

A Mathematical Scheme for Calculation of Lift of Planing Crafts with Large Mean Wetted Length and a Comparative Study of Effective Parameters

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Abstract

By the current paper, lift force that is an important characteristic of a planing hull is calculated. To accomplish this task, Morabito's mathematical model that has been previously derived for prediction of total pressure distribution on planing hulls is utilized. These equations are not restricted to any specific mean wetted length and can be used for planing hulls with large value of mean wetted length. Mathematical modeling has been accomplished by integrating the bottom pressure which leads to determination of the lift of planing hulls. Results of the presented method are compared against the results of Savitsky's formula and favorable agreement is displayed. Finally, a parametric study is conducted to examine the effects of various physical factors such as trim angle, deadrise angle, average wetted length, and speed coefficient on the lift force. Percentages of contribution of hydrodynamic and hydrostatic lifts are also investigated.

Key Word and Phrases

Lift, Mathematical Modeling, Planing Hulls, Empirical and Practical equations, Large Mean Wetted Length, Calm Water, Hydrodynamics, High Speed Crafts (HSC)

1. Introduction

Among different types of high speed crafts, planing hulls are the most common. Specific geometry of these crafts generates hydrodynamic lift on the bottom of the hull. This force and the buoyant force support weight of the vessel. On the other hand, this force reduces the wave making resistance of the boat. In order to compute the lift force, various methods have been proposed and different experimental studies have been performed. Earlier stages of these studies involved the modeling of planing crafts as a planing plate. In this context, Sottorf [1] studied forces of viscous and perfect fluid acting on planing plates. He also presented some schematics of longitudinal and transverse pressure distribution on these plates [2]. Wagner [3] on the other hand, tried to solve the water entry problem that is very useful in determining the loads on planing boats. He used potential theory and presented the velocity potential of the fluid for water entry of wedge shaped bodies. Korvin-Kroukovsky et al [4] used non-dimensional analysis and extracted an empirical formula for lift by some assumptions such as neglecting the surface tension. Pierson and Leshnover [5] analytically studied the pressure distribution and hydrodynamic forces acting on these hulls. In addition, Kapryan and Boyd [6] carried out a series of tests for measuring bottom pressure distribution of planing hulls with various deadrise angles. Because of many parametric plots, their efforts [6] were considered very useful. Furthermore, Shuford [7] conducted an experimental research in which hydrodynamic lift was investigated and some important results were presented.

In addition to all of the surveyed studies, Savitsky [8] developed a unique mathematical model for prediction of performance of planing boats. In his work, lift force and some other hydrodynamic characteristics were calculated by using empirical equations. Brown [9] also established an empirical relation for calculating the lift coefficient. Payne [10] analytically determined hydrodynamic force acting on the planing plates in steady and unsteady conditions. Later, he [11] presented a scheme in which pressure distribution and dynamic force can be computed by using thin wing methodology.

Numerical method is another approach for determining the lift force. One of the most important studies in this regard is the effort by Savander et al [12]. They used boundary element method (BEM) for computing the pressure distribution and lift force in steady planing problem. Numerical method was also utilized for examining the water entry problem by Zhao et al [13] and Song [14]. In the meantime, Ghadimi et al [15] used smooth particle hydrodynamic method (SPH) for determining the pressure distribution on planing plates. SPH method can also be used more generally for prediction of performance of planing hulls. On the other hand, Morabito [16] modified some practical equations of Smiley [17] and established a semi-empirical model for pressure distribution. Yen et al [18] also used both numerical and experimental model for investigating the lift of planing cylinders. Recently, Tavakoli et al [19] used Morabito's observation and modeled dynamic pressure distribution on the bottom of planing vessels. Subsequently, Ghadimi et al [20] proposed a computational procedure for determining three-dimensional total pressure acting on the bottom of the planing surfaces by means of Morabito's approach [16].

As useful as Savitsky's [8] empirical relation is, it can't be used for many of real planing hulls in which deadrise angle and beam are not fixed from the stern to bow. On the other hand, this equation can't be used for a planing hull with large value of L/b and can only be used for the condition $L/b > 4$. Accordingly, in this paper, hydrodynamic and hydrostatic lifts in planing hulls with large value of L/b is investigated. Subsequently, this scheme can be considered as the first step of investigating the planing boats with large value of L/b such as Series 65. In this context, existing equations of the pressure distribution which were derived in the mentioned studies are described and the proposed computational procedure by Ghadimi et al [20] will be utilized. These equations include hydrodynamic and hydrostatic pressure distributions and effect of transom stern on the pressure. The equation related to pressure distribution is used for determining the lift force. A computational procedure is presented for prediction of lift force of the planing hulls and a computer program is developed in MATLAB. The computer program is validated by comparing its outputs against empirical equations of Savitsky's model [8]. Effects of different parameters such as trim angle, mean wetted length, and deadrise angle on the lift force are investigated. Finally, percentages of contribution of hydrodynamic and hydrostatic lifts are studied for planing hulls with different deadrise angles and speed coefficients.

2. Mathematical Formulation

Weight of a planing hull is supported by the lift force which consists of two terms; hydrodynamic and hydrostatic terms. Lift coefficient, based on the beam squared, is denoted by C_L and lift coefficient, based on the bottom area, is denoted by $C_{L,s}$. These coefficients are obtained from equations (2.1) and (2.2) (Morabito ([18])).

$$C_L = \frac{L}{0.5\rho V^2 b^2} \quad (2.1)$$

$$C_{L,s} = \frac{L}{0.5\rho V^2 b^2 \lambda} \quad (2.2)$$

Here, L is the lift force acting on the bottom of the boat, ρ is the density of the water, V is the forward moving velocity, b is the beam and λ is the non-dimensional mean wetted length given by

$$\lambda = \frac{L_M}{b} \quad (2.3)$$

where L_M is the mean wetted length. As pointed out earlier, lift force consists of two terms; hydrodynamic lift which is generated by hydrodynamic pressure and hydrostatic lifts which is produced by the buoyant pressure and the submerged volume. Equation (2.4) was presented by Savitsky [8] for determining the lift force in planing plates:

$$C_{L0} = \tau^{1.1} \left(0.012\sqrt{\lambda} + \frac{0.0055\lambda^{2.5}}{C_v^2} \right) \quad (2.4)$$

In this equation, τ is the dynamic trim angle (degree) and C_V is the speed coefficient that can be computed by

$$C_V = \frac{V}{\sqrt{gb}} \quad (2.5)$$

where g is the gravitational acceleration. As evident in Eq. (2.4), lift force is a combination of two forces. First term $\tau^{1.1} 0.012 \lambda^{0.5}$ expresses hydrodynamic lift and is only a function of trim angle and mean wetted length. However, the second term is recognized as hydrostatic lift which is also a function of speed coefficient in addition to mean wetted length and trim angle. For determination of lift force acting on the bottom of the hull, Savitsky [8] introduced the relationship

$$C_{L\beta} = C_{L0} - 0.0065 \beta C_{L0}^{0.6} \quad (2.6)$$

where β is the deadrise angle (degree). Equations (2.4) and (2.6) are very useful in predicting the performance of planing hulls. However, they are restricted and can only be used for the following situations:

- $0.60 < C_V < 13.00$
- $2 < \tau < 15$
- $\lambda \leq 4$

On the other hand, by having values of hydrodynamic and hydrostatic pressure, lift force can be obtained through Morabito's [18] approach.

2.1. Hydrodynamic pressure

As mentioned earlier, first investigation of the pressure distribution was conducted by Sottorf [2]. He divided the three-dimensional pressure distribution into two segments [2]; longitudinal pressure distribution and transverse pressure distribution. His idea led many researchers to focus on pressure distribution from these points of views. By the current research, longitudinal view is adopted. Accordingly, pressure area of a planing boat must be explained. The projected area of a planing hull is illustrated in Fig.1. In this figure, two areas are obvious; the pressure area in which hydrodynamic forces and resistance are produced and the spray area in which whisker spray and its resistance are generated. These two areas are separated by a stagnation line which was introduced by Pierson and Leshover [4].

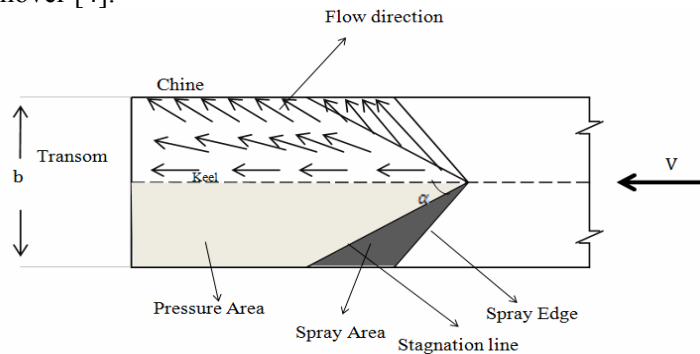


Fig.1 The projected Area of a planing hull [21].

Observations by Smiley [17] led him to draw a schematic of the pressure distribution, as shown in Fig.2. This figure reveals some facts about hydrodynamic pressure distribution. These facts are explained in detailed by Morabito [16] and listed as follows:

- (1) Existence of a maximum pressure that results from the intersection of keel and calm water line,
- (2) A peak pressure line which is near the stagnation line and a maximum pressure can be seen at the intersection of this line and a longitudinal section,

- (3) As the distance of the point from transom decreases, a decrease in longitudinal pressure distribution is observed,
- (4) Hydrodynamic pressure is zero at the transom stern,
- (5) Although value of pressure in maximum pressure line decreases as the distance from the center line increases, this reduction is very small in comparison to the reduction of pressure in longitudinal section, and
- (6) Value of hydrodynamic pressure in the chine is also zero.

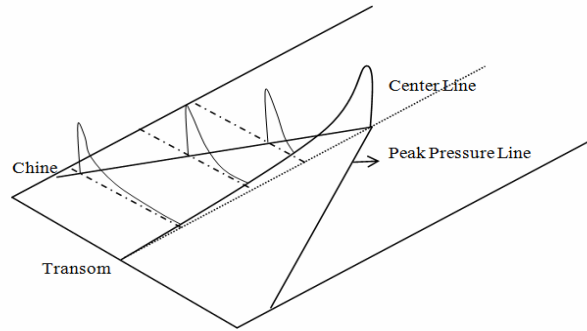


Fig.2 Three-dimensional hydrodynamic pressure distribution over the bottom of a planing hull [17].

Pressure distribution over the center line has a maximum value at the intersection of the center line and the peak pressure line and subsequently decreases as its distance from this intersection gets smaller. Morabito [16] proposed a new equation for determination of the dynamic pressure which can be applied to any type of longitudinal section. He applied boundary conditions like P_L/q at $X=0$ and at the transom for establishing this relation. Through his efforts, equations (2.7) through (2.12) were derived which is later used for computing the lift force.

$$\frac{P_L}{q} = P_T P_Y \frac{CX^{\frac{2}{3}}}{(X+K)} \quad (2.7)$$

$$P_T = \frac{(\lambda_y - X)}{(\lambda_y - X)^{1.4} + 0.05} \quad (2.8)$$

$$C = 0.006 P_Y \tau^{\frac{1}{3}} \quad (2.9)$$

$$K = \frac{C^{1.5}}{2.5888 \left(\frac{P_{\max}}{q}\right)_Y^{1.5}} \quad (2.10)$$

$$\frac{P_{Ystag}}{P_N} = [1.02 - 0.25Y^{1.4}] \frac{0.5 - Y}{0.51 - Y} \quad (2.11)$$

$$P_Y = [1.02 - 0.05(\beta + 5)Y^{1.4}] \frac{0.5 - Y}{0.51 - Y} \quad (2.12)$$

In the above equations, P_L is the hydrodynamic pressure, X is the non-dimensional distance from the intersection of calm water and keel. C and K are two dependent variables, P_T is the transom effect function and has a specific value in each longitudinal section, while P_Y is the effect of chine and also has a specific value in each longitudinal strip. Y is the non-dimensional distance from the keel and λ_y is the non dimensional wetted length of each longitudinal section introduced by Morabito [18].

2.2. Bouyant pressure

For a displacement hull, bouyant force is equal to the submerged volume. However, determination of this force in a planing craft is complicated. This complexity is caused by the transom stern and the chine. Shuford [7] assumed that this force is half of the submerged volume and Zarnick [22] another technique dealing with this issue. Morabito [16] expressed that hydrostatic pressure is affected by both transom and chine and can be obtained using equation (2.13).

$$\frac{P_B}{q} = \frac{\rho g H(X, Y)}{q} P_T P_Y \quad (2.13)$$

where $H(X, Y)$ denotes the depth.

2.3. Lift coefficient

Determination of the lift coefficient requires integration of the total pressure over the bottom of the planing hull. In this regard, Total pressure (P_{Total}) is determined by the summation of the hydrostatic and hydrodynamic forces as shown in equation (2.14). Subsequently, lift coefficient is calculated using equation (2.15).

$$\frac{P_{Total}}{q} = \frac{P_L}{q} + \frac{P_B}{q} \quad (2.14)$$

$$C_{LS} = \int_{-0.5}^{+0.5} \int_0^{\lambda_y} \frac{P_{Total}}{q} dx dy \cos \tau \quad (2.15)$$

Hydrostatic and hydrodynamic lift coefficients can also be calculated using equations (2.16) and (2.17). C_{LS} is considered as hydrostatic lift coefficient and C_{LD} is considered as the hydrodynamic lift coefficient.

2.4. Computational Procedure

After presenting mathematical model and its formulation, it is necessary to develop a computational procedure for determining the lift force. First, hydrodynamic and hydrostatic pressures are determined and then total pressure is computed using the proposed procedure by Ghadimi et al [20]. Accordingly, a mesh is generated on the bottom of the planing hull. Subsequently, bouyant and hydrodynamic pressures are determined in each longitudinal section. This computation procedure starts at the center line ($Y=0$) and finishes at the chine. On the other hand, pressure is calculated from the point $X=0$ to $X=\lambda_y$ for different longitudinal sections. Finally, lift force and hydrodynamic and hydrostatic coefficients are computed.

An algorithm is presented in Fig.3 and a computational program is developed. Inputs of the program are the deadrise and trim angles, speed coefficient, and mean wetted length. Lift force and pressure distribution are the outputs of the proposed model.

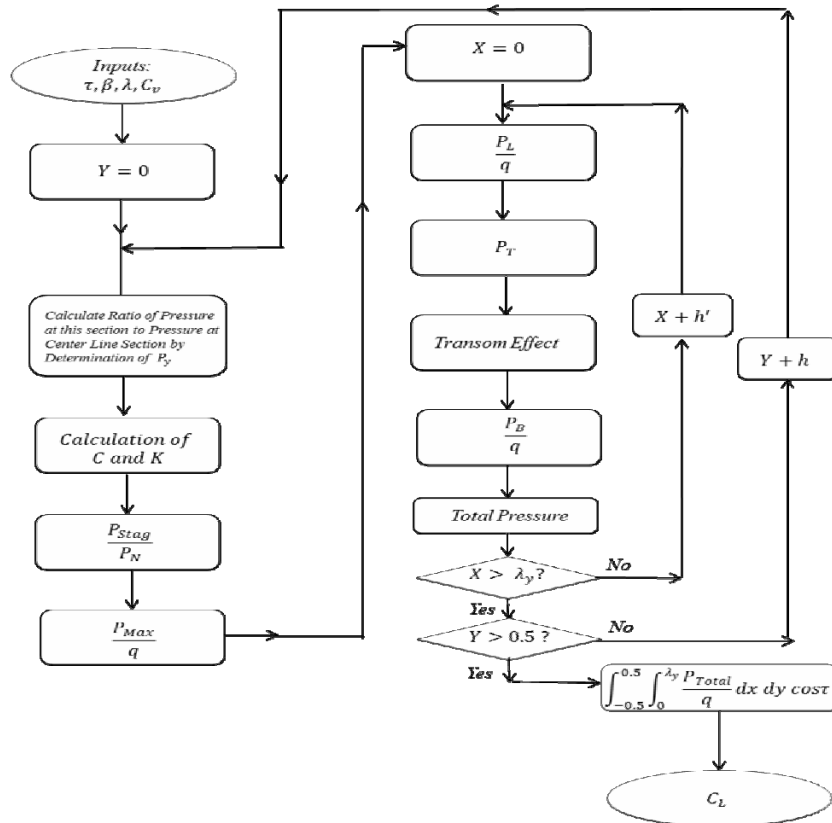
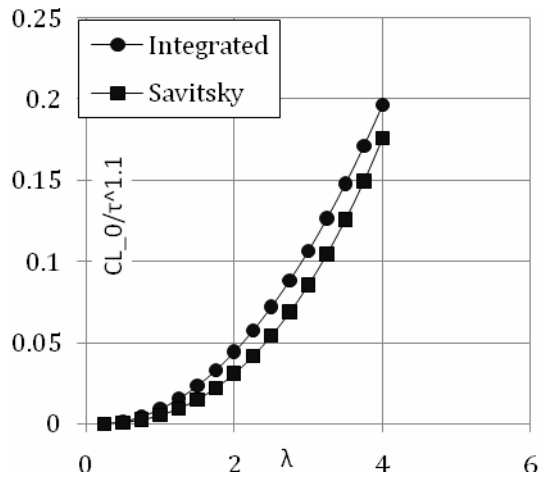


Fig.3 Computational algorithm for determining the lift force of the planing boats.

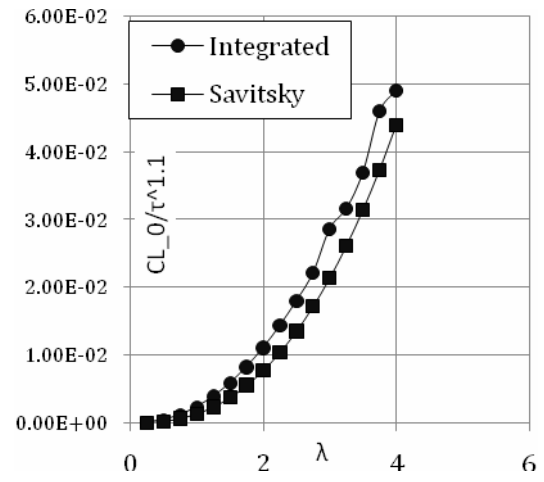
This computational method can also be used for $\lambda > 4$ and lift of the crafts with the mean wetted length can be computed. On the other hand, this computational method can easily be adjusted for the planing boats with variable deadrise angle and beam in their length.

3. Validation

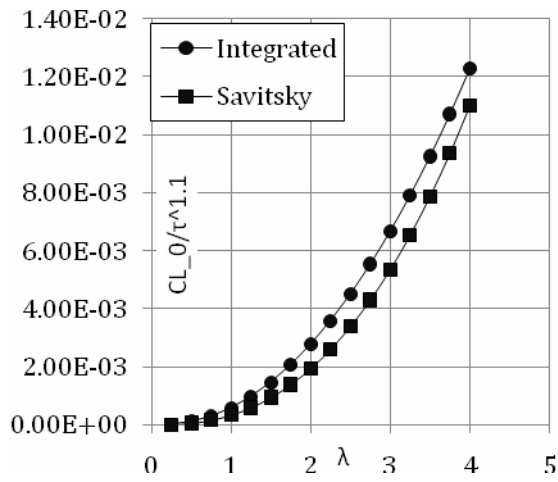
At this stage, it is necessary to validate the proposed method and the developed computer program. Hence, results of the presented model are compared against those by Savitsky's equation [8]. For a better comparison, values of $C_L/\tau^{1.1}$ are compared instead of lift coefficient. The predicted values of $C_L/\tau^{1.1}$ using the presented technique are compared against Savitsky's model at four various speed coefficients of $C_v = 1, 2, 4$ and 6 and are plotted via mean wetted length for a planing plate. Then, hydrostatic term and total lift are compared against Savitsky's equation, separately. Thus, predicted hydrostatic lift is compared against equation (2-3) and this comparison is illustrated in Fig.4. As evident in this figure, favorable accuracy is displayed. Total lift is computed and a comparison is illustrated in Fig.5. Favorable agreement is also achieved for the total lift of the planing plate.



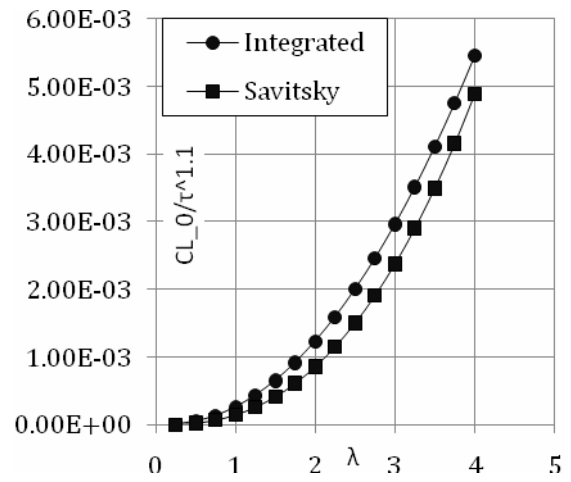
(a)



(b)



(c)



(d)

Fig.4 Comparison of the predicted hydrostatic term of $C_L / \tau^{1.1}$ against Savitsky's method.

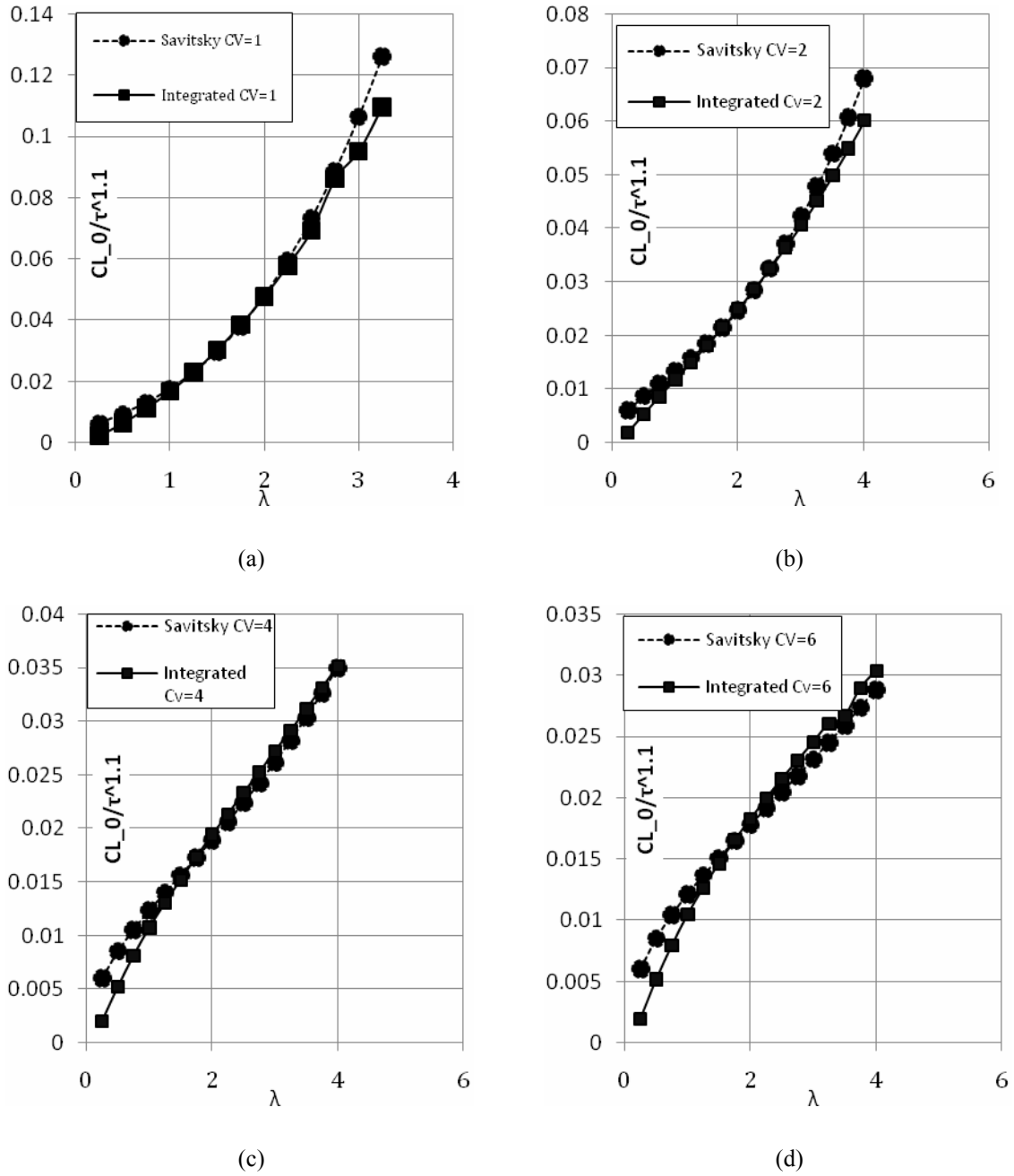


Fig.5 Comparison of the predicted total lift $C_L / \tau^{1.1}$ against Savitsky's method.

The predicted hydrodynamic pressure distribution is also compared against some of Kapryan and Boyad's [5] results and this comparison is shown in Figs.6 and 7. Therefore, values of P/q are determined in three different longitudinal sections. Nondimensional beam of these rows are $Y=0.025, 0.25, \text{ and } 0.475$. Predicted hydrodynamic pressures by the proposed method are very close to the experimental data of Kapryan and Boyd [6].

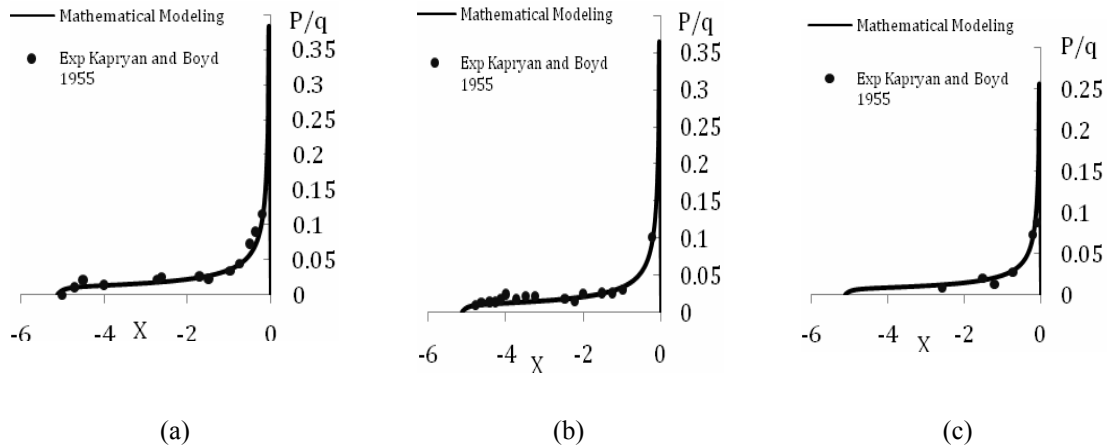


Fig.6 Predicted pressure distribution compared against Kapryan and Boyd’s [5] result for a planing plate with $\tau = 4$ and $\lambda = 5.14$ (a) $Y = 0.025$ (b) $Y = 0.25$ (c) $Y = 0.475$.

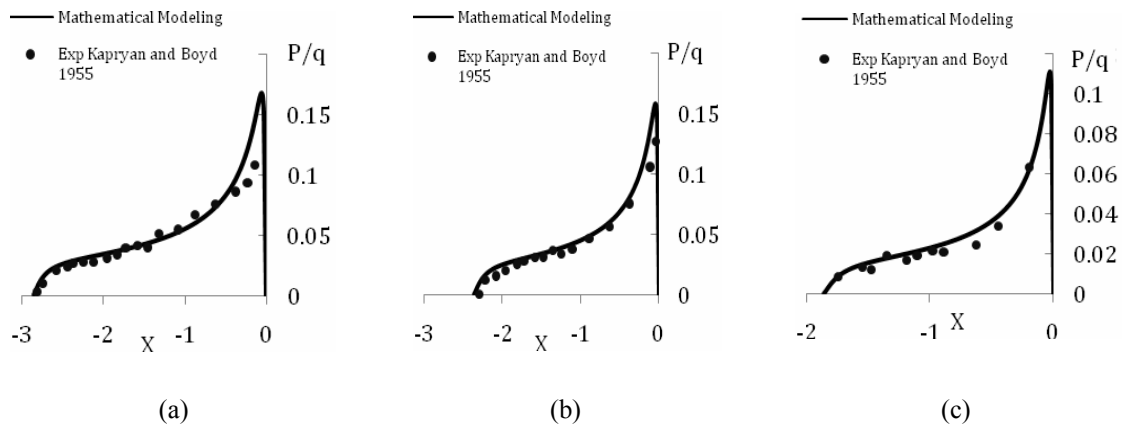


Fig.7 Predicted pressure distribution compared against Kapryan and Boyd’s [6] result for a planing hull with $\beta = 20$, $\tau = 4$ and $\lambda = 5.14$ (a) $Y = 0.025$ (b) $Y = 0.25$ (c) $Y = 0.475$.

4. Parametric Study : Results and Discussion

Effects of different parameters on lift are analyzed. These parameters include deadrise and trim angles, mean wetted length, and speed coefficients. On the other hand, contributions of hydrodynamic and hydrostatic lifts are investigated.

4.1. Hydrodynamic lift

First, effects of trim and dearise angles are investigated on the hydrodynamic lift force. Figure 8 shows the effect of increasing the trim angle on the hydrodynamic term in planing hulls. As evident in this figure, increasing the trim angle causes an increase in hydrodynamic lift, while slope of the resultant curve decreases with a decrease in the deadrise angle. Figure 9 illustrates the effect of deadrise angle on hydrodynamic lift. As shown in this figure, hydrodynamic lift decreases by increasing the deadrise angle and greater value of the hydrodynamic lift is observed for a planing hull with zero deadrise angle.

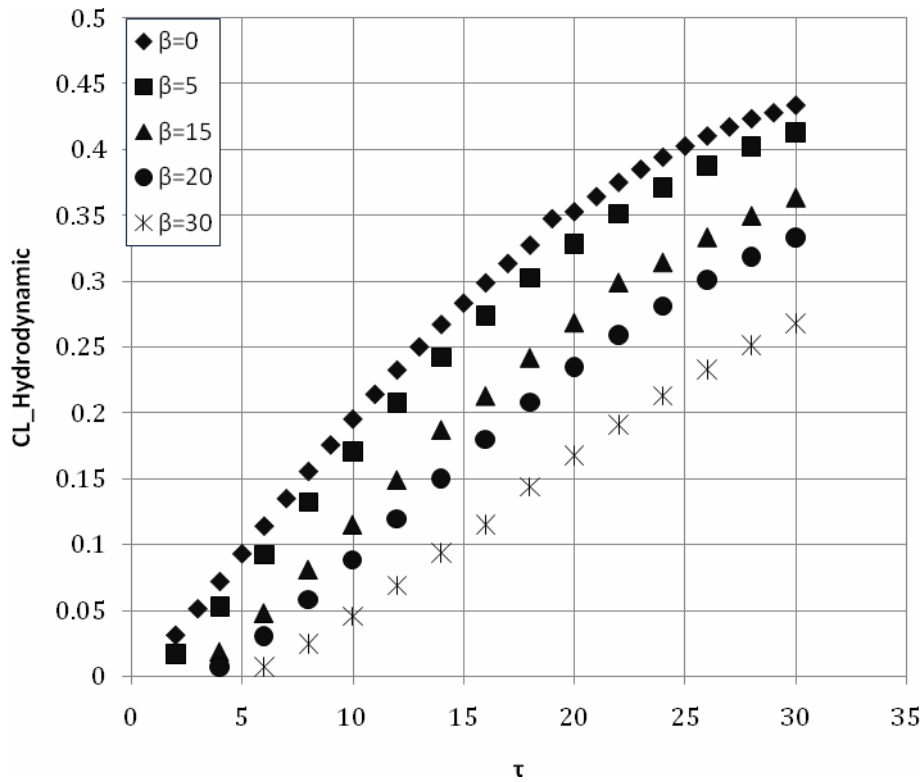


Fig.8 Effects of trim angle on hydrodynamic lift of planing hulls with $C_v = 4$, $\lambda = 1$ and various deadrise angles.

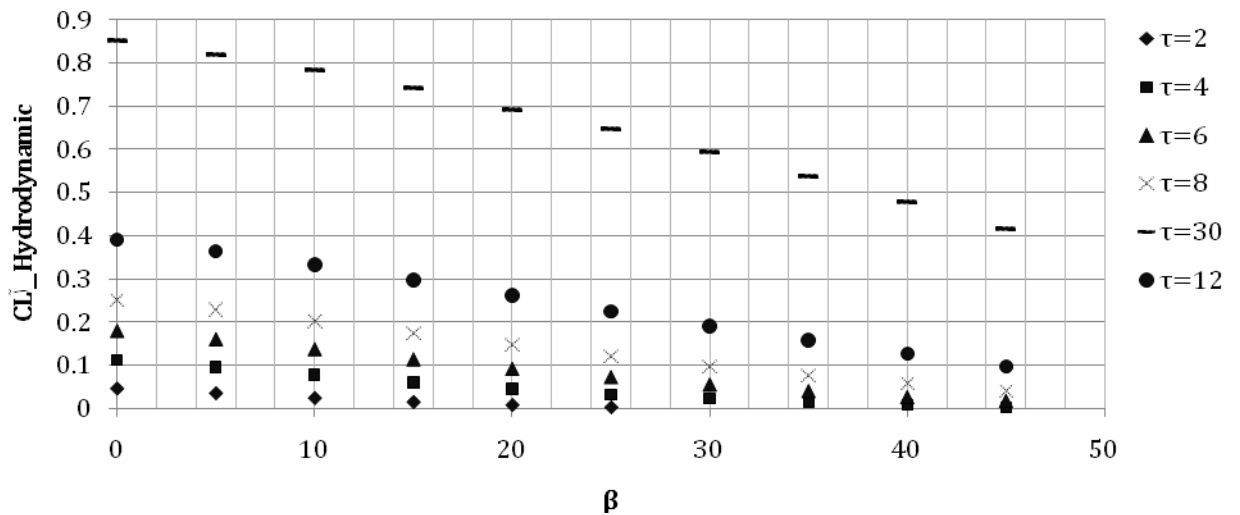


Fig.9 Effects of Deadrise angle on hydrodynamic lift with $C_v = 4$, $\lambda = 2.5$ and various trim angles.

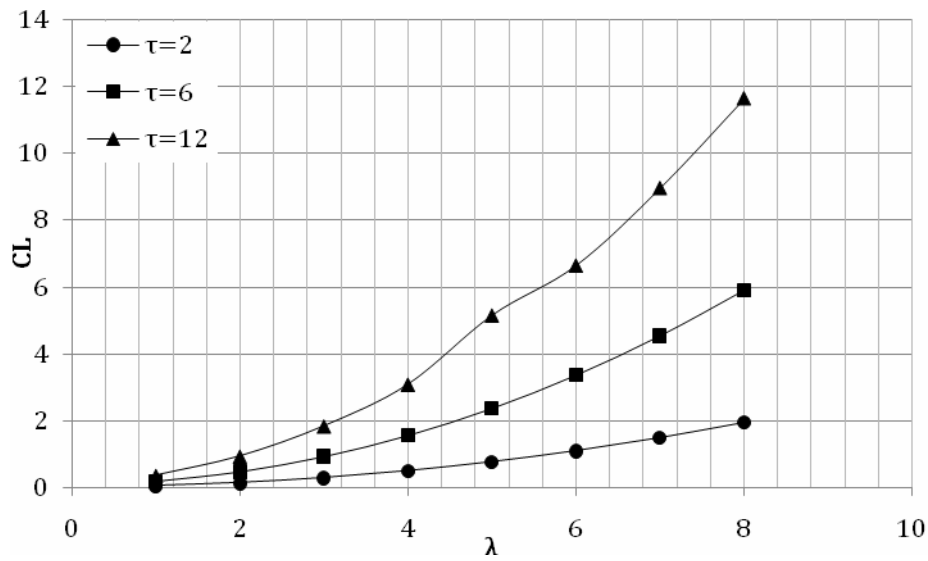
4.2. Total Lift

In order to investigate the effect of different parameters on the lift force, it is calculated using the proposed method for planing hulls with deadrise angle of 40° at three speed coefficients of $C_v=1, 2$ and 3 and three trim angles including $\tau=2, 6$ and 12 . In all of these cases, total lift

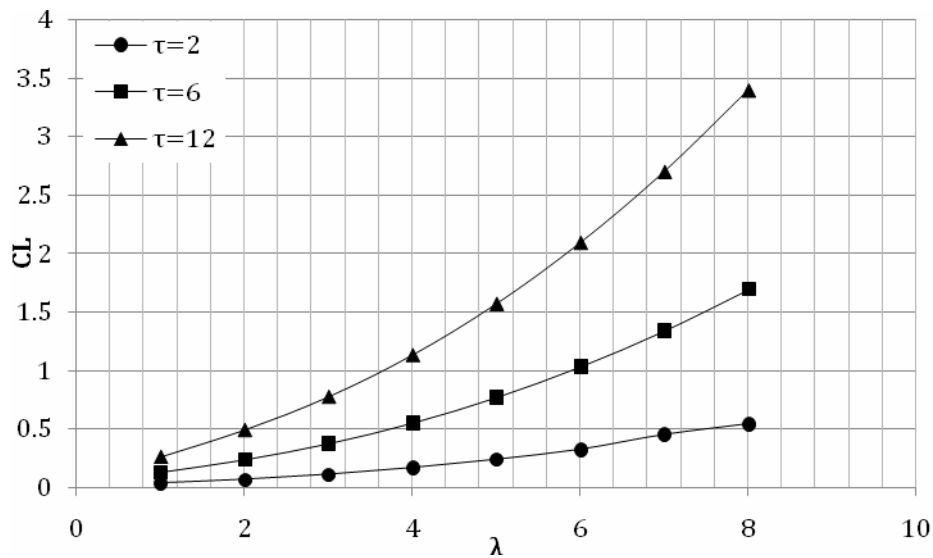
coefficient is computed for $1 \leq \lambda \leq 8$. Figures 10, 11 and 12 show the results for these deadrise angles. All of these figures illustrate that lift force increases by an increase in the mean wetted length.

Increasing speed coefficient while keeping other parameters constant causes a decrease in the lift force. A comparison between cases a, b, and c of each figure confirms this fact. This fact can also be found in Savitsky's formula. In his equation, there is an inverse relationship between hydrostatic lift and C_v^2 while there is no relation between C_v and hydrodynamic lift.

With regard to the trim angle, while other parameters are fixed, this parameter has a direct relation to the lift. Increasing the pressure distribution by increasing the trim angle is the main reason behind this change. All these cases include three trim angles and signify this fact. On the other hand, at higher trim angles, slope of the resultant plot of the lift via the mean wetted length is greater and the lift force increases, more intensely. Three illustrated plots in cases a, b, and c of each figure confirms this fact.



(a)



(b)

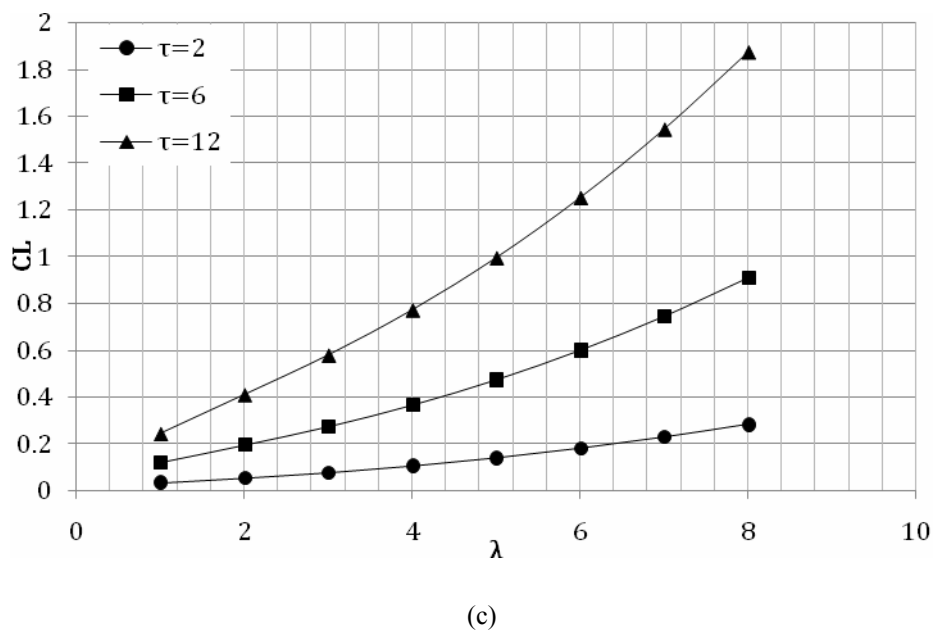
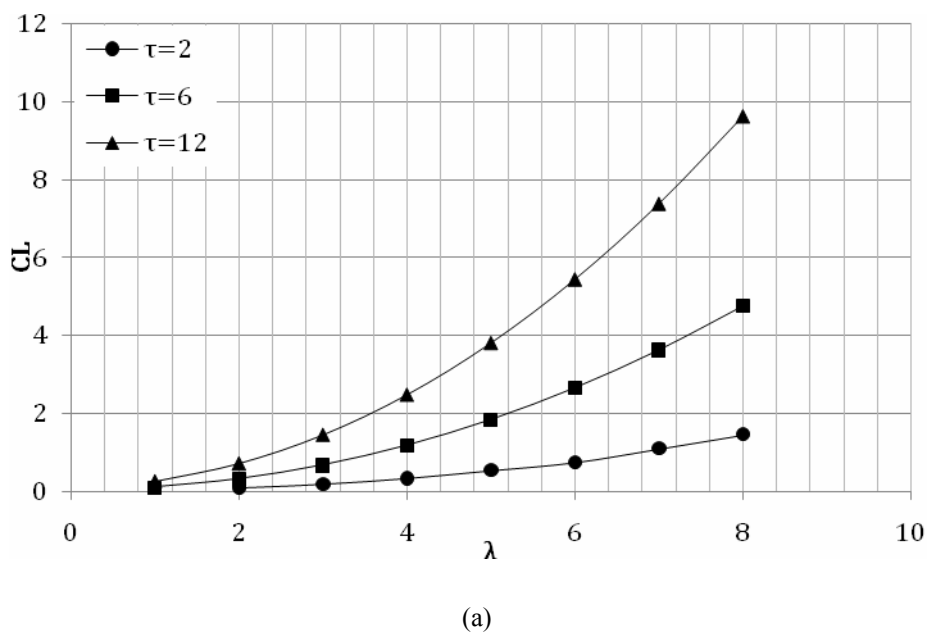
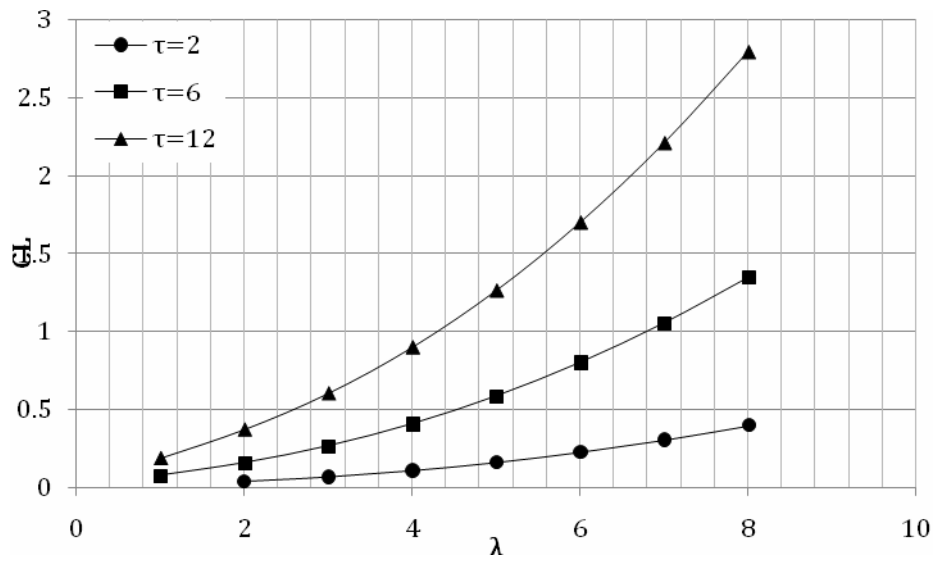
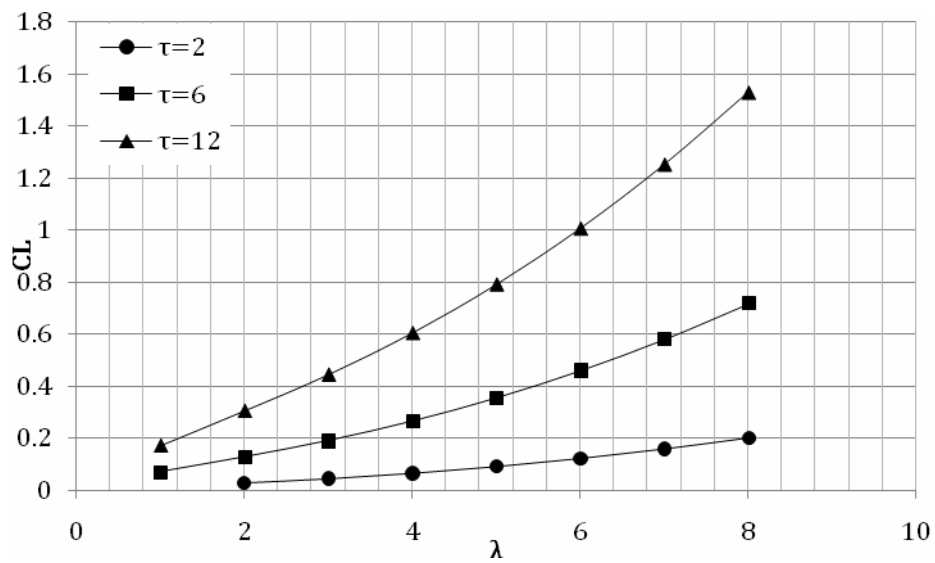


Fig.10 Total lift coefficient of planing hull with $\beta = 0$ (a) $C_V = 1$ (b) $C_V = 2$ and (c) $C_V = 3$.



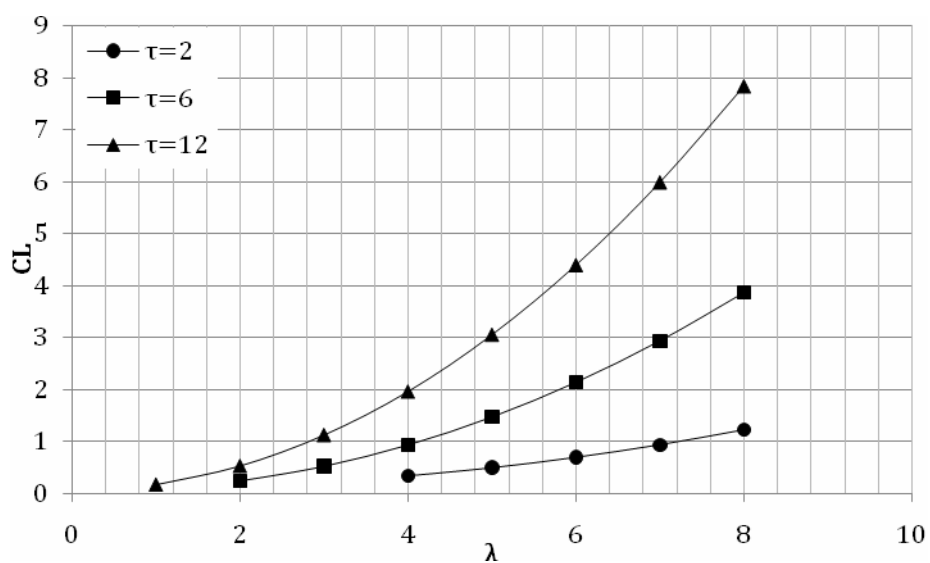


(b)

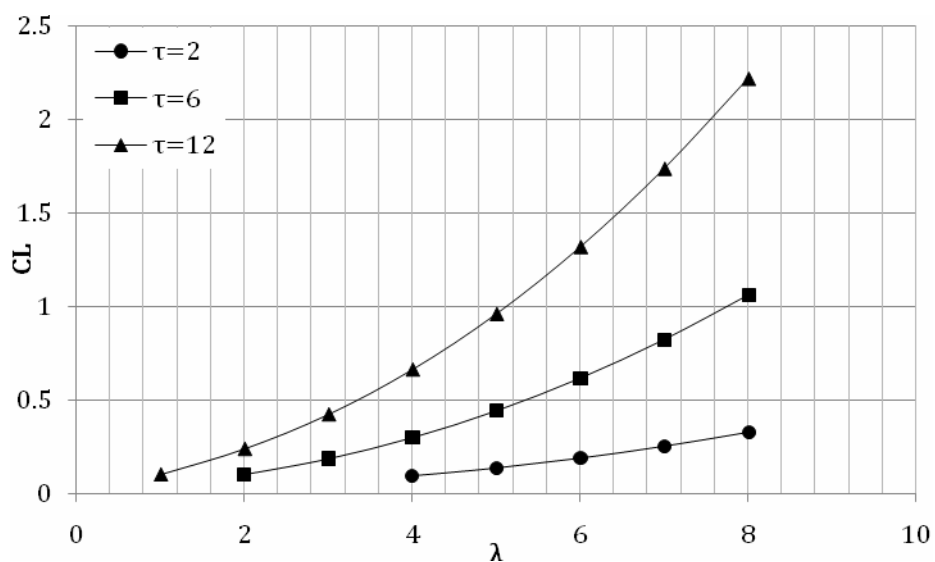


(c)

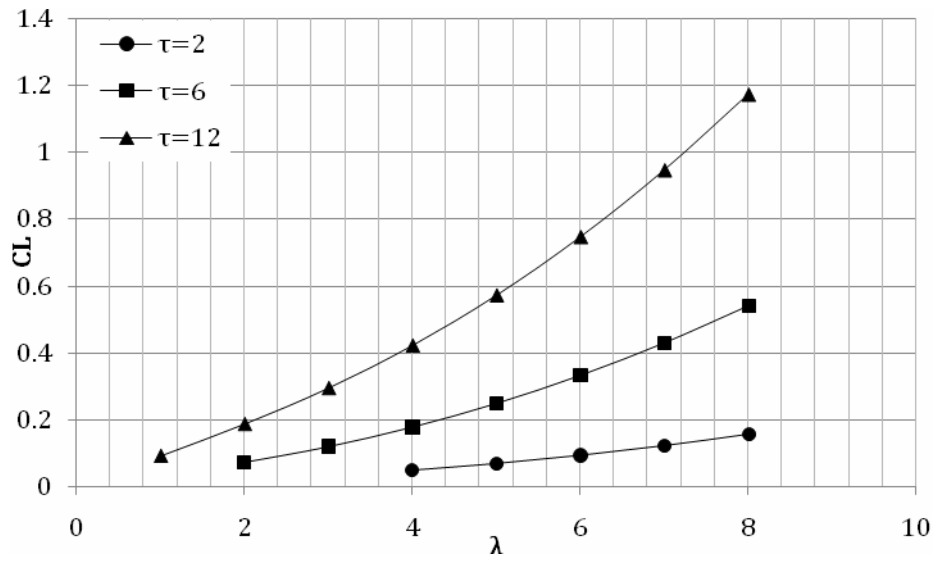
Fig.11 Total lift coefficient of planing hull with $\beta = 20$ (a) $C_V = 1$ (b) $C_V = 2$ and (c) $C_V = 3$.



(a)



(b)



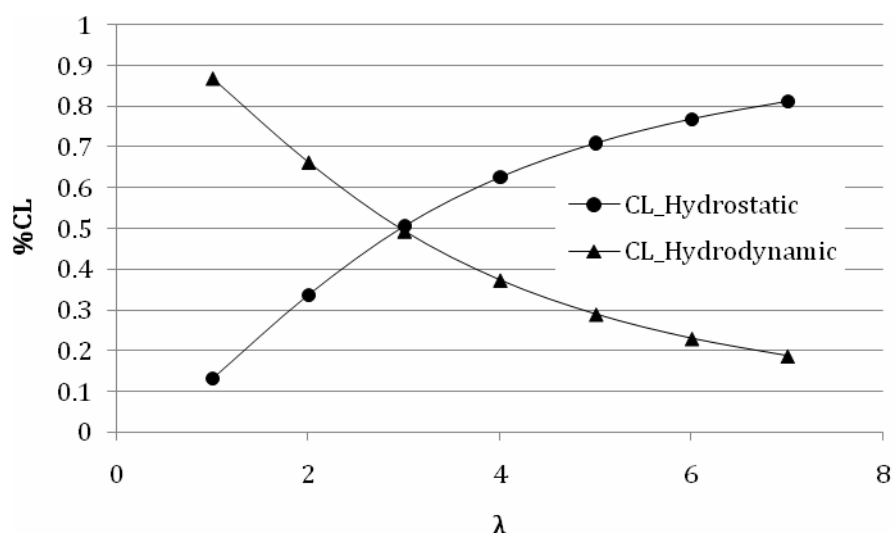
(c)

Fig.12 Total lift coefficient of planing hull with $\beta = 40$ (a) $C_V = 1$ (b) $C_V = 2$ and (c) $C_V = 3$.

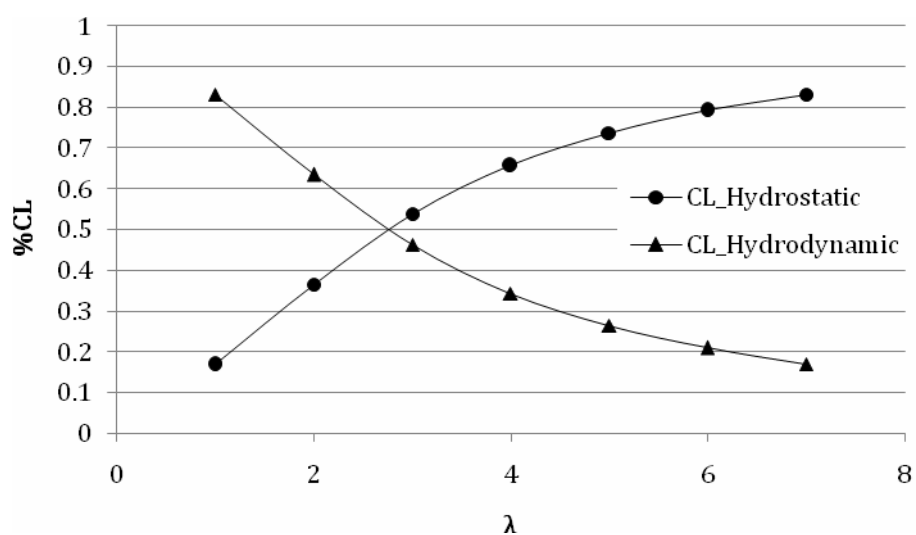
4.3. Contribution of hydrostatic and hydrodynamic lift components

The weight of a planing hull is supported by both hydrodynamic and buoyant pressure. Here, percentage of contribution of these two components is investigated. Accordingly, percentages of contribution of these components have been computed for a planing hull with zero deadrise angle and a planing hull with deadrise angle of 20 degrees at $C_V=2$ and $\tau=2$. The obtained results are displayed in Fig.13. Figure 13 indicates that for both planing hulls at $\lambda=1$, hydrodynamic term has the most contribution, while in the planing hull with $\beta=0$, this contribution is 86.9% and for the other planing craft ($\beta=20$), this contribution is 83.02%. For both cases, contribution of hydrostatic term increases by increasing of the mean wetted length. At a specific mean wetted length ($\lambda=3$), percentage of contribution of hydrostatic and hydrodynamic components for both hulls are equal. Furthermore, at $\lambda=7$, hydrostatic component has the most contribution. At this mean wetted length, percentage of contribution of hydrodynamic term for the planing boat with $\beta=20$ is 83.1%, while for the other hull ($\beta=0$), this contribution is about 81.2%.

Effect of speed coefficient on this contribution is also investigated and is shown in Fig.14 for a planing hull of $\beta=0$ at $\tau=4$ and at two different speed coefficients $C_V=2$ and 4. This figure proves that contribution of hydrodynamic lift at $C_V=2$ is larger than at $C_V=4$ and has the largest value at $\lambda=1$ which is approximately equal to 94%. Furthermore, at $C_V=4$, it can be concluded that contribution of hydrostatic term is larger at $\lambda \leq 6$ while contribution of hydrodynamic term becomes larger at a mean wetted length between 6 and 7, and at $C_V=2$, hydrostatic term becomes larger at $\lambda > 3$.



(a)



(b)

Fig.13 Percentages of contribution of hydrodynamic and hydrostatic lift coefficients for planing hulls with $C_V = 2$ and $\tau = 4$ (a) $\beta = 0$ and (b) $\beta = 20$.

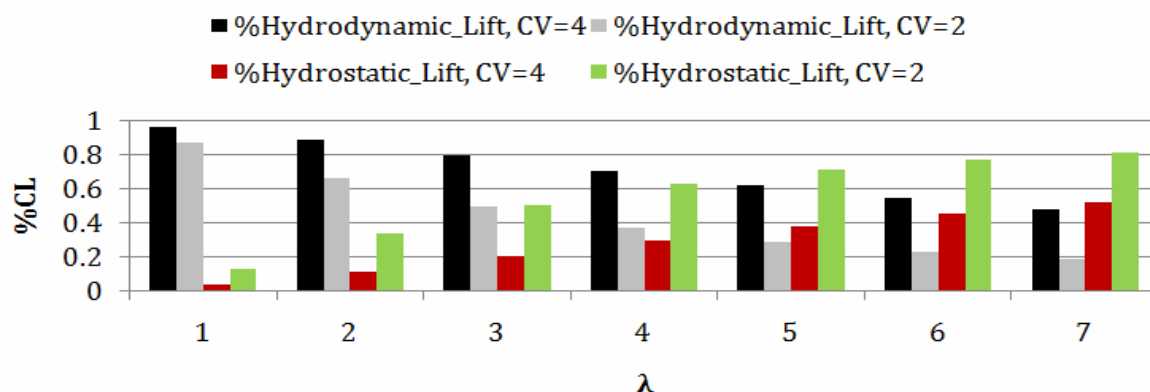


Fig.14 Percentages of contribution of hydrodynamic and hydrostatic lift coefficients for a planing hull with $\beta = 0$ and $\tau = 4$ (a) $C_V = 2$ and (b) $C_V = 4$.

5. Conclusions

By the present research, a mathematical procedure was developed and presented for calculation of lift force of the planing hulls. This method was based on semi-empirical equations of bottom pressure distribution over prismatic planing hulls and can also be used for hulls with $\lambda \geq 4$ and doesn't exhibit any limitation for the mean wetted length. To validate the proposed method and computational program, results of the developed code were compared against those by Savitsky's method [8] and favorable accuracy was displayed. On the other hand, the predicted pressure distributions were compared against the experimental results of Kapryan and Boyd [6] and accuracy of the prediction of hydrodynamic pressure was also verified.

Influences of some effective physical parameters are investigated and following observations are concluded:

- 1) Increasing the trim angle causes an increase in the hydrodynamic component. This effect is minimal at large deadrise angles.
- 2) Increasing the deadrise angle would cause an increase in the hydrodynamic component of the lift coefficient.
- 3) Mean wetted length has direct relation with the total lift coefficient.
- 4) Increasing the running trim angle causes an increase in the total lift, while other parameters are kept fixed. On the other hand, the slope of the resultant plot has a direct relation with the trim angle.
- 5) Hydrostatic lift coefficient has an inverse relation with CV.
- 6) Percentage of contribution of hydrostatic and hydrodynamic component is investigated for two different deadrise angles at CV=2 and 4, and $\tau=4$. It is shown that contribution of hydrodynamic lift decreases by an increase in the mean wetted length.
- 7) A study on percentages of contribution of both components is performed for a planing hull with zero deadrise angle at two different speed coefficients and it is concluded that at larger speed coefficients, contribution of hydrodynamic lift is greater.

This scheme can be considered as a first step of a method for investigating the performance and dynamic motion of planing hulls with large values of L/b or planing hulls with variable deadrise angles and beam in their length. Dynamic motion of these crafts and prediction of their performance including prediction of the running trim angle and resistance are the future plans of our study.

Nomenclature

b	Beam of the planing hull		A variable for determination of pressure distribution
C	A variable for determination of pressure distribution	K	
C_{L0}	Lift Coefficient of planing plates	P_B	Hydrostatic pressure
C_L	Lift Coefficient based on beam squared	P_L	Pressure distribution over the planing hull
$C_{L,S}$	Lift Coefficient based on bottom area	P_{Max}	Maximum pressure acting on the bottom of a planing hull
C_{LS}	Hydrostatic component of lift coefficient	P_T	Transom effect function
C_{LD}	Hydrodynamic component of lift coefficient	P_{Total}	Total pressure
$C_{L\beta}$	Lift Coefficient of planing hulls - non-zero deadrise angles (Savitsky's equation)	P_Y	Transverse pressure distribution ratio
$C_V = V/\sqrt{gB}$	Speed coefficient	$\frac{P_{YStag}}{P_N}$	Ratio of pressure over stagnation line to pressure due to V_{∞}
g	Gravity acceleration	V	Velocity of the craft
H	Depth	$X = x/b$	Non-dimensional longitudinal distance from the stagnation line
L_M	Mean wetted length	x	Longitudinal distance from the stagnation line
		$Y = y/b$	Non-dimensional lateral distance from center line

y	Lateral distance from the center line		length
$\alpha = \tan^{-1}\left(\frac{\pi \tan \tau}{2 \tan \beta}\right)$	Angle between stagnation line and keel	λ_y	Non-dimensional distance from the transom stern at each longitudinal section
α_w	Angle between calm water line and center line	ρ	Density of water
β	Deadrise angle	τ	Trim angle
$\lambda = L_M / B$	Non-dimensional mean wetted		

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