risk of breaking them during handling.
2) The same molds can be used to cast specimens for both fracture and strength tests.

3) In determining the fracture parameters of cement-based materials, the contribution of the weight of the specimen can be ignored, which is not true of notched beams.

Tests in which cubical and cylindrical specimens are used to study concrete fracture can be classified as compact-tension (CT) / wedge-splitting (WS) tests and split-tension tests. CT and WS tests have been performed on cubical and cylindrical specimens with edge notches. Although the CT and WS tests were originally developed for the FCM, they can also be used with the TPM and the double-K models [11, 12]. The split-tension test has been used over the last decade to indirectly test the tensile strength of quasi-brittle materials such as concrete and rock [13] and to study concrete fracture. Modeer [14] and Rocco et al. [15] used splitting-cube/cylinder specimens with the FCM. Tang et al. [16] and Yang et al. [17] used pre-notched splitting specimens to investigate effective crack models, and Ince [18-20] used cube and diagonal cube specimens with a central notch for the same purpose.

In this study, CT specimens and WS specimens with edge notches were used to investigate the two-parameter model. The fracture parameters of this model were determined using the peak-load method. For this purpose, six series of historical test results obtained for CT and WS specimens were used. The results of this study indicate that the fracture parameters of quasi-brittle materials can be determined from analysis of CT and WS test results by the peak-load method.

2. Fracture Mechanics of Cementitios Materials

In the 1970s, experimental investigations of the fracture mechanics of cement-based materials such as paste, mortar and concrete revealed that classical LEFM was not valid for quasi-brittle materials [2]. The inapplicability of LEFM is a result of the relatively large inelastic zone in concrete, the fracture process zone (FPZ), that occurs ahead of and in the vicinity of the tips of major cracks. Several nonlinear fracture mechanics models have been developed to characterize the FPZ.

These fracture models can be categorized as either cohesive crack models, including the fictitious crack model developed by Hillerborg et al. [3] and the crack band model developed by Bazant and Oh [4], or effective crack models, including the two-parameter model (TPM) developed by Jenq and Shah [5], the effective crack model developed by Nallathambi and Karihaloo [6], the size effect model developed by Bazant and Kazemi [7], the double-K model developed by Xu and Reinhardt [8], and the double-G model developed by Xu and Zhang [9]. Cohesive crack models simulate the FPZ with a closing pressure that diminishes near the crack tip, while effective crack models simulate the FPZ with an effective crack length. The aim of each approach is to determine the critical crack extension, defined as $\Delta a = a_c - a_0$, where a_c and a_0 are the critical crack length at the peak load and the initial crack length, respectively, during the load–displacement response or the load–crack mouth opening displacement response of the structure. Additionally, a_c depends on a structure's size because it tends to converge to a_0 as the size increases [7]. Therefore, nonlinear fracture approaches require at least two experimentally determined fracture parameters to simulate concrete fracture.

A concrete structure fails, according to the TPM, when the stress intensity factor K_I and the crack opening displacement $CTOD$ reach their critical values, K_{Ic}^{s} and $CTOD_c$, respectively. These fracture parameters can be determined from the following LEFM equations:

$$K_{Ic}^{s} = \sigma_{Nc}\sqrt{\pi a_c}\,Y(\alpha_c) \tag{2.1}$$

$$CTOD_c = \frac{4\sigma_{Nc} a_c}{E_c'} V_1(\alpha_c) M(\alpha_c, \beta = a_0/a_c) \tag{2.2}$$

where σ_{Nc} is the nominal failure stress; $\alpha_c = a_c/d$; d is the structural size; $E_c' = E_c/(1-\nu^2)$ for plane strain; $E_c' = E_c$ for plane stress; E_c is Young's modulus for cement-based materials; ν is Poisson's ratio; and Y, V_1, and M are dimensionless functions that depend on the geometry of the structure and the load type. Because values of the Y, V_1, and M functions can be found in LEFM handbooks [21], the TPM can be easily applied in structural analysis. In this approach, the fracture parameters are deduced from one of two experimental methods: the compliance method, proposed by RILEM [22], and the peak-load method, proposed by Tang et al. [16]. In the first method, the fracture parameters are determined from the relationship between the load and the crack mouth opening displacement (P-CMOD) of centrally edge-notched three-point bending specimens using closed-loop test equipment, as shown schematically in Fig. 1. The critical crack length a_c is calculated from two values taken from the P-CMOD curve: the initial compliance C_i and the unloading compliance C_u, measured at approximately 95% of the peak load (P_c) in the descending branch (Fig. 1), using Eq. (2.3):

$$a_c = a_0 \frac{C_u V_1(a_0/d)}{C_i V_1(a_c/d)} \tag{2.3}$$

The Young's modulus of concrete E_c can also be calculated using the initial compliance C_i or the unloading compliance C_u.

Because the peak-load method does not require complicated testing equipment, it is simpler than the method proposed by RILEM for determining fracture parameters for the TPM. Nevertheless, the peak-load method requires three or more distinct specimens, due to the inherent variation in concrete properties. This is true for both methods. These specimens may be identical in size, but their initial crack lengths may differ. Alternatively, the initial crack lengths may be equal, but the sizes of the specimens may differ. For each specimen tested, the following equations can be written according to the TPM:

$$K_I^i\left(\sigma_{Nc}^i, a_c^i\right) = K_{Ic}^s, \qquad CTOD^i\left(\sigma_{Nc}^i, a_c^i\right) = CTOD_c \qquad (i = 1, 2) \tag{2.4}$$

where i denotes the i^{th} specimen. Consequently, the fracture parameters can be determined by simultaneously solving four nonlinear equations. However, three or more distinct specimens must be tested to ensure statistically valid results because random errors always exist in measured values of σ_{Nc}^1 and σ_{Nc}^2. In this case, a statistical procedure is used to calculate K_{Ic}^s and $CTOD_c$.

Fracture test specimens, such as three-point bending beams, eccentric compression prisms and notched split-tension specimens, i.e., cubes, cylinders, diagonal cubes, and holed cylinders, can be used with the peak-load method [16-19], while three-point bending beams and WS specimens are used with the compliance method. Additionally, for geometrically similar structures ($\alpha_0=a_0/d$=constant), Eq. (2.1) can be simplified to the following form [23]:

$$\sigma_{Nc} = \frac{K_{Ic}^s}{\left[Y(\alpha_0) + Y'(\alpha_0)\dfrac{\Delta a_c}{d}\right]\sqrt{\pi(a_0 + \Delta a_c)}} \tag{2.5}$$

where Δa_c is the critical crack extension, and $Y'(\alpha_0)$ denotes the derivative with respect to α_0. This formula is identical in form to the size effect model recommended by Bazant and Kazemi [7]. However, Eq. (2.5) can only be solved by using a nonlinear algorithm, while the formula of the size effect model can also be solved by the linear regression analysis. Although the concept of geometrical similarity can be used to simplify the size effect model, such similarity is not a fundamental condition describing the size effect in concrete structures according to the TPM [23].

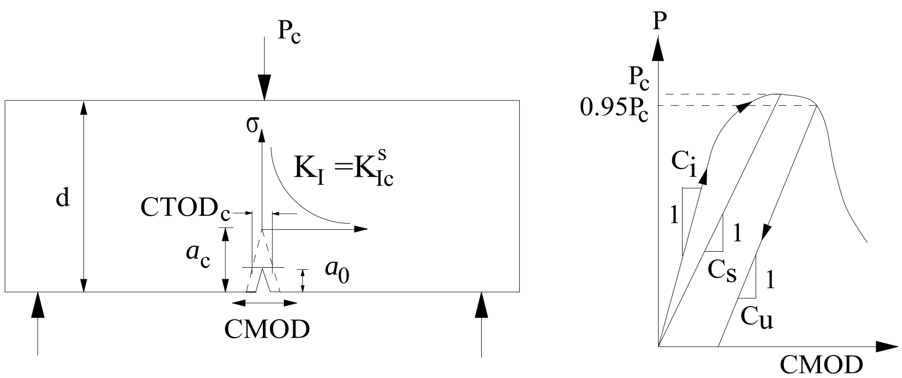

Fig. 1 Determination of concrete fracture parameter values according to TPM

Similarly, the double-K fracture model describes the behavior of cracked concrete structures in terms of fracture parameters such as the unstable stress intensity factor, K_{Ic}^{un}, and the initiation stress intensity factor, K_{Ic}^{ini}. The parameter K_{Ic}^{un} can be calculated in a manner similar to that described for the TPM using Eqs. (2.1) and (2.3). However, the value of a_c in the double-K model is calculated from the secant compliance C_s at the peak load, as shown in Fig 1. Tests on beams and wedge-splitting specimens have shown that the values of K_{Ic}^{s} and $CTOD_c$ determined with the TPM agree well with the values of K_{Ic}^{un} and $CTOD_c$ determined with the double-K model [24]. In addition, Ince [20] performed tests on split-tension specimens (cylinders, cubes and diagonal cubes) and concluded that the fracture parameters of the double-K model could be determined using the peak-load method.

Nevertheless, the TPM, the effective crack model developed by Nallathambi and Karihaloo [6] and the size effect model developed by Bazant and Kazemi [7] give essentially equivalent results. Experimental studies have revealed that the critical stress intensity factor (K_{Ic}) is reasonably well correlated in both the TPM and the effective crack model [25]. In addition, K_{Ic}^{s} (or K_{Ic}^{un}, as described above) values obtained with the TPM can be transformed into fracture energy in the size effect model using the well-known LEFM relation:

$$G_f = \left(K_{Ic}^{s}\right)^2 / E_c' \tag{2.6}$$

Similarly, the $CTOD_c$ parameter can be transformed into the effective fracture process zone length (c_f) in the size effect model using Eq. (2.7) [26]:

$$c_f = \frac{\pi E_c'}{32 G_f} CTOD_c^2 \tag{2.7}$$

The fracture energy parameter G_F in the fictitious crack model corresponds to the entire area under the stress separation curve. This parameter is not identical to G_f in the size effect model. Statistical investigations by Bazant and Becq-Giraudon [26] have shown that the ratio G_F/G_f is approximately 2.5.

3. A Historical Overview of the Compact-Tension Test and Wedge-Splitting Test

Although notched-beam specimens have been widely used in concrete fracture mechanics, CT and WS specimens have some advantages over beams, such as compactness and lightness (Fig. 2a). CT specimens were initially used by Wittmann et al. [27] to determine the fracture energy G_F and evaluate the strain-softening behavior of cement-based materials. However, as shown in Fig. 2b, Hillemeier and Hilsdorf [28] had previously performed tests on wedge-loaded CT specimens, including needle bearing to perform more stable fracture tests than CT tests. Subsequently, Linsbauer and Tschegg [29] developed a similar splitting test for cube-shaped samples. Brühwiler and Wittmann [10] proposed a popular wedge-splitting test, which has been used in recent years in concrete fracture testing with test specimens modified from those proposed by Linsbauer and Tschegg [29].

A WS specimen can be considered a compact form of the three-point bending beam, as shown in Fig. 3a [11]. WS specimens with grooves were developed for use as CT specimens, as shown in Fig. 3b. WS and CT testing can be conducted on both cylindrical and cubical specimens. The use of cylindrical specimens, which can be obtained from existing structures by coring, offers the great advantage of estimating the fracture properties of existing structures based on fracture mechanics. In WS testing, the load is applied to the specimen by means of a wedge and a loading device with roller bearings, as illustrated in Fig. 3c. The horizontal load P_H acts on the rollers because of the vertical load (P_V) on the wedge, as shown in Fig. 3d. Friction forces also occur between the rollers and the wedge. However, the friction forces can be ignored when the wedge angle θ=15°. The horizontal load can be calculated as follows:

$$P_H = \frac{1-\mu \tan\theta}{2(\mu + \tan\theta)} P_V \approx \frac{P_V}{2\tan\theta} \tag{3.1}$$

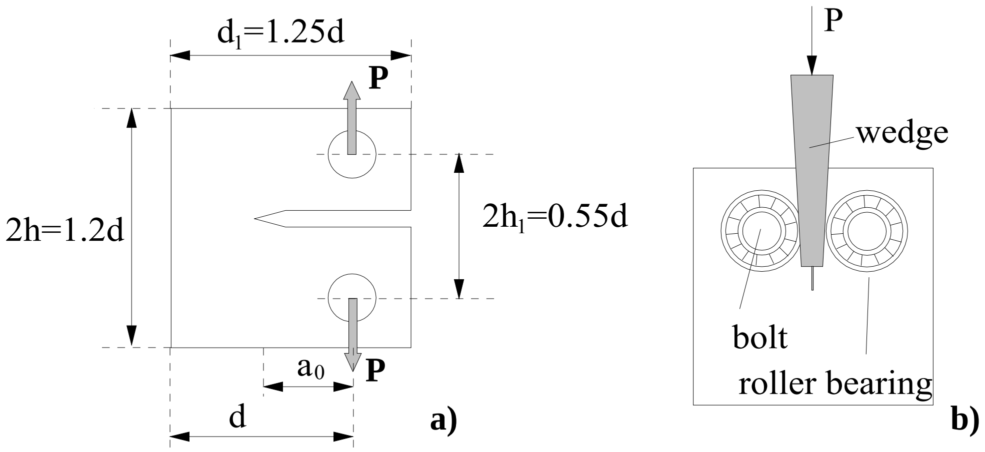

Fig. 2 **a)** Standard compact tension specimen **b)** Hillemeier and Hilsdorf [28] test set-up

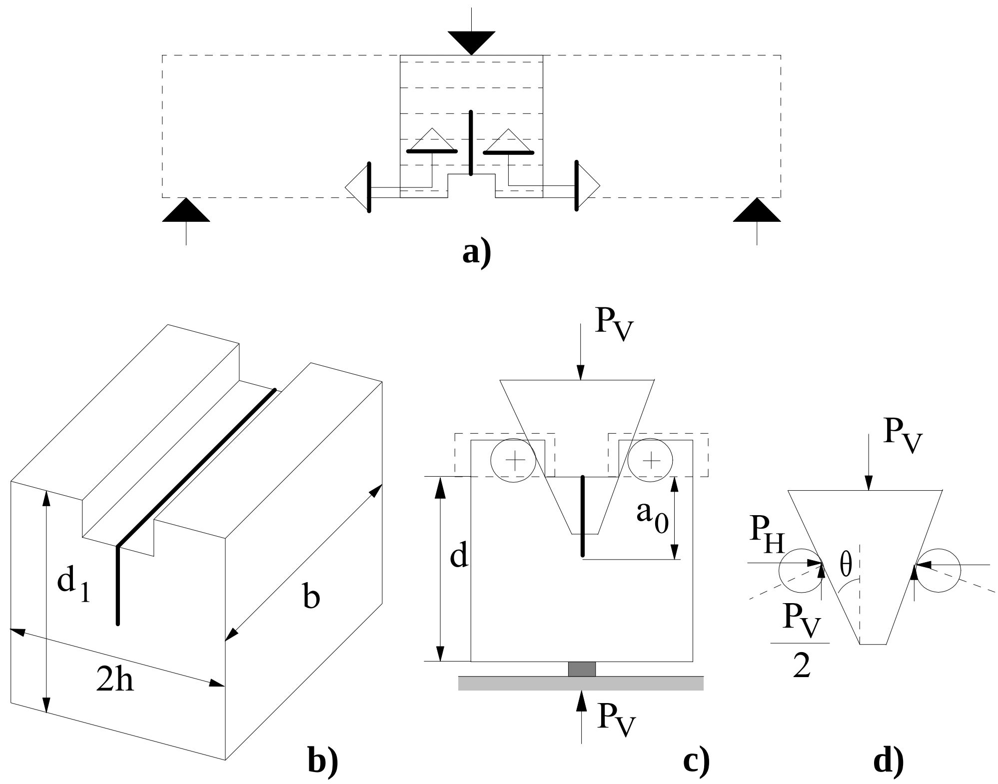

Fig. 3 Wedge-splitting test **a)** Wedge-splitting specimen as “compact” three-point bending specimen [11] **b)** Specimen configuration **c)** Loading **d)** Wedge forces

where μ is the coefficient of friction between the rollers and the wedge. The approximate formula in Eq. (3.1) can be used for θ=15°.

The vertical load P_V and the crack opening displacement COD, which is measured at the loading level or at the axles of the rollers, are recorded in the fracture tests. By considering the similarities in loading conditions and specimen geometries between CT test and WS test, the same LEFM formulas can be used for the effective crack models [12, 30]. For CT and WS specimens, the stress intensity factor is given as follows:

$$K_I = \sigma_N \sqrt{d}Y(\alpha) \tag{3.2}$$

where $\sigma_N = P/bd$, α is the relative crack length (a/d), and the dimensionless function $Y(\alpha)$ is given by the following equation [31]:

$$Y(\alpha) = \frac{(2+\alpha)\left(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4\right)}{(1-\alpha)^{3/2}} \tag{3.3}$$

The crack opening displacement COD can be calculated as follows [32]:

$$COD = \frac{P}{bE'_c} V_1(\alpha) \tag{3.4}$$

$$V_1(\alpha) = \left(\frac{1+\alpha}{1-\alpha}\right)^2 \left(2.163 + 12.219\alpha - 20.065\alpha^2 - 0.992\alpha^3 + 20.609\alpha^4 - 9.9314\alpha^5\right) \tag{3.5}$$

The accuracy of Eq. (3.3) is ±0.5 % for $\alpha > 0.2$, while the accuracy of Eq. (3.5) is ±0.5 % for $0.2 \le \alpha \le 0.95$. To calculate the critical crack tip opening displacement value of WS and CT specimens, the dimensionless function M can be determined from the following formula, which is the same as that used with three-point bending specimens [33, 34]:

$$M(\alpha_c, \beta = a_0/a_c) = \sqrt{(1-\beta)^2 + (1.081 - 1.149\alpha_c)\left[\beta - \beta^2\right]} \tag{3.6}$$

4. Application of the Peak-Load Method to the CT and WS Tests

Experimental studies have revealed that the fracture parameters of concrete are particularly influenced by four material parameters: the compressive strength (f'_c), the maximum aggregate size (d_{max}), the water–cement ratio (w/c), and the aggregate type [26, 35, 36]. The fracture parameters of concrete can also be affected by other material parameters, such as the type of cement, the aggregate–sand ratio, the porosity, the curing conditions, etc. Bazant and Becq-Giraudon [26] suggested relationships between the fracture parameters of the size effect model (G_f, c_f) and these material parameters, based on 238 data sets available in the literature.

Ince [35, 36] applied artificial neural network (ANN) modeling, a powerful tool used to solve many civil engineering problems, to predict the fracture parameters of the TPM (K_{Ic}^s and $CTOD_c$) and the effective crack model (K_{Ic}^e**)** based on the aggregate type (river gravel or crushed stone), w/c, d_{max} and f'_c. The ANN results reveal that the value of the critical stress intensity factor K_{Ic}^s (or K_{Ic}^e**)** is higher for a mix with crushed aggregate than for a mix with rounded aggregate. In addition, the results showed a strong correlation between f'_c and K_{Ic}^s (or between w/c and K_{Ic}^s, based on the strong correlation between f'_c and w/c), whereas $CTOD_c$ was nearly independent of f'_c and w/c. Furthermore, both $CTOD_c$ and K_{Ic}^s were found to increase with d_{max}.

On the basis of these previous studies, six series of CT and WS test data taken from the literature were analyzed according to the peak-load method based on the TPM. These series of test data were obtained using various test methods and various concrete mixes in various laboratories [9, 10, 24, 37-39]. Table 1 summarizes the geometric properties (b=specimen width, d=characteristic size, h=specimen height, and a_0/d=relative initial crack length), the compressive strength of concrete, f'_c, and the ultimate horizontal load, P_H, of the test specimens. In the same table, n=the number of tested specimens and the specimen type (WS or CT) are also reported. It should be noted that the experiment set-up used by Xu and Zhang [9] was a wedge-splitting configuration used to test compact-tension specimens, similar to the set-up used in the study by Hillemeier and Hilsdorf [28], as shown in Fig. 2b. This set-up is referred to as “WS on CT” in Table 1.

Table 1 CT and WS test results reported in the literature

Reference	Specimen type	n	f_c' (MPa)	$b\times d\times 2\,h$ (mm)	a_0/d	P_H (kN)
Zhu [37]	WS	3	41.80	100×200×200	0.3	6.350
		5			0.4	5.440
		3			0.5	4.573
		3			0.6	3.073
Xu et al. [24]	WS	7	40.18	200×170×400	0.353	11.000
		5			0.471	7.024
		8			0.588	4.587
		9			0.706	2.383
Brühwiler and Wittmann [10]	CT	6	42.90	120×300×360	0.5	7.292
		6		120×600×720		12.525
		6		120×1200×1440		20.712
Xu et al. [38]	CT	1	39.14	200×1500×1500	0.4	60.01
		1		200×2000×2000		69.76
		1		200×2500×2500		89.54
		1		200×3000×3000		93.21
Zhao et al. [39]	WS	2	30.47	150×150×150	0.5	3.488
		3		200×200×200		6.003
		5		300×300×300		16.008
		6		450×450×450		36.137
Xu and Zhang [9]	WS on CT	5	23.52	198×200×240	0.4	10.376
		5		202×300×360		13.975
		3		203×400×480		20.343
		5		215×600×720		28.221
		6		237×800×960		34.913
		6		253×1000×1200		43.580

In the peak-load method, the fracture parameter K_{Ic}^{s} that causes the smallest standard deviation in the $CTOD_c$ is determined because at least two tests are required. However, as previously mentioned, at least three groups must be tested to obtain statistically valid results because random errors always exist in measured nominal strength values. In this method, specimens of the same size are grouped with respect to their relative initial crack lengths, and specimens with the same relative initial crack lengths are grouped with respect to their sizes. Table 1 was designed in this form. Because P_H values reported in Table 1 refer to average ultimate horizontal loads in the literature and they were computed by considering these groups in this study. The following statistical procedures were used to evaluate the TPM, K_{Ic}^{s} and $CTOD_c$ according to the method described by Tang et al. [16], Yang et al. [17], and Ince [18, 19].

The initial crack length and the nominal strength were calculated for each group. The K_{Ic}^{s}–$CTOD_c$ relationships were determined for each group using Eqs. 3.2-3.6. Four examples using data from the studies by Zhu [37], Xu et al. [24], Brühwiler and Wittmann [10], and Xu and Zhang [9] are shown in Figs. 4 and 5. Subsequently, the average K_{Ic}^{s}–$CTOD_c$ curve was obtained for these groups, as indicated by the solid curved lines in Figs. 4 and 5. The sample standard deviation of the groups was then calculated as follows:

$$s\left(K_{Ic}^{s}\right)=\sqrt{\frac{\sum_{i=1}^{n}\left(\overline{CTOD_c}-CTOD_{ci}\right)^2}{n-1}} \tag{4.1}$$

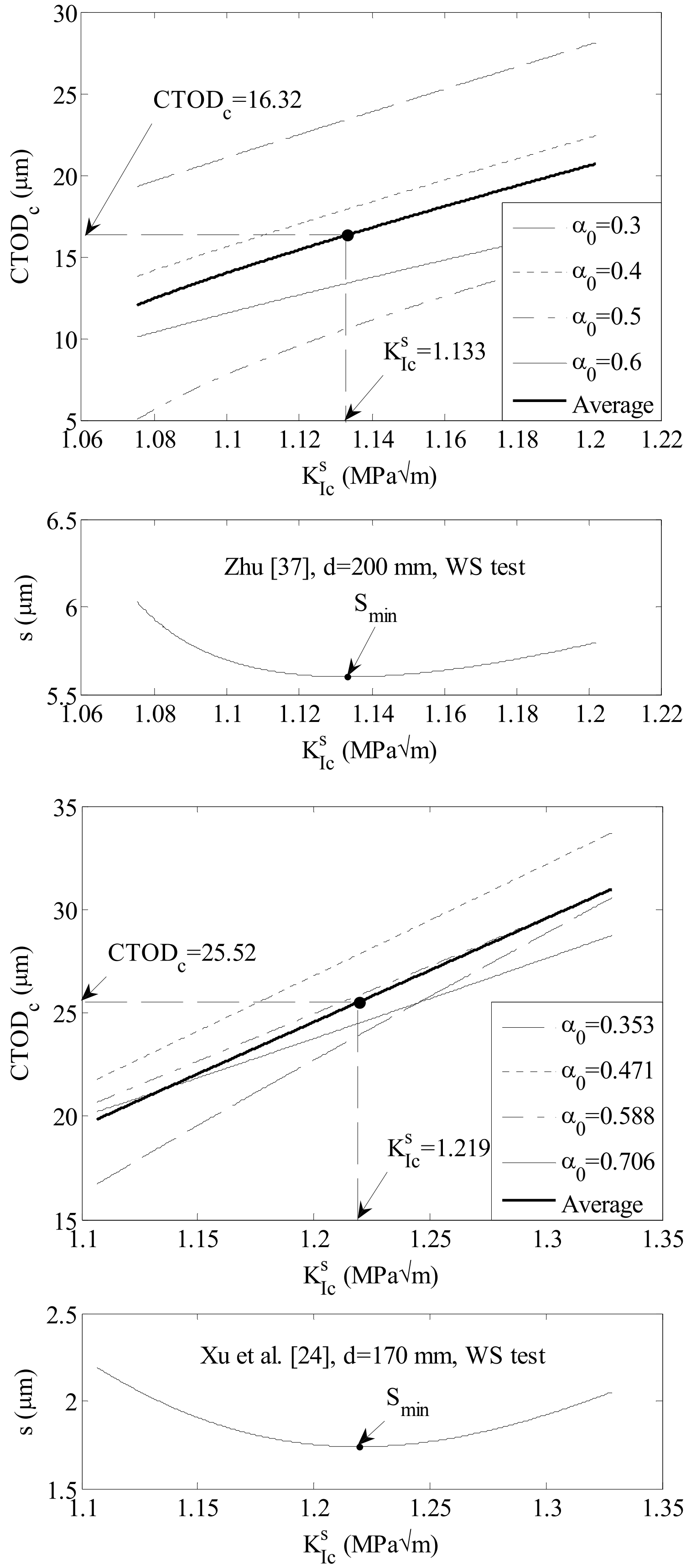

Fig. 4 Applications of the peak-load method to WS specimens with different relative initial crack lengths.

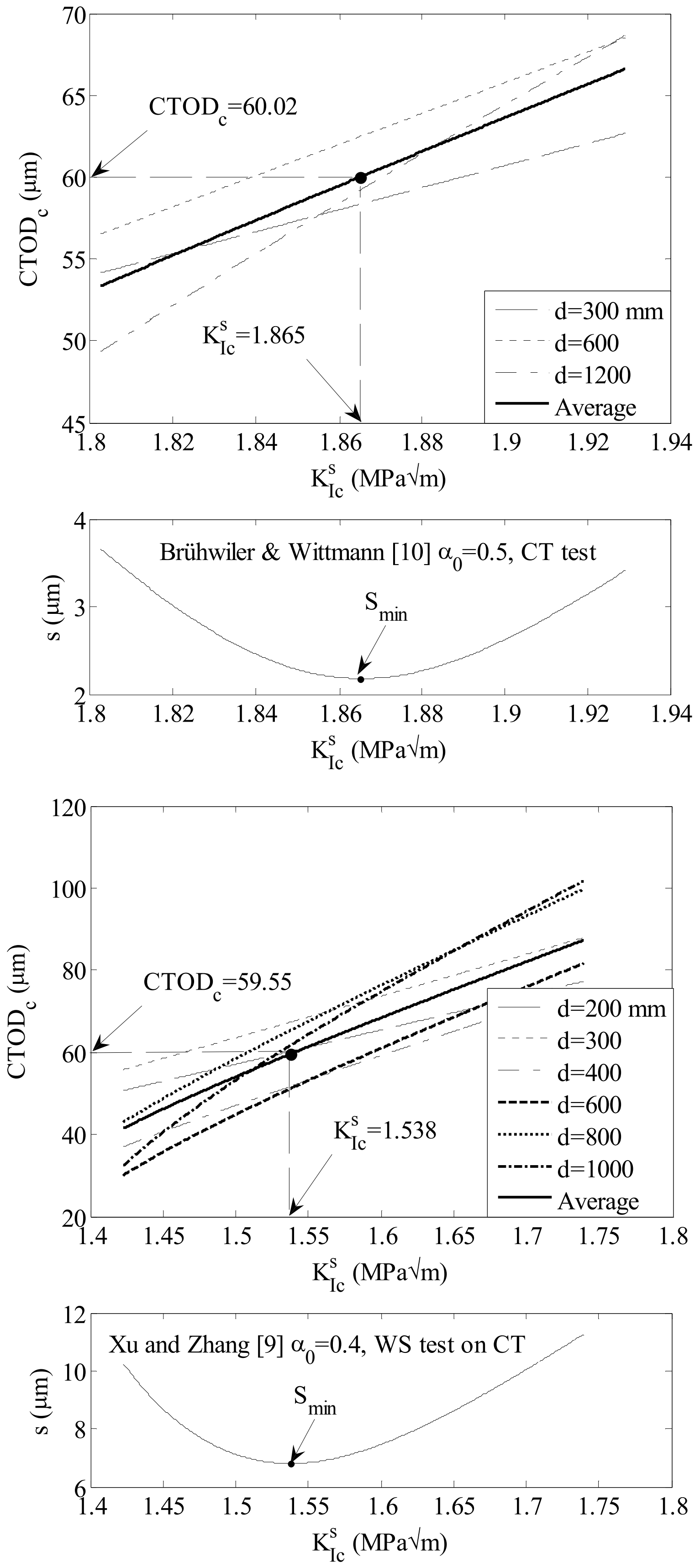

Fig. 5 Applications of the peak-load method to CT and WS specimens with different specimen depths.

where n is the number of groups, $\overline{CTOD_c}$ is the average value of $CTOD_c$ for all groups, and $CTOD_{ci}$ is the value of $CTOD_c$ for the i^{th} group. As shown in Figs. 4 and 5, the value of K_{Ic}^s corresponding to the minimum value of s was taken as the value of the fracture parameter K_{Ic}^s. The value of the fracture parameter $CTOD_c$ was obtained by substituting this value of K_{Ic}^s into the average K_{Ic}^s–$CTOD_c$ curve. For instance, the values of the K_{Ic}^s and $CTOD_c$ parameters were found to be 1.133 MPa√m and 16.32 μm at the minimum s value (s_{min}) in the study by Zhu [37] (Fig. 4a). In this study, the value of Young's modulus of concrete in Eq. (3.4) was determined according to ACI-318 [40] as follows:

$$E_c = 4734\sqrt{f_c'} \tag{4.2}$$

where E_c and f_c' are in MPa. Although this formula is commonly used to determine concrete fracture parameter values in the peak-load method, this formula may produce a large scatter. Therefore, Young's modulus can also be determined from testing of cylindrical specimens using strain gauges whose length is three times the maximum aggregate size.

Indeed, the determination of the fracture parameters of TPM is a nonlinear optimization problem with two-variable, as shown Eq. (2.4). However, the peak-load method reduces this to a one-variable nonlinear optimization problem by minimizing the following function:

$$f\left(K_{Ic}^s\right) = \sum_{i=1}^{n}\left(\overline{CTOD_c} - CTOD_{ci}\right)^2 \tag{4.3}$$

Consequently, there is no difference between f in Eq. (4.3) and s in Eq. (4.1) since $f = (n-1)s^2$ with n as a constant [17, 41-43].

In Table 2, the calculation results for the peak-load method, based on the TPM, and the results reported in the literature, based on the double-K model, are summarized and compared. Note that the literature values in the 1st, 3rd, 4th and 5th rows were taken from the study by Xu and Reinhardt [12], while the other values were taken from the related references.

In this study, CT and WS specimens with different specimen depths were also analyzed according to Eq. (2.5), which gives a general expression of the size effect on the nominal strength. The Levenberg–Marquardt algorithm based on Eq. (2.5) was used to determine the fracture parameters (K_{Ic}^s and Δa_c) of geometrical similar specimens in the last four series in Table 1.

The results of these non-linear analyses are reported in Table 2, in which the values of $CTOD_c$ were obtained from the values of Δa_c by using the LEFM formulas. As shown in Table 2, the results obtained with the peak-load method are close to the results obtained with the double-K model and the size effect approach based on the TPM. However, as shown in Table 2, the values of $CTOD_c$ have much greater than the values of K_{Ic}^s since the determination of this parameter is very sensitive to the precision of the measurements. Therefore, Tang et al. [16] developed the peak-load method to reduce discrepancies in measuring $CTOD_c$ [23].

Table 2 Comparison of results obtained with the peak-load method and the results of other methods

Reference	Double-K Model		TPM (Eq. (2.5))		Peak-Load Method	
	K_{Ic}^{un} (MPa√m)	$CTOD_c$ (μm)	K_{Ic}^s (MPa√m)	$CTOD_c$ (μm)	K_{Ic}^s (MPa√m)	$CTOD_c$ (μm)
Zhu [37]	1.211-1.656	12.34-48.92	-	-	1.133	16.32
Xu et al. [24]	0.942-1.506	8-31	-	-	1.219	25.52
Brühwiler and Wittmann [10]	1.440-2.228	32.7-111.3	1.634	40.31	1.865	60.02
Xu et al. [38]	2.468-2.694	111.61-167.55	2.272	100.09	2.608	168.81
Zhao et al. [39]	1.186-1.671	32.58-53.25	2.014	78.65	2.041	95.06
Xu and Zhang [9]	1.498-1.729	35.35-97.92	1.439	45.22	1.538	59.55

5. Conclusions

In recent years, splitting specimens such as compact-tension, wedge-splitting and cylindrical/cubical split-tension specimens have been widely preferred over beams for use in concrete fracture testing. Compact-tension and wedge-splitting test results were used for the first time in this study to determine the fracture parameters of concrete using the peak-load method based on the two-parameter fracture model. The following conclusions can be drawn from the results of this investigation:

1) Notched split-tension specimens, namely, cubes, cylinders, diagonal cubes, holed cylinders, eccentric compression prisms and beams, have been used with the peak-load method. The results of this study indicate that the peak-load method can be successfully applied to compact-tension and wedge-splitting specimens.
2) Many structural laboratories do not have sophisticated testing equipment such as closed-loop testing systems and displacement-controlled testing machines. The peak-load method offers the great advantage of requiring measurement of only the maximum load applied to specimens to determine the values of the fracture parameters of concrete. Another advantage of the peak-load method is that it applies a specific statistical analysis procedure to test data to determine the values of the fracture parameters of concrete. On the other hand, the peak-load method simplifies a two-variable nonlinear optimization problem to one-variable nonlinear optimization problem to determine the fracture parameters of the two-parameter model.
3) The size effect approach based on the TPM was initially applied to compact-tension and wedge-splitting test in this study. One of the main requirements in this approach is the need to test samples with different sizes. However, the specimens of the same size but with different crack lengths can additionally be used in the peak-load method. This leads to great advantage for estimating the fracture parameters of concrete.
4) The fracture parameters of concrete required for the two-parameter model were analyzed in this study. However, the results obtained can easily be adapted to other concrete fracture models, such as the size effect model, the effective crack model, the double-K model and the fictitious crack model, using the formulas given in the section 2.

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