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Non-linear Heat Transfer Analysis for Fin Profiles with Temperature dependent Thermal Conductivity & Heat Transfer Coefficient

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Abstract

By the current paper, the natural convection of a non-Newtonian fluid flow between two parallel vertical flat plate under the influence of non-Newtonian nature of the heat transfer fluid will be investigated by using analytical and numerical methods. Heat transfer by natural convection occurs in many engineering problems and applications such as geothermal systems, heat exchangers, etc. These phenomena are modeled by ordinary or partial differential equations. In most cases, practical solutions to these problems cannot be applied, so the above equations are solved with the use of specific techniques. Recently, much attention has been devoted to this technique to build analytical solutions, collocation and vim methods in order to solve equation boundary conditions to be applied in this case. The impact of physical parameters such as velocity and temperature profiles are intended to establish the results obtained and are compared with exact methods, which will be displayed on tables and charts. The results show the high accuracy of the numerical method which is investigated. So, this method can be further applied to other linear and nonlinear equations which are widely used in applied science and engineering.

Key Word and Phrases

Collocation Method (CM), Natural Convection, Non-Newtonian Fluid, Numerical Method (NM), Gallerkin Method (GM).

1. Introduction

Convection heat transfer occurs when a gas or liquid is in direct contact with a solid object and the heat or not to heat. The phenomenon of the moving fluid molecules and the temperature changes that alter the physical properties. The physical displacement caused the displacement of the fluid mixture and the resulting transfer of heat energy. Handling processes in gases and liquids, the molecules they have the ability to move, could be possible. Displacement fluid molecules depends on various factors and the nature of the work. The physical properties of the fluid contact surface area, the geometric object, the temperature difference between the fluid and other parameters, the effective heat transfer coefficient. The heat transfer process is very complex and requires high precision survey. Thermal convection occurs in two forms, the free movement of natural or forced convection, the free movement of the displacement, the only reason of the different layers of the fluid density difference caused by temperature changes occur.

Thus, the density of the fluid in the lower layers is reduced and then heated fluid is moved upward. The amount of fluid that moves the fluid material, the temperature difference and the volume of fluid in the space in which it is located. But the forcible displacement of the external force on the fluid (such as wind Ed, the fan, pumps or fans) can displace the fluid molecules and consequently the heat transfer. Natural convection is widely used in engineering problems and physical phenomena such as geothermal systems, heat exchangers, reactors, catalytic, chemical, fiber and seed insulation, flat packed, oil and nuclear waste repositories. Besides, Newtonian and non-Newtonian fluid flow, heat transfer method relocation through two parallel infinite vertical plate have been investigated by several authors. The natural convection problem between vertical flat plates for a certain class of non-Newtonian fluids has been carried out by Bruce and Na [1]. Other laminar natural convection problems involving heat transfer have also been studied [2]. However, Rajagopal presented a complete thermodynamic analysis of the constitutive functions [3]. There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation (CM), Galerkin (GM) and Least Square (LSM) are examples of the WRMs. Stern and Rasmussen [4] used Collocation Method for solving a third order linear differential equation. Vaferi et al. [5] studied the feasibility of applying of Orthogonal Collocation Method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami [6] used the Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently Least Square Method is introduced by Aziz and Bouaziz [7] and was applied for prediction of the performance of a longitudinal fin [8].

As it is well known, most scientific problems such to Natural Convection Flow of a Non-Newtonian Fluid Between Two Vertical Flatand other fluid mechanic problems are inherently nonlinear. In most cases, these problems do not admit analytical solution, so these equations should be solved using some special techniques. Over the recent decades, much attention has been devoted to the newly developed methods to construct an analytic solution of equation; such as Perturbation techniques which are too strongly dependent upon the so-called “small parameters” [9]. Many other different methods have introduced to solve nonlinear equation such as the Adomian’s decomposition method [10], homotopy perturbation method (HPM) [11]-[13], variational iteration method (VIM) [14], differential transformation method [15], homotopy asymptotic method (HAM) [16]-[19].optimal homotopy asymptotic method (OHAM) [20] and collocation method (CM) [21],[22].

Many advantages of CM compared to other analytical and numerical methods make it more valuable and motivate researchers to use it for solving problems. It has the following benefits: First, unlike all previous analytic techniques, it solves the equations directly and no simplifications needs. Second, unlike all previous analytic techniques it does not need to any perturbation, linearization or small parameter versus homotopy perturbation method (HPM) and parameter perturbation method (PPM). Third, unlike homotopy asymptotic method, it does not need to determine the auxiliary function and parameter versus HAM. The main purpose of the present investigation is to apply collocation method and vim to find approximate solutions of the velocity profile on to Natural Convection Flow of a Non-newtonian Fluid Between Two Vertical Flatland. A clear conclusion can be drawn from the numerical method’s (NUM) results that the collocation method provides highly accurate solutions for nonlinear differential equations.

2. Problem Statement and Mathematical Formulation

A schematic diagram of the problem is shown in Fig. 1. It consists of two flat plates that can be positioned vertically. A non-Newtonian fluid is in two flat plates a distance $2b$ apart. The walls at $x = +b$ and $x = -b$ are held at constant temperatures Θ_2 and Θ_1, respectively, where $\Theta_1 > \Theta_2$ .This difference in temperature causes the fluid near the wall at $x = -b$ to rise and the fluid near the wall at $x = +b$ to fall. The equation of motion is as following : (for more details see [3])

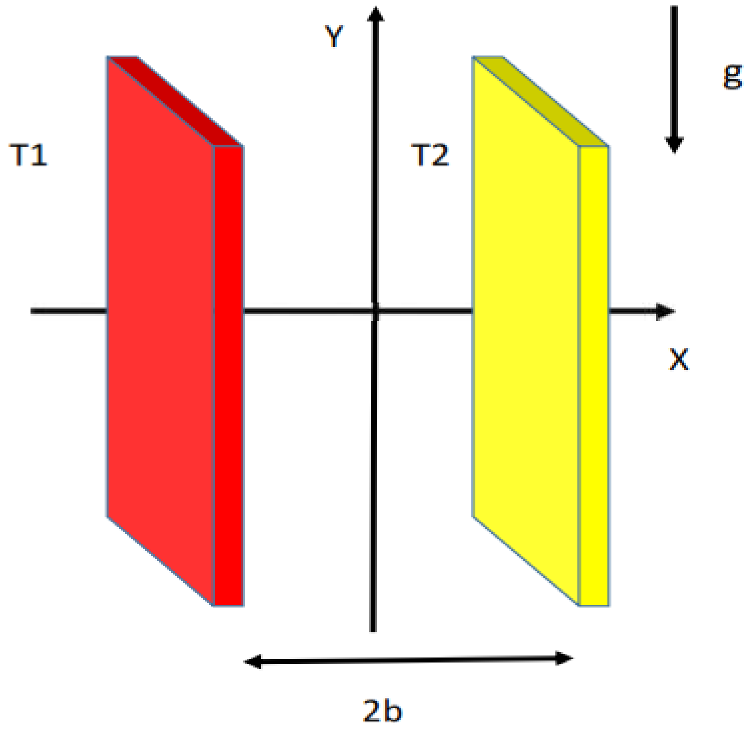

Fig. 1 Schematic of the Three- dimensional Problem

$$\mu\frac{d^2v}{dx^2}+6\beta_3\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2}+\rho_0\gamma\left(\theta-\theta_m\right)g=0 \tag{2.1}$$

and the energy equation as follows:

$$\mu\left(\frac{dv^2}{dx}\right)+2\beta_3\left(\frac{dv}{dx}\right)^2+k\frac{d^2\theta}{dx^2}=0, \tag{2.2}$$

$$v=\frac{v}{v_0},\eta=\frac{x}{b},\theta=\frac{\Theta-\Theta_m}{\Theta_1-\Theta_2}, \tag{2.3}$$

The Navier–Stokes and Energy equations can be reduced to the following pair of ordinary differential equations [3]:

$$\frac{d^2\nu}{d\eta^2}+6\delta\left(\frac{d\nu}{d\eta}\right)^2\frac{d^2\nu}{d\eta^2}+\theta=0, \tag{2.4}$$

$$\frac{d^2\theta}{d\eta^2}+E.pr\left(\frac{d\nu}{d\eta}\right)^2+2\delta E.\Pr\left(\frac{d\nu}{d\eta}\right)^4=0, \tag{2.5}$$

where:

$$E=\frac{V_0^{\ 2}}{c(\Theta_1-\Theta_2)}, \tag{2.6}$$

$$Pr=\frac{\mu c}{k}, \tag{2.7}$$

and:

$$\delta=\frac{6\beta_3 V_0^2}{\mu b^2}, \tag{2.8}$$

where *c* is the specific heat of the fluid. Also, the appropriate boundary conditions are:

$$\nu=0, \quad \theta=\frac{1}{2} \quad \text{at} \quad \eta=-1, \tag{2.9}$$

$$\nu=0, \quad \theta=-\frac{1}{2} \quad \text{at} \quad \eta=1, \tag{2.10}$$

By the following sections, we shall solve the system of Eqs. (4) and (5) by using the CM and GM. The equations are coupled and highly nonlinear**.**

3. Collocation Method and Galerkin Method

Analysis of the Collocation Method

Suppose we have a differential operator D acting on a function u to produce a function p:

$$D(u(x))=p(x) \tag{3.1}$$

We want to approximate u by a function $\tilde{u}$, which is a linear combination of basic unctions chosen from a linearly independent set. That is:

$$u \cong \tilde{u} = \sum_{i=1}^{n} C_i \varphi_i \tag{3.2}$$

Now, when substituted into the differential operator ,D, the result of the operations is not, in general, $p(x)$. Hence an error or residual will exist:

$$E(x) = R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{3.3}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is:

$$i = 1, 2, \ldots\ldots\ldots, n \qquad \int_x R(x) W_i(x) = 0 \tag{3.4}$$

where the number of weight functions Wi are exactly equal the number of unknown constants Ci in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants Ci. For collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $w_i(x) = \delta(x - x_i)$. The Dirac δ function has the property that:

$$\delta(x - x_i) = \begin{cases} 1 \\ 0 \end{cases} \tag{3.5}$$

and residual function in eqn (16) must force to be zero at specific points.

4. The CM Applied to the Problem

It should be noted that the trial solution must satisfies the boundary conditions, so the trial solution can be written as:

$$v(\eta) = C_1(\eta - \eta^3) + C_2(\eta - \eta^5) \tag{4.1}$$

$$\theta(\eta) = -\frac{\eta}{2} + C_1(\eta - \eta^3) + C_2(\eta - \eta^5) \tag{4.2}$$

By introducing the above Eqs., then the residual function will be found and by substituting the residual function into Eqs. a set of equations with seven equations and seven unknown coefficients will be appeared. Consequently, by solving this system of equations, coefficients c1-c4 will be determined. By using GM, when *Pr=1, E=1, δ=0.5,* following equations will be determined for temperature distribution and velocities for natural convection flow of a non-newtonian fluid between two vertical flat plates:

$$v(\eta) = -0.08284499189\eta + 0.08226481124\eta^3 + 0.0005801806472\eta^5 \tag{4.3}$$

$$\theta(\eta) = -0.4992267246\eta - 0.00198878770\eta^3 + 0.001215511426\eta^5 \tag{4.4}$$

5. Galerkin Method

Galerkin method is one of the weighted residual methods. For perception main idea of GM, suppose a differential operator *D* acting on a function *u* to produce a function:

$$D(u(x)) = p(x) \quad (5.1)$$

We want to approximate *u* by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is:

$$u \approx \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \quad (5.2)$$

Now, when substituted into the differential operator, D, the result of the operations is not, in general, *p(x)* . Hence an error or residual will exist:

$$E(x) = R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \quad (5.3)$$

The notion in the Collocation is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x) W_i(x) = 0 \quad (5.4)$$

where the number of weight functions W_i are exactly equal the number of unknown constants c_i in $\tilde{u}$.

The result is a set of *n* algebraic equations for the unknown constants *ci*. For GM the derivative of the approximating function or trial function is used for finding weighted function. In this method weight functions are:

$$i = 1,2,3,.......W_i = \frac{\partial \tilde{u}}{\partial c_i} \quad (5.5)$$

6. The GM Applied to the Problem

It should be noted that the trial solution must satisfies the boundary conditions, so the trial solution can be written as $\nu(\eta)$.

$$\theta(\eta) = -\frac{\eta}{2} + C_3(\eta - \eta^3) + C_5(\eta - \eta^3) \quad (6.1)$$

$$\nu(\eta) = C_1(\eta - \eta^3) + C_2(\eta - \eta^5) \quad (6.2)$$

By introducing the above Eqs., then the residual function will be found and by substituting the residual function into Eqs. a set of equations with seven equations and seven unknown coefficients will be appeared. Then by solving this system of equations, coefficients c1-c4 will be determined. By using GM, when *Pr=1,E=1,δ=0.5*, following equations will be determined for temperature distribution and velocities for natural convection flow of a non-newtonian fluid between two vertical flat plates.

$$\theta(\eta) = -0.4990280612\eta - 0.0007453269563\eta^3 - 0.0002266117539\eta^5 \quad (6.3)$$

$$\nu(\eta) = -0.08272162474\eta + 0.08373866920\eta^3 - 0.001017044465\eta^5 \quad (6.4)$$

Table 1 The result of the CM, GM and NUM method for Pr=1,E=1,δ=0.5

	$\theta(\eta)$			$v(\eta)$		
η	*CM*	*GM*	*NM*	*CM*	*GM*	*NM*
-1	0.0000000028	0.00000000051	0.00000000	0.5000000000	0.0500000000	0.50000000
-0.8	0.02396629657	0.23636336629	0.23058251	0.4000013397	0.3396783105	0.40071781
-0.6	0.03189268105	0.31624507678	0.03135636	0.2998710451	0.2995944866	0.30114634
-0.4	0.02786710779	0.02773978912	0.02771888	0.0199805525	0.1996612456	0.02015466
-0.2	0.01590169423	0.01587474102	0.016170032	0.0998608662	0.9981164738	0.01870444
0.0	0.00000000000	0.00000000000	0.000075121	0.0000000000	0.0000000000	0.02001042
0.2	0.01590169423-	0.01587474105	0.01474168-	0.0998608662-	0.9981164738-	-0.09812529
0.4	0.02786710779-	0.02773978961-	-0.02650959	0.0199805525-	0.1996612456-	-0.19845211
0.6	0.03189268105-	0.02773978961-	-0.03049585	0.2998710451-	0.2995944866-	-0.29885930
0.8	0.02396629657-	-0.02363636299	-0.02262939	0.4000013397-	0.3396783105-	-0.39992921
1	0.0000000028-	-0.00000000051	0.000000000	0.5000000000-	0.0500000000-	-0.50000000

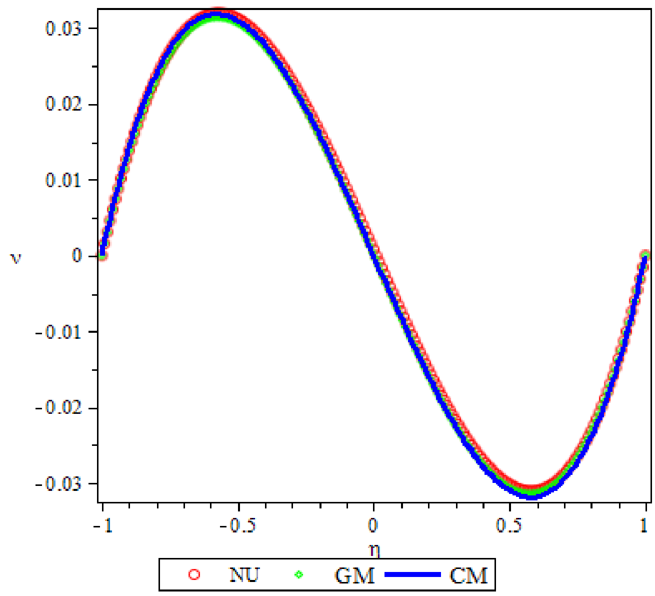

Fig. 2 The comparison between the Numerical, GM and CM solutions for $v(\eta)$ when Pr=1,E=1,δ=1

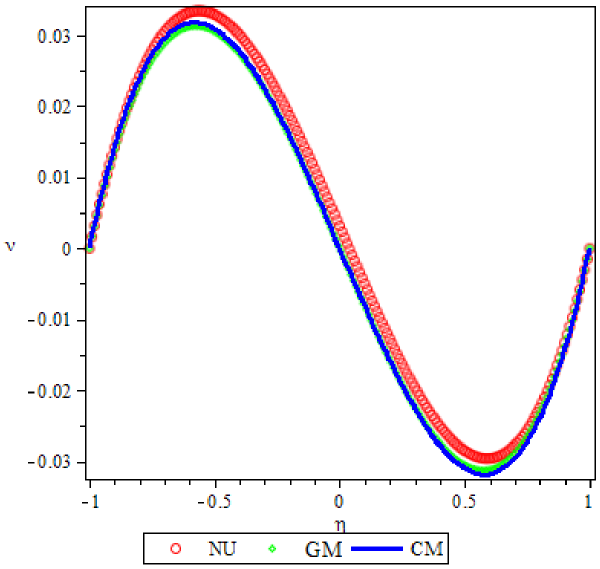

Fig. 3 The comparison between the Numerical, GM and CM solutions for $\nu(\eta)$ when Pr=4,E=1,δ=1

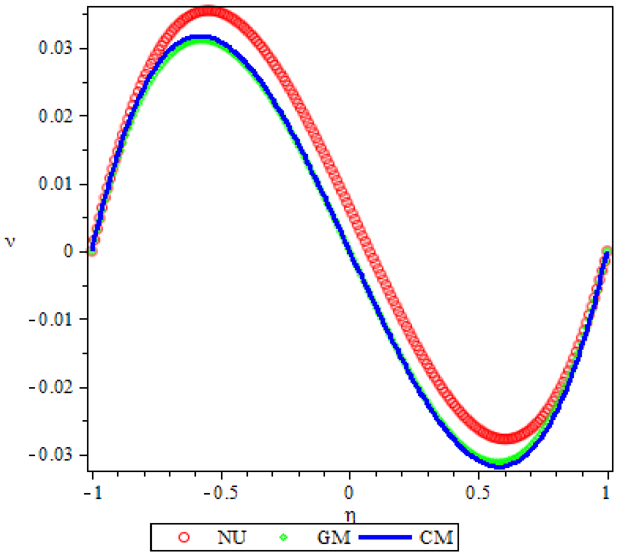

Fig. 4 The comparison between the Numerical, GM and CM solutions for $\nu(\eta)$ when Pr=8,E=1,δ=1

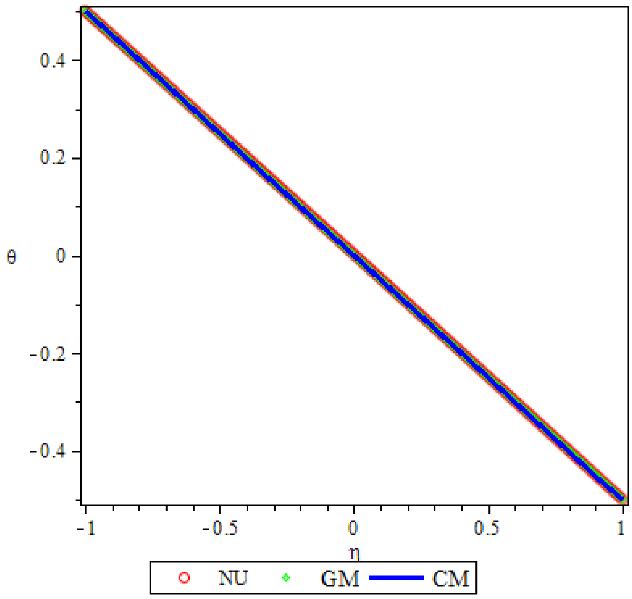

Fig. 5 The comparison between the Numerical, GM and CM solutions for $\theta(\eta)$ when Pr=1,E=1,δ=1

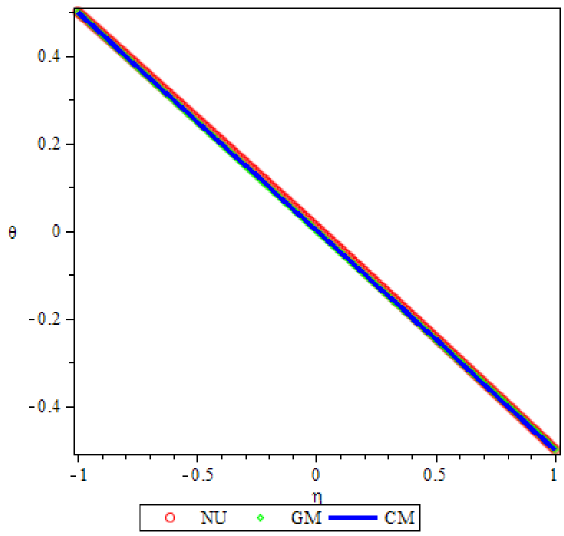

Fig. 6 The comparison between the Numerical, GM and CM solutions for $\theta(\eta)$ when Pr=4,E=1,δ=1

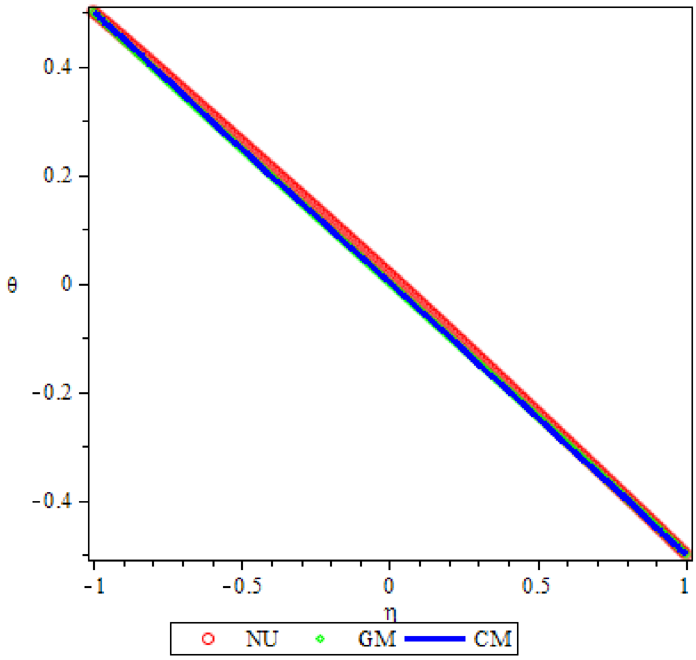

Fig. 7 The comparison between the Numerical, GM and CM solutions for $\theta(\eta)$ when Pr=8,E=1,δ=1

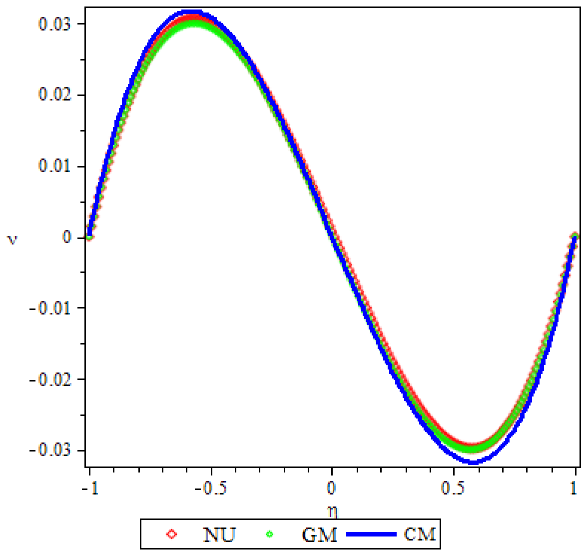

Fig. 8 The comparison between the Numerical, GM and CM solutions for $v(\eta)$ when Pr=1,E=1,δ=4

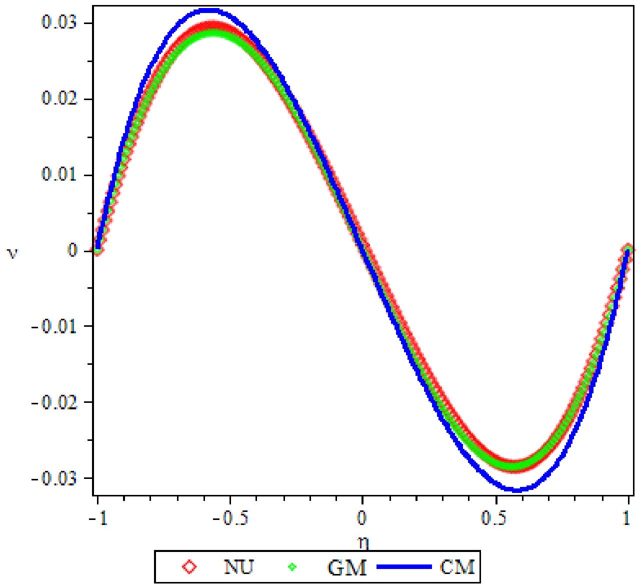

Fig. 9 The comparison between the Numerical, GM and CM solutions for $\nu(\eta)$ when Pr=1,E=1,δ=8

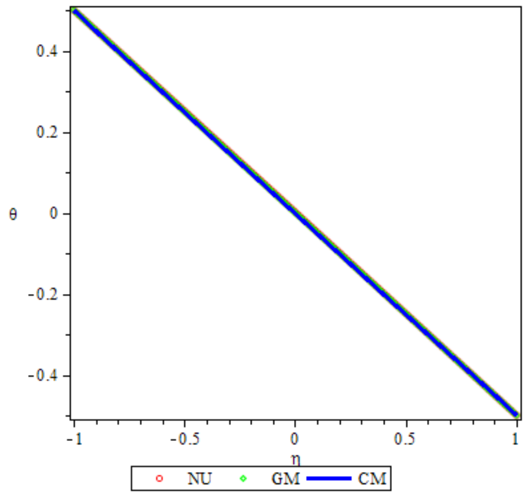

Fig. 10 The comparison between the Numerical, GM and CM solutions for $\theta(\eta)$ when Pr=1,E=1,δ=4

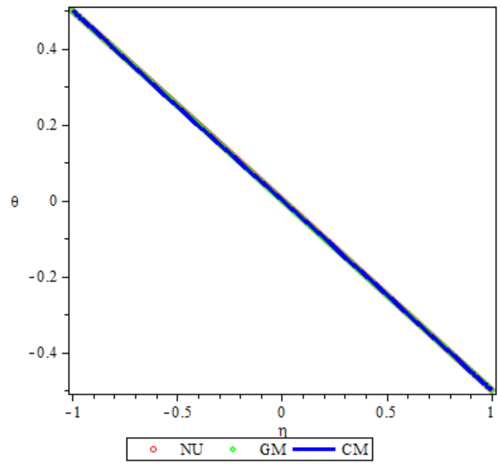

Fig. 11 The comparison between the Numerical, GM and CM solutions for $\theta(\eta)$ when Pr=1,E=1,δ=8

7. Conclusions

In this study, collocation Method and Galerkin Method are applied to solve the problem of natural convection of an incompressible fluid of third grade between two infinite parallel vertical plates field. First of all a comparison between the applied methods, CM and GM methods are investigated. Figures 2 to 4 show the effect of PR number on speed dimensionless parameter. Figures 8 and 9 show the effect of the speed parameter of the delta. The Delta also regularly increasing speed increased in all three methods. Figures 5 and 6 show the temperature effect on the PR number that the approximation of the CM and GM corresponded very well with numerical methods. The results in Table 1 show the high accuracy of CM and GM. As a main outcome from the present study, it is observed that the results of CM and specially GM are in excellent agreement with numerical ones, so they can be used for finding analytical solutions in science and engineering problems simplicity. The Table and figures clearly show that the results by Collocation Method and Galerkin Method are in excellent agreement with the Numerical solution. The observed good agreement between the present method and numerical method results shows that these methods are a powerful approach for solving nonlinear differential equations such as this problem.

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