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Application of Non-linear Analytical Methods to Flow between Porous Walls with Variable Permeability

M. Movahedi, D.D. Ganji
Department of Mechanical Engineering, Babol University of Technology, Babol, Iran
movahedi_mm@yahoo.com
ddg_davood@yahoo.com

Abstract
By the current research, we focus on the viscous flow driven by small porous wall of two variable permeable walls. Most fluid mechanics problems such as two-dimensional viscous flow between porous walls with permeability and other fluid mechanic problems are inherently nonlinear. The problem is first reduced to a nonlinear differential equation that is later solved both numerically and analytically. Except a limited number of these problems, most of them do not have analytical solution. Therefore, these nonlinear equations should be solved using other methods. By the present article the problem, Adomian's Decomposition Method and Collocation Method and Least Square Method are employed to compute an approximation to the solution of the system of nonlinear differential equations governing the problem. Comparisons are made between the fourth-order Runge–Kutta numerical method (NUM) and the results of the Adomian's Decomposition Method (ADM) and Collocation Method (CM) and Least Square Method (LSM). The results reveal that these methods are very effective and simple and can be applied for fluid nonlinear problems.

Key Word and Phrases
Nonlinear Equation, Two-dimensional Viscous Flow, Weak Permeability, Adomian's Decomposition Method (ADM), Collocation Method (CM), Least Square Method (LSM).

1. Introduction

Investigations on particular fluid transport in biological organisms are often concerned with the flow of a particular fluid inside a vessel with permeable walls. The flow behavior inside the lymphatics exhibits a similar character. In such models, circulation is induced by successive contractions of two thin sheets that cause the downstream convection of the sandwiched fluid. Seepage across permeable walls is clearly important to the mass transfer between blood, air and tissue (Chang et al. [1]). Its assessment can serve to better understand the function of biological filters such as kidneys and lungs. Therefore, the Navier–Stokes equations for a semi-infinite tube were reduced to a single differential equation (Cox and King [2]). This 'exact' reduction was precipitated by a viable similarity transformation in space and time. Their equation was then solved numerically and used to explain the principal flow characteristics. In a later study by Goto and Uchida [3], the similarity analysis was repeated for a contracting tube with permeable walls.

The first study by Uchida and Aoki [4] constituted a visible improvement over former studies of peristaltic pumping that considered either inviscid flows (Jones [5]) or pulsating flows inside infinitely long, valveless tubes (cf. Lighthill [6]; Jaffrin and Shapiro [7]; Shapiro et al. [8]; Fung and Yih [9]).The second study by Goto and Uchida not only captured the effects of viscosity, wall contraction, and realistic body length, but also accounted for the effect of wall permeability. Their ODE was hence capable of embracing the equations produced by Yuan and Finkelstein [10], and Terrill and Thomas [11] for a permeable tube with stationary walls. In principle, results obtained by other investigators could be reproduced from Goto and Uchida's differential equation by setting its wall expansion rate to zero. Under the assumption of weak permeability and slow wall motion, the current study focuses on obtaining an asymptotic solution for the equivalent slit flow problem. In the process, the reader will be exposed to the complete development of the governing equation in rectilinear coordinates. This will be followed by an asymptotic solution that can be corroborated by numeric. Our planar solution will be carried out without imposing any of the overly simplifying assumptions used in previous studies (e.g. Wang [12]; Bhatnagar [13]).The flow of Newtonian and non-Newtonian fluids in a porous surface channel has attracted the interest of many investigators in view of its applications in engineering practice, particularly in chemical industries. Examples of these are the cases of boundary layer control, transpiration cooling and gaseous diffusion. Theoretical research on steady flow of this type was initiated by Berman [14] who found a series solution for the two-dimensional laminar flow of a viscous incompressible fluid in a parallel-walled channel for the case of a very low cross-flow Reynolds number. After his work, this problem has been studied by many researchers considering various variations in the problem, e.g., Choi et al. [15] and references cited therein. For the case of a converging or diverging channel with a permeable wall, if the Reynolds number is large and if there is suction or injection at the walls whose magnitude is inversely proportional to the distance along the wall from the origin of the channel, a solution for laminar boundary layer equations can be obtained [16]. Except a limited number of these problems, most of them do not have analytical solution. Therefore, these nonlinear equations should be solved using other methods[17]-[19]. The Adomian's Decomposition Method (ADM) and Collocation Method (CM) and Least Square Method (LSM) are well-known method to solve the nonlinear equations. They were shown by many authors that these methods provide improvements over existing numerical techniques.

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation (CM) and Least Square (LSM) are examples of the WRMs. Stern and Rasmussen [20] used collocation method for solving a third order linear differential equation. Vaferi et al. [21] studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami [22] used the Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently least square method is introduced by Aziz and Bouaziz [23] and is applied for prediction of the performance of a longitudinal fin [24]. They found that least squares method is simple compared with other analytical methods. In this study, we explore the analytical solutions to flow between porous walls with different permeability, especially walls with weak permeability and will make a comparison between the ADM and CM and LSM analytical methods and numerical results solved by fourth-order Runge–Kutta method.

2. Mathematical Formulation

As shown in Fig.1, consider the laminar, isothermal, and incompressible flow in a rectangular domain bounded by two permeable surfaces that enable the fluid to move during successive [25]. One side of the cross section, representing the distance ($2a$) between the walls is taken to be smaller than the other two (W and L). Both walls are assumed to have equal permeability and to expand uniformly at a time dependent rate $\dot{a}$. Furthermore, the origin $\hat{x}=0$ is assumed to be the center of the classic squeeze film problem. This enables us to assume flow symmetry about $\hat{x}=0$ [26].

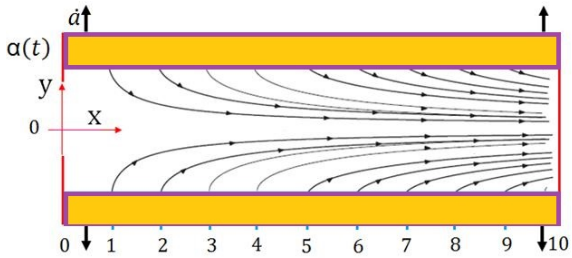

Fig.1 Schematic of the problem to flow between porous walls with variable permeability.

Under these assumptions, the equations for continuity and motion become

$$\frac{\partial \hat{u}}{\partial \hat{x}}+\frac{\partial \hat{v}}{\partial \hat{y}}=0 \tag{2.1}$$

$$\frac{\partial \hat{u}}{\partial \hat{t}}+\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}=-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{x}}+\upsilon\nabla^2\hat{u} \tag{2.2}$$

$$\frac{\partial \hat{v}}{\partial \hat{t}} + \hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{y}} + \upsilon \nabla^2 \hat{v} \tag{2.3}$$

where $\hat{p}$, ρ , υ and t are the dimensional pressure, density, kinematic viscosity, and time. Auxiliary conditions can be specified such as

$$\hat{u}\left(\hat{x},0\right) = 0,\ \hat{v}\left(a\right) = -\hat{v}\left(w\right) = -\dot{a}/c$$

$$\frac{\partial \hat{u}}{\partial \hat{y}}\left(\hat{x},0\right) = 0\ ,\ \hat{v}\left(0\right) = 0,\ \hat{u}\left(0,\hat{v}\right) = 0 \tag{2.4}$$

After some modification and special variable [10], we have

$$\frac{d^4F}{dy^4} + \alpha(y\frac{d^3F}{dy^3} + 3\frac{d^2F}{dy^2}) + Re(\frac{dF}{dy})(\frac{d^3F}{dy^3}) - ReF\frac{d^2F}{dy^2} = 0 \tag{2.5}$$

with the boundary conditions

$$\frac{d^2F}{dy^2}\Big|_{y=0} = 0, F(0) = 0, \frac{dF}{dy}\Big|_{y=1} = 0, F(1) = 1 \tag{2.6}$$

where a prime denotes differentiation with respect to y : Note that Berman's [1] well-known ODE can be viewed as a special case of Eq. (2.6) with $\alpha = 0$.

3. Collocation Method

3.1. Principles of Method

Suppose we have a differential operator D acting on a function u to produce a function p [27]

$$D\left(u\left(x\right)\right) = p\left(x\right) \tag{3.1.1}$$

We want to approximate u by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \tag{3.1.2}$$

Also, when substituted into the differential operator, D , the result of the operations is not, in general, $p\left(x\right)$. Hence an error or residual will exist:

$$E\left(x\right) = R\left(x\right) = D\left(\tilde{u}\left(x\right)\right) - p\left(x\right) \neq 0 \tag{3.1.3}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is [28]

$$\int_x R(x)\,W_i(x) = 0 \quad i = 1, 2, \dots, n \tag{3.1.4}$$

where the number of weight functions W_i are exactly equal the number of unknown constants c_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants c_i . For collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is $W_i\left(x\right) = \delta\left(x - x_i\right)$. The Dirac δ function has the property that [27], [28]

$$\delta\left(x - x_i\right) = \begin{cases} 1 & if\ x = x_i \\ 0 & Otherwise \end{cases} \tag{3.1.5}$$

Furthermore, the residual function in Eq. (3.1.3) must force to be zero at specific points.

3.2. Application of CM to Flow between Porous Walls

Consider the trial function as:

$$\begin{aligned}\mathrm{F}(y) = -0.5y^3 + \frac{1}{2}y + c_1(-\frac{1}{2}y^4 + \frac{3}{4}y^3 - \frac{1}{4}y) + c_2(-\frac{1}{2}y^5 + \frac{3}{4}y^4 - \frac{1}{8}y^3 - \frac{1}{8}y) \\ + c_3(-\frac{1}{2}y^6 + \frac{3}{4}y^5 - \frac{1}{8}y^4 - \frac{1}{16}y^3 - \frac{1}{16}y) \\ + c_4(-\frac{1}{2}y^7 + \frac{3}{4}y^6 - \frac{1}{8}y^5 - \frac{1}{16}y^4 - \frac{1}{32}y^3 - \frac{1}{32}y)\end{aligned} \tag{3.2.1}$$

which satisfies the boundary condition in Eq. (2.6) and set it into Eq. (3.1.3), residual function, $R(c_1,c_2,c_3,c_4,y)$, is found as equation in appendix A.

On the other hand, the residual function must be close to zero. For reaching this importance, four specific points in the domain $t \in [0,1]$ should be chosen. These points are:

$$y_1 = \frac{1}{5}\ ,\ y_2 = \frac{2}{5}\ ,\ y_3 = \frac{3}{5}\ ,\ y_4 = \frac{4}{5} \tag{3.2.2}$$

Finally by substitutions these points into the residual function $R(c_1,c_2,c_3,c_4,x)$, a set of four equations and four unknown coefficients are obtained. After solving these unknown parameters (c_1,c_2,c_3,c_4), the equation will be determined (see Eq. (3.1.5)). Using collocation method with described coefficients, the solution of Eq. (2.5) and for example for $Re = 5$ and $\alpha = -0.5$ is as follows:

$$\begin{aligned}F(y) = 1.495729315y - 0.5877770416y^3 + 0.2183738605y^4 \\ - 0.1658305662y^5 + 0.05327182098y^6 - 0.01376738986y^7\end{aligned} \tag{3.2.3}$$

$$\begin{aligned}F'(y) = 1.495729315 + 1.495729315y - 1.763331125y^2 + 0.2857184004y^3 \\ - 0.6107789705y^4 + 0.1538003597y^5 - 0.04309990804y^6 \\ - 0.013767338986y^7\end{aligned} \tag{3.2.4}$$

4. Least Square Method

4.1. Principles of Method

If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of : [28]

$$S = \int_x R(x)R(x)dx = \int_x R^2(x)dx \tag{4.1.1}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is :

$$\frac{\partial S}{\partial c_i} = 2\int_x R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{4.1.2}$$

Comparing with Eq. (3.1.4), the weight functions are seen to be: [29]

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{4.1.3}$$

However, the ''2'' coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the least squares method are just the derivatives of the residual with respect to the unknown constants:

$$W_i = \frac{\partial R}{\partial c_i} \tag{4.1.4}$$

4.2. Application of LSM to Flow between Porous Walls

Consider the following trial function,

$$\mathrm{F}(y) = -0.5y^3 + \frac{1}{2}y + c_1(-\frac{1}{2}y^4 + \frac{3}{4}y^3 - \frac{1}{4}y) + c_2(-\frac{1}{2}y^5 + \frac{3}{4}y^4 - \frac{1}{8}y^3 - \frac{1}{8}y) \tag{4.2.1}$$

From Eq. (3.1.3) and using trial function, residual function is:

$$\begin{aligned} R(c_1, c_2, \mathrm{y}) = {} & (-0.3750000000\,\mathrm{Re}c_1c_2 + 7.50000000\,\mathrm{Re}c_1 - .750000000\,\mathrm{Re}c_2 - 12c_1 + 18c_2 \\ & - 4.50\,\mathrm{Re}) + (18\alpha c_1 - 3\alpha c_2 + 27.75000000\,\mathrm{Re}c_2 - 25.50000000\,\mathrm{Re}c_1 \\ & + 4.125000000\,\mathrm{Re}c_1^2 - 2.343750000\,\mathrm{Re}c_2^2 - 60c_2 - 12.0\alpha + 4.50\,\mathrm{Re} \\ & - 2.625000000\,\mathrm{Re}c_1c_2)\,\mathrm{y} + (-30\alpha c_1 + 45\alpha c_2 - 56.25000000\,\mathrm{Re}c_2 \\ & - 4.50000000\,\mathrm{Re}c_1 + 80625000000\,\mathrm{Re}c_1^2 + 5.156250000\,\mathrm{Re}c_2^2 \\ & + 4.50\,\mathrm{Re} + 5.625000000\,\mathrm{Re}c_1c_2)y^2 + (-60\alpha c_2 - 23.25000000\,\mathrm{Re}c_2 \\ & + 37.5000000\,\mathrm{Re}c_1 - 46.12500000\,\mathrm{Re}c_1^2 - 10.53125000\,\mathrm{Re}c_2^2 \\ & - 4.50\,\mathrm{Re} + 60.87500000\,\mathrm{Re}c_1c_2)y^3 + (75.00000000\,\mathrm{Re}c_2 \\ & + 46.50000000\,\mathrm{Re}c_1^2 + 72.75000000\,\mathrm{Re}c_2^2 - 15.0\,\mathrm{Re}c_1 \\ & - 188.2500000\,\mathrm{Re}c_1c_2)y^4 + (167.6250000\,\mathrm{Re}c_2^2 - 22.50000000\,\mathrm{Re}c_2 \\ & + 159.7500000\,\mathrm{Re}c_1c_2)y^5 + (127.5000000\,\mathrm{Re}c_2^2 - 35\,\mathrm{Re}c_1c_2)y^6 \\ & - 25\,\mathrm{Re}c_2^2y^7 \end{aligned}$$

(4.2.2)

After introducing coefficients to residual function, Eq. (4.2.2), and solving Eq. (4.1.2), coefficients of Eq. (4.2.1) will be calculated and the solution of Eq. (2.5) and for example $Re = 5$ and $\alpha = -0.5$ will be :

$$F(y) = 1.536724298y - 0.7923302701y^3 + 0.4377633501y^4 - 0.821573776y^5 \tag{4.2.3}$$

$$\begin{aligned} F'(y) = {} & 1.490368409 - 2.126875741y^2 + 2.641523680y^3 - 4.135768282y^4 \\ & + 4.019343814y^5 - 1.484899444y^6 - 2.576310006y^7 + 3.374924126y^8 \\ & - 0.7119115750y^9 - 0.8249538805y^{10} + 0.4035989470y^{11} \\ & + 0.02095344261y^{12} - 0.03593411405y^{13} + 0.004554611986y^{14} \end{aligned}$$

(4.2.4)

5. Adomian's Decomposition Method

5.1. Principles of Method

Consider equation $F; u(t) = g(t)$, where F represents a general nonlinear ordinary or partial differential operator including both linear and nonlinear terms. The linear terms are decomposed into $L + R$, where L is easily invertible (usually the highest order derivative) and R is the remainder of the linear operator. Thus, the equation can be written as: [30]

$$Lu + Nu = Ru + g \tag{5.1.1}$$

where Nu indicates the nonlinear terms. By solving this equation for Lu, since L is invertible, it can be written:

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{5.1.2}$$

If L is a second-order operator, $L-1$ is a twofold indefinite integral. By solving Eq. (5.1.2):

$$u = A + Bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{5.1.3}$$

where A and B are constants of integration and can be found from the boundary or initial conditions. Adomian's method assumes that the solution u can be expanded into infinite series as:

$$u = \sum_{n=0}^{\infty} u_n \tag{5.1.4}$$

Also, the nonlinear term Nu will be written as:

$$Nu = \sum_{n=0}^{\infty} A_n \tag{5.1.5}$$

where A_n are the special Adomian polynomials. By specified A_n, next component of u can be determined:

$$u_{n+1} = L^{-1}\sum_{n=0}^{n} A_n . \tag{5.1.6}$$

Finally, after some iteration and getting sufficient accuracy, the solution can be expressed by Eq. (5.1.3). In Eq. (5.1.3), the Adomian polynomials can be generated by several means. Here the following recursive formulation has been used:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]\right]_{\lambda=0}, \quad n = 0,1,2,3,\ldots \tag{5.1.7}$$

Since the method does not resort to linearization or assumption of weak nonlinearity, the solution generated is in general more realistic than those achieved by simplifying the model of the physical problem.

5.2 Application of ADM to Flow Between Porous Walls

Following the Adomian decomposition analysis, the linear operator is defined as:

$$LF = F'''' + 3\alpha F'' \tag{5.2.1}$$

Furthermore, the linear operator is defined as:

$$NF = \alpha y F''' + ReF'F''' - ReF'F'' \tag{5.2.2}$$

For nonlinear differential equations, the nonlinear operator $NF = P(y)$ is represented by an infinite series of the so-called Adomian polynomials:

$$P(y) = \sum_{m=0}^{\infty} A_m , \tag{5.2.3}$$

and:

$$A_m = 0.5yF_m''' + 1.5F_m'' - 2.5\left(\sum_{n=0}^{m} F_{m-n}''' F_n'''\right) + 5\left(\sum_{n=0}^{m} F_{m-n}'' F_n'''\right) \tag{5.2.4}$$

Hence, from Adomian polynomials A_0, A_1, A_2 and A_3 follows:

$$A_0 = 0.5yF_0''' + 1.5F_0'' - 2.5F_0'''F_0''' + 5F_0''F_0'''$$
$$A_1 = 0.5yF_1''' + 1.5F_1'' - 2.5F_1'''F_0''' - 2.5F_0'''F_1''' + 5F_1''F_0''' + 5F_0''F_1'''$$
$$A_2 = 0.5yF_2''' + 1.5F_2'' - 2.5F_2'''F_0''' - 2.5F_1'''F_1''' - 2.5F_0'''F_2''' + 5F_2''F_0''' + 5F_1''F_1''' + 5F_0''F_2'''$$
$$A_3 = 0.5yF_3''' + 1.5F_3'' - 2.5F_2'''F_1''' - 2.5F_1'''F_2''' - 2.5F_0'''F_3''' - 2.5F_3'''F_0''' + 5F_1''F_2''' + 5F_2''F_1''' + 5F_0''F_3''' + 5F_3''F_0'''$$

(5.2.5)

The Adomian method defines the solution $F(y)$ by the series:

$$F(y) = \sum_{m=0}^{\infty} F_m(y) \tag{5.2.6}$$

Applying the inverse operator L^{-1} to Eq. (5.2.1), thus:

$$F_0(y) = 1.49y - 0.709y^3 \tag{5.2.7}$$

The solution of this equation, will be as follows:

$$\begin{aligned}
F(y) &= 1.490368409y - 0.7089585803y^3 - 0.2641523679\alpha \operatorname{Re} y^4 + (0.1417917161\alpha \\
&- 0.05283047358 \operatorname{Re} - 0.07873686886\alpha^2 \operatorname{Re}^2)y^5 + (0.02513111343\alpha \operatorname{Re} \\
&- 0.02624562295\alpha \operatorname{Re}^2 + 0.079245771036\alpha^2 \operatorname{Re} - 0.01955782365\alpha^3 \operatorname{Re}^3)y^6 \\
&+ (0.01077047718 \operatorname{Re} - 0.02025595944\alpha^2 + 0.02675329824\alpha^2 \operatorname{Re}^2 \\
&- 0.001874687354 \operatorname{Re}^2 + 0.01257868418\alpha \operatorname{Re} + 0.02812031031\alpha^3 \operatorname{Re}^2 \\
&- 0.008381924423\alpha^2 \operatorname{Re}^3)y^7 + (0.01404548158\alpha \operatorname{Re}^2 - 0.01570694589\alpha^2 \operatorname{Re} \\
&- 0.01367931905\alpha^3 \operatorname{Re} - 0.001047740553\alpha \operatorname{Re}^3 + 0.007733085334\alpha^2 \operatorname{Re}^2 \\
&+ 0.01594890822\alpha^3 \operatorname{Re}^3)y^8 + (-0.004188518905\alpha \operatorname{Re} - 0.002333167899\alpha^2 \operatorname{Re}^2 \\
&+ 0.001337664912 \operatorname{Re}^2 - 0.001816921049\alpha^2 \operatorname{Re} + 0.002250662160\alpha^3 \\
&+ 0.000468671838\alpha \operatorname{Re}^2 + 0.009746555022\alpha^2 \operatorname{Re}^3 - 0.01738964386\alpha^3 \operatorname{Re}^2 \\
&- 0.00003880520568 \operatorname{Re}^3)y^9 + (-0.001463532591\alpha \operatorname{Re}^2 \\
&- 0.006814659580\alpha^2 \operatorname{Re}^2 + 0.001628117714\alpha \operatorname{Re}^3 - 0.003698561166\alpha^3 \operatorname{Re}^3 \\
&+ 0.004682165776\alpha^3 \operatorname{Re})y^{10} + (-0.0001619719864 \operatorname{Re}^2 \\
&- 0.0005566847918\alpha \operatorname{Re}^2 + 0.0009392436333\alpha^2 \operatorname{Re} - 0.002643882108\alpha^2 \operatorname{Re}^3 \\
&+ 0.00008155691703 \operatorname{Re}^3 + 0.002128774678\alpha^3 \operatorname{Re}^2)y^{11} \\
&+ (0.0009858116432\alpha^2 \operatorname{Re}^2 - 0.00050336308037\alpha \operatorname{Re}^3 \\
&+ 0.0002563201551\alpha^3 \operatorname{Re}^3)y^{12} + (0.0002000085887\alpha^2 \operatorname{Re}^3 \\
&- 0.00002807466787 \operatorname{Re}^3 + 0.00009033053086\alpha \operatorname{Re}^2)y^{13} \\
&+ 0.00004106755892\alpha \operatorname{Re}^3 y^{14} + 0.000002429126393 \operatorname{Re}^3 y^{15}
\end{aligned} \tag{5.2.8}$$

And for $Re = 5$ and $\alpha = -0.5$:

$$F(y)=1.490368409y-0.70895803y^3+0.6603809200y^4$$
$$-0.8271536565y^5+0.6698906357y^6-0.2121284920y^7$$
$$-0.3220387508y^8+0.3749915695y^9-0.07119115750y^{10}$$
$$-0.07499580732y^{11}+0.03363324558y^{12}+0.001611803278y^{13}$$
$$-0.002566722432y^{14}+0.0003036407991y^{15}$$

(5.2.9)

$$F'(y)=1.536724298-2.376990810y^2+1.751053400y^3-0.910786880y^4 \quad (5.2.10)$$

6. Fourth order Runge–Kutta Method (NUM)

To validate the analytical results, the variable permeability (α) and Reynolds number (Re) for flow between porous walls is compared with the numerical solution. The numerical solution is performed using the algebra package Maple 17.0, to solve this problem for $F(y)$ and $F'(y)$. The package Maple 17.0 uses a fourth order Runge–Kutta–Fehlberg procedure for solving nonlinear B–V problems. The algorithm is proved to be precise and accurate in solving a wide range of mathematical and engineering problems especially flow porous cases. It can be seen from Fig.2 and Fig.3 and Table1 and Table2 that the results show high accuracy in comparison among present analytical and numerical solution [31].

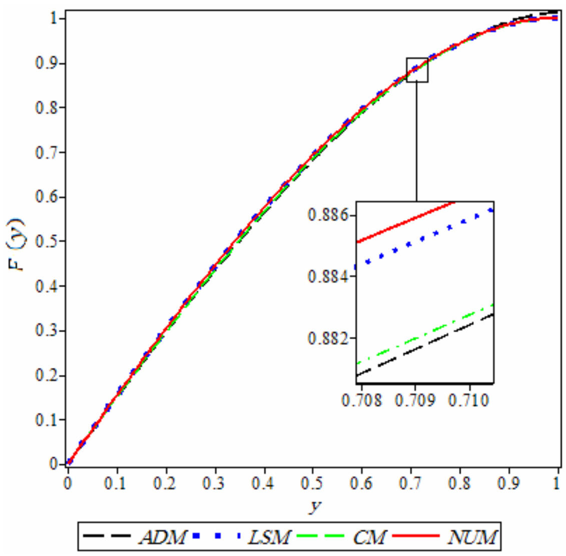

Fig.2 Comparison between ASM, LSM, CM and NUM for solving $F(y)$ when $Re=5$ and $\alpha=-0.5$.

Table 1 Comparison between Numerical and ADM and LSM and CM results for $F(y)$ when $Re = 5, \alpha = -0.5$.

y	$F(y)$				Error (%)		
	NUM	ADM	LSM	CM	ADM	LSM	CM
0	0. 000000000	0	0	0	0	0	0
0.1	0.152867390	0.152942938	0.152922056	0.149005386	0.000494206	0.00035760	0.025263756
0.2	0.301551019	0.301686302	0.301648351	0.294743212	0.000448624	0.00032277	0.022575970
0.3	0.442606060	0.442776442	0.442727615	0.434250499	0.000384952	0.00027464	0.018878100
0.4	0.573196123	0.573374564	0.573322035	0.564761906	0.000311309	0.00021967	0.014714365
0.5	0.690875804	0.691037958	0.690988659	0.683583503	0.000234708	0.00016335	0.010555155
0.6	0.793373238	0.793501229	0.793460814	0.787984067	0.000161325	0.00011038	0.006792731
0.7	0.878372650	0.878457523	0.878429517	0.875096988	9.66253E-05	6.4741E-05	0.003729240
0.8	0.943296910	0.943339761	0.943324879	0.941825837	4.54268E-05	2.965E-05	0.001559501
0.9	0.985090087	0.985101869	0.985097525	0.984746650	1.19603E-05	7.551E-06	0.000348635
1	1	1	1	0.999999999	0	0	9E-10

Table 2 Comparison between Numerical and ADM and LSM and CM results for $F'(y)$ when $Re = 5, \alpha = -0.5$.

y	$F'(y)$				Error (%)		
	NUM	ADM	LSM	CM	ADM	LSM	CM
0	1.536153985	1.490368409	1.536724311	1.495729315	0.029805330	0.000371300	0.026315506
0.1	1.514112220	1.471366082	1.514614376	1.478889684	0.028231816	0.000331700	0.023262831
0.2	1.453855213	1.420974718	1.454195841	1.430953503	0.022616073	0.000234300	0.015752401
0.3	1.362554538	1.345095347	1.362696195	1.354604201	0.012813572	0.000104000	0.005834876
0.4	1.245207297	1.246060759	1.245157039	1.251152013	0.000685398	4.0361E-05	0.004774077
0.5	1.104636120	1.123815238	1.104434085	1.120744070	0.017362385	0.000182900	0.014582132
0.6	0.941489160	0.976400851	0.941197154	0.962505101	0.037081352	0.000310150	0.022322020
0.7	0.754240098	0.800387702	0.753930180	0.774608738	0.061184236	0.000410900	0.027005512
0.8	0.539188141	0.591549122	0.538931207	0.554279453	0.097110782	0.000476520	0.027988955
0.9	0.290458021	0.345617931	0.290312392	0.297725086	0.189906650	0.000501380	0.02501933
1	0	0	0	0	0	0	0

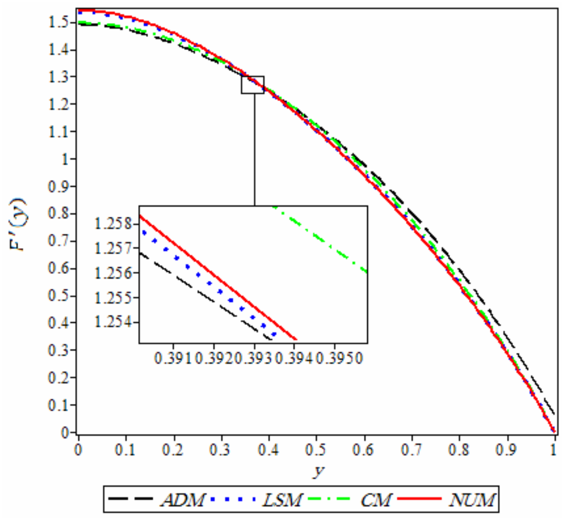

Fig.3 Comparison between ASM, LSM, CM and NUM for solving $F'(y)$ when $Re = 5$ and $\alpha = -0.5$.

7. Result and Discussion

In this study, we compared the results of analytical solution and the numerical solution to flow between porous walls with different permeability.

Tables 1 and 2 confirm that LSM has lower errors compares to ADM and CM, so it is more accurate and reliable than ADM and CM. in another applicable physical problem.

Figs 2 and 3, exhibits and compares the approximate solution for equations (2.5) and (2.6) obtained for $Re = 5$ and $\alpha = -0.5$ values using ADM and CM and LSM and NUM for $F(y)$ and $F'(y)$. Results show that LSM is near to numerical solution.

Fig 4, illustrates the effect of α on $F(y)$ when $Re = 1$. For example, in y= 0.09; values of $F(y)$ for $\alpha = 0, \alpha = 0.5, \alpha = 1, \alpha = 3, \alpha = 5$ and $\alpha = 7$ are 0.138, 0.142, 0.148, 0.171, 0.192 and 0.197, respectively. This means that the values of $F(y)$ are increased by increasing the permeability and by decreasing permeability values are reduced.

Fig 5, illustrates the effect of α on $F'(y)$ when $Re = 1$.For example, in y= 0.2; values of $F(y)$ for $\alpha = 0, \alpha = 0.5, \alpha = 1, \alpha = 3, \alpha = 5$ and $\alpha = 7$ are 1.48, 1.52, 1.63, 1.74, 1.93 and 2.01, respectively. According to the curves obtained from the analytical methods, when permeability is lowest, the function $F'(y)$ has the greatest value at the beginning, but with moving to point $y = 1$, the lowest level is reaching. Furthermore, we can find point in which the amount of the highest permeability and lowest permeability was equal.

Fig 6, show effect of various Reynolds number (Re) on $F(y)$ and when $\alpha = -0.5$.

Fig 7 , show effect of various Reynolds number (Re) on $F'(y)$ and when $\alpha = -0.5$.

Results show that various Reynolds number is low on $F(y), F'(y)$.

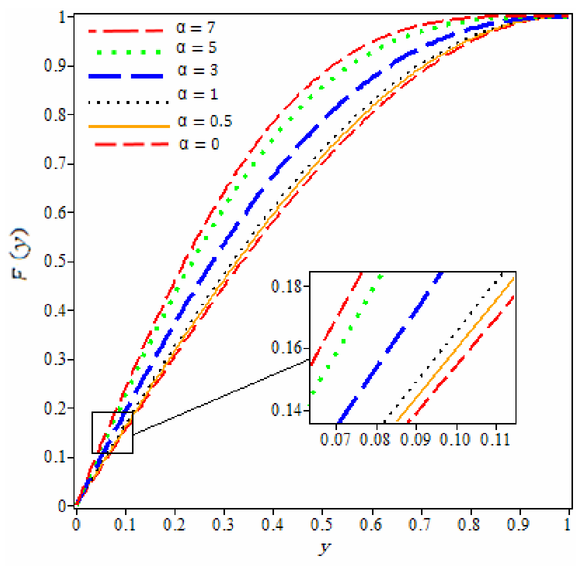

Fig.4 Effect of various α on $F(y)$ when $Re = 1$.

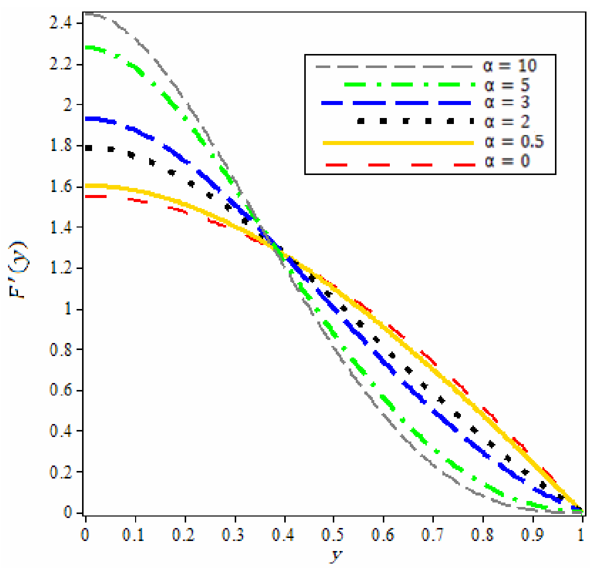

Fig.5 Effect of various α on $F'(y)$ when $Re = 1$.

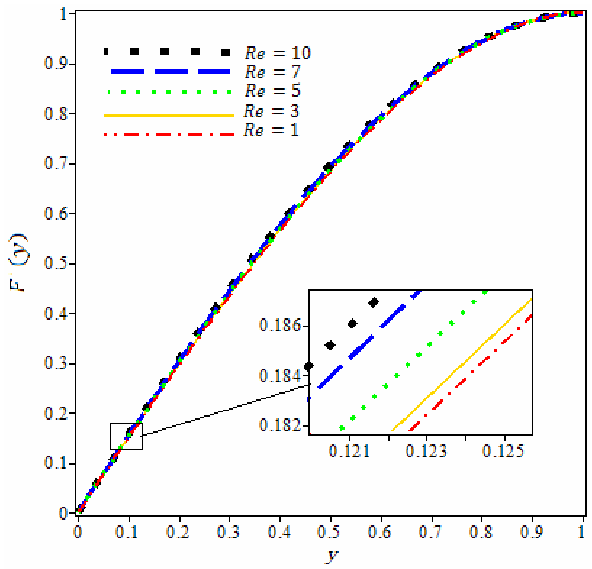

Fig.6 Effect of various Reynolds number (Re) on $F(y)$ when $\alpha = -0.5$.

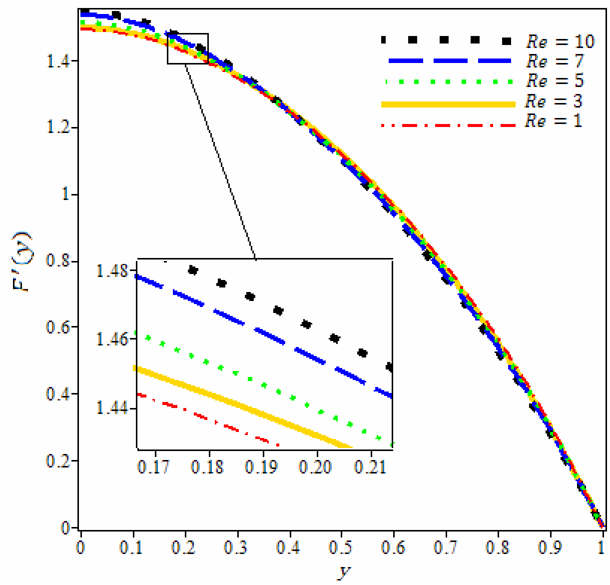

Fig.7 Effect of various Reynolds number (Re) on $F'(y)$ when $\alpha = -0.5$.

8. Conclusions

Results clearly show that ADM and CM and LSM which were applied to flow between porous walls with variable permeability problem, were capable of solving them with successive rapidly convergent approximations without any restrictive assumptions or transformations causing changes in the physical definition of the problem. As a main outcome from the present study, it is observed that the results of LSM are more accurate than ADM and CM and they are in excellent agreement with numerical ones, so LSM can be used for finding analytical solutions of this nonlinear equation with high accuracy. Finally, it has been attempted to show the capabilities and wide–range applications of the ADM and CM and LSM in comparison with the numerical solution off low between porous walls with different permeability.

Nomenclature

a	Distance
A	Constant
ADM	Adomian decomposition method
B	Constant
CM	Collocation Method
c_i	Constants
F	Function
t	Time
L	Linear terms
LSM	Least Square Method
n	Number of iteration
N	Nonlinear terms
NUM	Numerical method
p	Porosity parameter
Re	Reynolds number
$R(x)$	Residual function
u, v	Dimensionless components velocity in x and y directions, respectively
$\tilde{u}$	Trial function
W_i	Weighted function
x	Axial coordinate

Greek symbols

α	Permeability
ρ	Density of the fluid
θ	Cylindrical coordinates
υ	Kinematic viscosity
φ	Porosity

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Appendix A

$$
\begin{aligned}
&R(c_1,c_2,c_3,c_4,\mathrm{y})=\\
&(0.046874\,\mathrm{Re}c_1c_2+0.09375\,\mathrm{Re}c_2c_3+0.023437\,\mathrm{Re}c_3c_4-0.09375\,\mathrm{Re}c_1c_4\\
&-4.50\,\mathrm{Re}-0.1875\,\mathrm{Re}c_1c_3+0.09375\,\mathrm{Re}c_2^2-1.125\,\mathrm{Re}c_1^2-0.375\,\mathrm{Re}c_3\\
&+7.5\,\mathrm{Re}c_1-0.75\,\mathrm{Re}c_2+0.005859375\,\mathrm{Re}c_4^2+0.0234375\,\mathrm{Re}c_3^2\\
&-0.1875\,\mathrm{Re}c_4-12c_1-1.5c_4-3c_3+18c_2)+(-0.421875\,\mathrm{Re}c_2c_4+4.50\,\mathrm{Re}\\
&-60c_2+0.041015625\,\mathrm{Re}c_4^2-2.34375\,\mathrm{Re}c_2^2+0.1640625\,\mathrm{Re}c_3c_4-15c_4\\
&+0.1640625\,\mathrm{Re}c_3^2-2.0625\,\mathrm{Re}c_4-0.84375\,\mathrm{Re}c_2c_3-4.125\,\mathrm{Re}c_3-3\alpha c_2\\
&-2.625\,\mathrm{Re}c_1c_2-25.5\,\mathrm{Re}c_1+0.84375\,\mathrm{Re}c_1c_4-12\alpha+27.75\,\mathrm{Re}c_2-1.5\alpha c_4\\
&-0.75\,\mathrm{Re}c_1^2+1..6875\,\mathrm{Re}c_1c_3+90c_3)y+(0.228515625\,\mathrm{Re}c_4^2+45\alpha c_2\\
&-3.75\alpha c_4-3.09375Rc_2c_3-7.5\alpha c_3-30\alpha c_1+5.625\,\mathrm{Re}c_1c_2+4.5\,\mathrm{Re}\\
&-13.6875\,\mathrm{Re}c_1c_3+270c_4+8.625\,\mathrm{Re}c_1^2-0.9609375\,\mathrm{Re}c_3c_4+70.875\,\mathrm{Re}c_3\\
&-180c_3+5.15625\,\mathrm{Re}c_2^2-4.5\,\mathrm{Re}c_1-2.8359375\,\mathrm{Re}c_3^2-9.5625\,\mathrm{Re}c_4\\
&+2.203125\,\mathrm{Re}c_2c_4-56.25\,\mathrm{Re}c_2+0.65625\,\mathrm{Re}c_1c_4)y^2+(-46.125\,\mathrm{Re}c_1^2\\
&-60\alpha c_2-2.7578125\,\mathrm{Re}c_3c_4-4.5\,\mathrm{Re}-10.53125\,\mathrm{Re}c_2^2+14.4375\,\mathrm{Re}c_1c_3\\
&-420c_4+60.875\,\mathrm{Re}c_1c_2+37.5\,\mathrm{Re}c_1-13.515625\,\mathrm{Re}c_2c_4-107.625\,\mathrm{Re}c_3\\
&+90\alpha c_3+5.46875\,\mathrm{Re}c_2c_3+141.1875\,\mathrm{Re}c_4-23.25\,\mathrm{Re}c_2-25.28125\,\mathrm{Re}c_1c_4\\
&-2.720703125\,\mathrm{Re}c_4^2-15\alpha c_4+5.367187500\,\mathrm{Re}c_3^2)y^3+(46.5\,\mathrm{Re}c_1^2\\
&+130.125\,\mathrm{Re}c_1c_3+75\,\mathrm{Re}c_2-105\alpha c_3-180\,\mathrm{Re}c_4+5.25\,\mathrm{Re}c_3c_4\\
&+5.0625\,\mathrm{Re}c_4^2+72.75\,\mathrm{Re}c_2^2-60\,\mathrm{Re}c_3-9.75\,\mathrm{Re}c_3^2-31.125\,\mathrm{Re}c_2c_3\\
&+157.5\alpha c_4-15\,\mathrm{Re}c_1-188.25\,\mathrm{Re}c_1c_2+14.4375\,\mathrm{Re}c_2c_4\\
&+20.0625\,\mathrm{Re}c_1c_4)y^4+(-7.6640625\,\mathrm{Re}c_4^2+245.4375\,\mathrm{Re}c_1c_4-122.625\,\mathrm{Re}c_4\\
&-12\,\mathrm{Re}c_1^2-167.625\,\mathrm{Re}c_2^2+132.75\,\mathrm{Re}c_3+159.75\,\mathrm{Re}c_1c_2-340.125\,\mathrm{Re}c_1c_3\\
&-60.5625\,\mathrm{Re}c_2c_4+264.375\,\mathrm{Re}c_2c_3-24.28125\,\mathrm{Re}c_3c_4-168\alpha c_4-22.5\,\mathrm{Re}c_2\\
&-17.90625\,\mathrm{Re}c_3^2)y^5+(-12.203125\,\mathrm{Re}c_4^2-65.9375\,\mathrm{Re}c_3c_4+215.25\,\mathrm{Re}c_4\\
&-31.5\,\mathrm{Re}c_3+255.75\,\mathrm{Re}c_1c_3+127.5\,\mathrm{Re}c_2^2-554.375\,\mathrm{Re}c_2c_3+216.9375\,\mathrm{Re}c_3^2\\
&-35\,\mathrm{Re}c_1c_2-565.625\,\mathrm{Re}c_1c_4+451.0625\,\mathrm{Re}c_2c_4)y^6+(387\,\mathrm{Re}c_1c_4\\
&-53.6875\,\mathrm{Re}c_4^2-428.25\,\mathrm{Re}c_3^2-48\,\mathrm{Re}c_1c_3-25\,\mathrm{Re}c_2^2-42Rc_4+387\,\mathrm{Re}c_2c_3\\
&-869\,\mathrm{Re}c_2c_4+688.5\,\mathrm{Re}c_3c_4)y^7+(514.3125\,\mathrm{Re}c_4^2-63\,\mathrm{Re}c_1c_4+563.25\,\mathrm{Re}c_2c_4\\
&-67.5\,\mathrm{Re}c_2c_3-1275.75\,\mathrm{Re}c_3c_4+281.25\,\mathrm{Re}c_3^2)y^8+(-45\,\mathrm{Re}c_3^2+791.25\,\mathrm{Re}c_3c_4\\
&-87.5\,\mathrm{Re}c_2c_4-910.625\,\mathrm{Re}c_4^2)y^9+\left(-115.5\,\mathrm{Re}c_3c_4+540.75\,\mathrm{Re}c_4^2\right)y^{10}\\
&-73.5\,\mathrm{Re}c_4^2y^{11}
\end{aligned}
$$