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Numerical Comparison of two Reproducing Kernel Methods for solving Nonlinear Generalized Regularized Long Wave Equation

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Abstract
Reproducing kernel method (RKM) of [1] is an effective method, but it has a very cumbersome process when we practise Gram-Schmidt orthogonalization. In this paper, it is the first time to solve the nonlinear generalized regularized long wave (GRLW) equation by another RKM which avoid this process. The aim of the paper is to present a numerical comparison of two methods when they are applied in nonlinear transverse waves in shallow water and magnetohydrodynamic waves in plasma. Tables 1-2 and Figures 1-8 of numerical experiment 1-2 show that the absolute errors by present method are more tiny, which indicate the present method is more simple and accurate.

Key Word and Phrases
Nonlinear, Reproducing Kernel, Numerical Comparison, GRLW Equation

1. Introduction

The nonlinear generalized regularized long wave (GRLW) equation is a partial differential equation which has been used to model a variety of phenomena such as nonlinear transverse waves in shallow water, ion-acoustic and magnetohydrodynamic waves in plasma, and phonon packets in nonlinear crystals [2]. The GRLW equation has been studied extensively, both analytically and numerically by various methods. Some solitary wave solutions for GRLW equations have been derived by based on the separation of the temporal and spatial derivatives [3]-[4].

By using the Adomian decomposition method, [5]-[6] gave the numerical solution of the GRLW equation. In addition, Petrov-Galerkin method [7] can also solve the GRLW equation effectively.
As it is known, reproducing kernel method is a accurate method have been researched by many people. In [1], a numerical method by using a reproducing kernel function has be used for solving the GRLW equation, but it has a very cumbersome process when we practise Gram-Schmidt orthonomalization. By the current research, it is the first time to solve the nonlinear GRLW equation by another reproducing kernel method which avoid this process.

In this paper, we consider the GRLW equation can be written as: [1]

$$\begin{cases} \dfrac{\partial u}{\partial t} + \alpha \dfrac{\partial u(x,t)}{\partial x} + \beta u \dfrac{\partial u}{\partial x} - \mu \dfrac{\partial^2 u}{\partial x^2} - \delta \dfrac{\partial^3 u}{\partial x^2 \partial t} = f(x,t), \quad (x,t) \in (a,b)\times(0,T) \\ u(x,0) = \tilde{u}_0(x,0),\ u(a,t) = \gamma_a(t),\ u(b,t) = \gamma_b(t), \end{cases} \tag{1.1}$$

where $\alpha, \beta, \mu, \delta$ are nonnegative constants. $\tilde{u}_0(x,0),\ \gamma_a(t),\ \gamma_b(t)$ are sufficiently smooth functions. In order to solve (1.1) by RKHSM, the initial boundary value conditions are homogenized. Let:

$$y=(x-1)/(b-a),\, u_0(y)=\tilde{u}_0(a+(b-a)y),\, v(y,t)=u(y,t)-w(y,t)-u_0(y)+w_0(y),$$
$$w(y,t)=\gamma_a(t)(1-y)+\gamma_b(t)y,\ w_0(y)=w(y,0),$$

so (1.1) can be transformed to (1.2):

$$\begin{cases}\dfrac{\partial v}{\partial t}+\alpha\dfrac{\partial v(y,t)}{\partial y}+\beta v\dfrac{\partial v}{\partial y}-\mu\dfrac{\partial^2 v}{\partial y^2}-\delta\dfrac{\partial^3 v}{\partial y^2\partial t}=f(y,t,v,\dfrac{\partial v}{\partial y}), \quad (y,t)\in(0,1)\times(0,T)\\ v(y,0)=\tilde{v}_0(y,0),\ v(a,t)=\gamma_a(t),\ u(b,t)=\gamma_b(t).\end{cases} \tag{1.2}$$

where:

$$f(y,t,v,\frac{\partial v}{\partial y})=f(y,t)-\frac{\partial w}{\partial t}-\frac{\alpha}{b-a}\frac{\partial(w+u_0-w_0)}{\partial y}+\frac{\mu}{(b-a)^2}\frac{\partial^2 u_0}{\partial y^2}-\frac{\beta}{b-a}(v+w+u_0-w_0)$$
$$\frac{\partial(v+w+u_0-w_0)}{\partial y}.$$

2. Reproducing Kernel Method

2.1. Space $W_2^1[0,1]=\{u(x)\,|\,u \textit{ is} \text{ one-variable absolutely continuous function }, u'\in L^2[0,1]\}$

An inner product is defined as:

$$\langle u(x),v(x)\rangle_{W_2^1}=u(0)v(0)+\int_0^1 u'(x)v'(x)dx, \quad u(x),v(x)\in W_2^1[0,1] \tag{2.1}$$

The reproducing kernel is:

$$R_x(y)=\begin{cases}1+x, & y>x\\ 1+y, & x>y\end{cases} \tag{2.2}$$

2.2. Space $W_2^2[0,1]=\{u(x)\,|\,u,u' \text{ are one-variable absolutely continuous functions, } u(0)=0.\ u'\in L^2[0,1]\}$

An inner product is defined as:

$$<u(x),v(x)>_{W_2^2}=\sum_{i=0}^{1}u^{(i)}(0)v^{(i)}(0)+\int_0^1 u''(x)v''(x)dx,\ u(x),v(x)\in W_2^2[0,1] \tag{2.3}$$

The reproducing kernel is:

$$K_x(t)=\begin{cases}tx+tx^2/2-x/6, & t>x\\ -t^3/6+tx+t^2x/2, x>t\end{cases} \tag{2.4}$$

2.3. Space $W_2^3[0,1]=\{u(x)\,|\,u,u',u'' \text{ are one-variable absolutely continuous functions, } u(0)=u(1)=0.\ u''\in L^2[0,1]\}$

An inner product is defined as:

$$<u(x),v(x)>_{W_2^3}=\sum_{i=0}^{2}u^{(i)}(0)v^{(i)}(0)+\int_0^1 u'''(x)v'''(x)dx,\ u(x),v(x)\in W_2^3[0,1] \tag{2.5}$$

The reproducing kernel is:

$$K_x(y)=\begin{cases}K(x,y), & y>x\\ K(y,x), & x>y\end{cases} \tag{2.6}$$

$$K(x,y) = -\frac{(1+115x+40x^2)(1+115y+40y^2)}{18720} + \frac{1}{120}x(x^4+120y-5x^3y+30xy^2+10x^2y^2),$$
$$K(y,x) = -\frac{(1+115x+40x^2)(1+115y+40y^2)}{18720} + \frac{1}{120}y(y^4+10x^2y(3+y)-5x(-24+y^3)).$$

3. Exact Solution and Approximate Solution

3.1. Analytical Solution

In order to solve (1.1), let:

$$(Lu)(x,t) = F(x,t) \tag{3.1}$$

where L : $H(D) \to H_1(D)$ is a bounded linear operator[3], $D=[0,1]\times[0,1]$, L^{-1} is existent, $H(D)=W_2^3[0,1]\otimes W_2^3[0,1]$ and $H_1(D)=W_2^1[0,1]\otimes W_2^1[0,1]$ is the reproducing kernel of $K_{(\xi,\eta)}(x,t)$ and $\overline{K}_{(\xi,\eta)}(x,t)$ [8]. So, the solution of (3.1) is the solution of (1.2).

Let:

$$\varphi_i(x,t) = \overline{K}_{(x_i,t_i)}(x,t), \quad \psi_i(x,t) = L^*\varphi_i(x,t), \quad B\phi = b, \tag{3.2}$$

where $b=[\psi_1(x,t),\psi_2(x,t),...]^T$, $\phi=[\zeta_1,\zeta_2,...]^T$, $B=(L\psi_i(x,t)|_{(x,t)=(x_j,t_j)})_{i,j=1,2,...}$, L^* is the adjoint operator. If B^{-1} is existent, then $(L\zeta_j(x,t)|_{(x,t)=(x_i,t_i)})_{i,j=1,2,...}$ is an identity matrix. If $\{x_i,t_i\}_{i=1}^{\infty}$ is dense on D, $\psi_i\{x_i,t_i\}_{i=1}^{\infty}$ is a complete function system in $H(D)$, then an analytical solution of (3.1) is:

$$u(x,t) = \sum_{j=1}^{\infty} F(x_j,t_j)\zeta_j(x,t) \tag{3.3}$$

3.2. Approximate Solution

If (1.1) is nonlinear, the analytical solution can be written by:

$$u(x,t) = \sum_{j=1}^{\infty} F(x_j,t_j,u(x_j,t_j))\zeta_j(x,t) \tag{3.4}$$

The approximate solution to (1.1) can be obtained using the following method.

We can give initial function $u_1(x,t)\in H(D)$, using the form (3.4), an iterative sequence is constructed by:

$$u_n(x,t) = \sum_{j=1}^{\infty} F(x_j,t_j,u_{n-1}(x_j,t_j))\zeta_j(x,t), \ n=2,... \tag{3.5}$$

The convergence of $u_n(x,t)$ has been proved by [3].

4. Numerical Experiment

Experiment 1. To show the numerical comparison of two methods, we solve the equation in [1] by the present method. The equation has a true solution which is:

$$u_T(x,t) = 3c\sec h^2(k[x-x_0-vt]),$$

where $v=1+\beta c$, $k=\sqrt{\beta c/(4\delta v)}$, $c=0.1$, $x_0=0$, $\alpha=\beta=\delta=1$, $\mu=0$, $a=40$, $b=60$. $T=1$.

In this calculation, the truncated approximate solution can be written as:

$$u_{m,n}(x,t)=\sum_{j=1}^{m} f(x_j,t_j,u_{n-1}(x_j,t_j),\frac{\partial u_{n-1}(x_j,t_j)}{\partial x})\zeta_j(x,t),\ n=2,3,....$$

We choose $m=23, n=3,$ by Mathematica 7.0, the approximate solution $u_{23,3}(x,t)$ is obtained, and the numerical comparison of two methods are tabulated in Table 1. The true solution $u(x,t)$ and absolute errors $|u(x,t)-u_{23,3}(x,t)|$ are given in Figures 1 & 2. Figure 3 represents the 40 approximate solutions(red points) and the true solution(the blue line) at t=1. Besides, Figure 4 is the absolute errors $|u(x,1)-u_{23,3}(x,1)|$.

Table 1

(x,t)	$u_T(x,t)$	$u_{23,3}(x,t)$	$\lvert u(x,t)-u_{23,3}(x,t)\rvert$ ([1])
(-32,1)	5.5574E—04	5.04097E—06	1.1546E—04
(-16,1)	6.3896E—03	2.97735E—04	2.3370E—03
(0,1)	2.9190E—01	9.16134E—05	8.7824E—03
(16,1)	1.3135E—02	1.22966E—04	2.0659E—03
(32,1)	1.0787E—04	4.06209E—06	9.9975E—03
(48,1)	1.2933E—04	5.32068E—06	1.2847E—04
(56,1)	7.7681E—07	1.80332E—06	3.6981E—06
(-32,20)	1.0190E—07	4.41744E—04	6.2540E—04
(0,20)	1.5749E—03	5.82901E—04	2.9713E—03
(48,20)	4.7236E—04	4.34467E—04	3.2430E—03
(56,20)	4.2367E—07	1.89941E—05	1.8868E—03

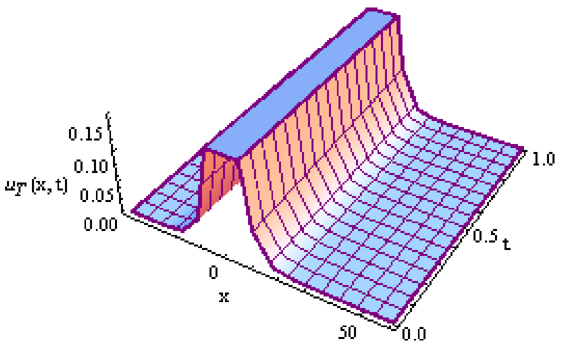

Fig.1 $u_T(x,t)$ of Experiment 1.

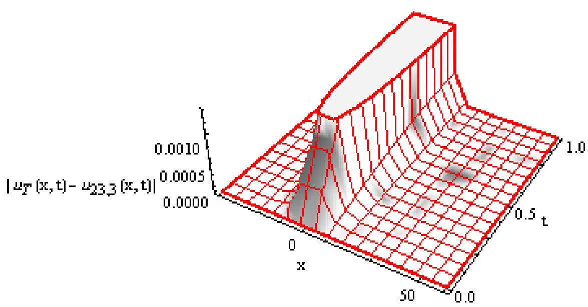

Fig.2 $|u_T(x,t)-u_{23,3}(x,t)|$ of Experiment 1.

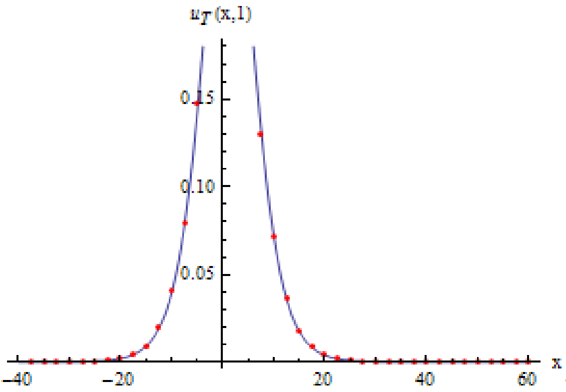

Fig.3 $u_{23,3}(x,1)$ (40 red points) and $u_T(x,1)$ (blue line) of Experiment 1.

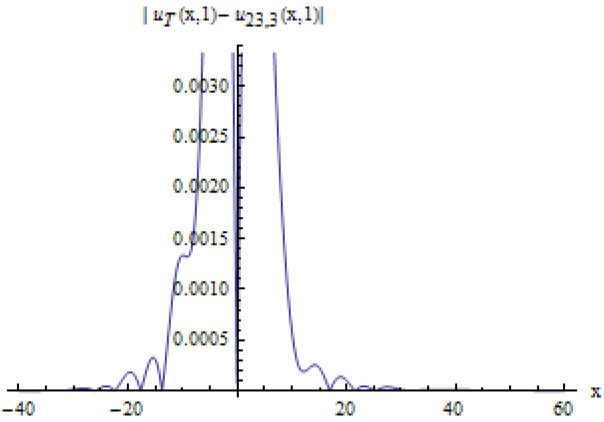

Fig.4 $|u_T(x,1)-u_{23,3}(x,1)|$ of Experiment 1.

Experiment 2. To show the accuracy of the present method, another experiment is given. In (1.1), let:

$$\alpha=\beta=\delta=\mu=1,\ a=0,\ b=1,\ T=1.\ f(x,t)=e^t(\sin\pi x+\pi\cos\pi x(1+e^t\sin\pi x)),$$

a true solution of this problem which is $u_T(x,t)=e^t\sin\pi x$. As the same steps in Experiment 1, we choose m=10, n=3. By Mathematica 7.0, the approximate solutions $u_{10,3}(x,t)$ and absolute errors $|u(x,t)-u_{10,3}(x,t)|$ are tabulated in Table 2. The true solutions $u(x,t)$ and absolute errors $|u(x,t)-u_{10,3}(x,t)|$ are given in Figures 5 & 6. Figure 7 represents the 40 approximate solutions (red points) and the true solution(the blue line) at t=1, it is obvious that they are in good agreement with each other, and Figure 8 is the absolute errors $|u(x,1)-u_{10,3}(x,1)|$.

Table 2

Note	True solutions	Approximate solutions	Absolute errors
(0.1,0.1)	0.34152	0.34252	1.00359E—03
(0.2,0.2)	0.71792	0.72110	3.17251E—03
(0.3,0.3)	1.09206	1.09650	4.44549E—03
(0.4,0.4)	1.41881	1.42239	3.57665E—03
(0.5,0.5)	1.64872	1.64944	7.13790E—04
(0.6,0.6)	1.73294	1.73000	2.93703E—03
(0.7,0.7)	1.62916	1.62334	5.81595E—03
(0.8,0.8)	1.30814	1.30143	6.71103E—03
(0.9,0.9)	0.76006	0.75505	5.00878E—03

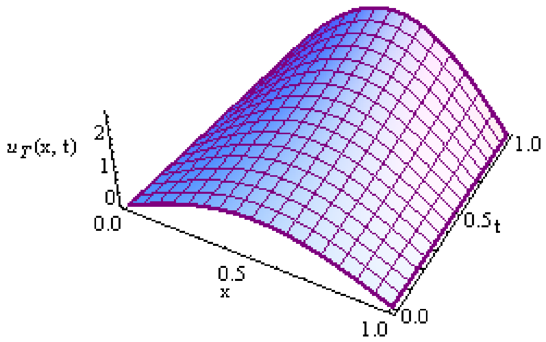

Fig. 5 $u_T(x,t)$ of Experiment 2.

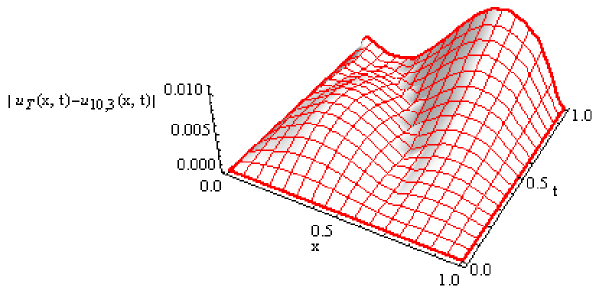

Fig.6 $|u_T(x,t)-u_{10,3}(x,t)|$ of Experiment 2.

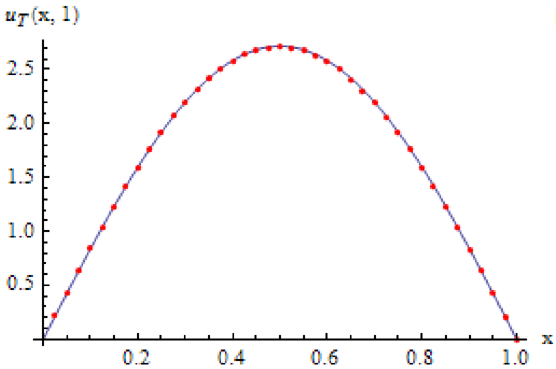

Fig. 7 $u_{10,3}(x,1)$ (40 red points) and $u_T(x,1)$ (blue line) of Experiment 2.

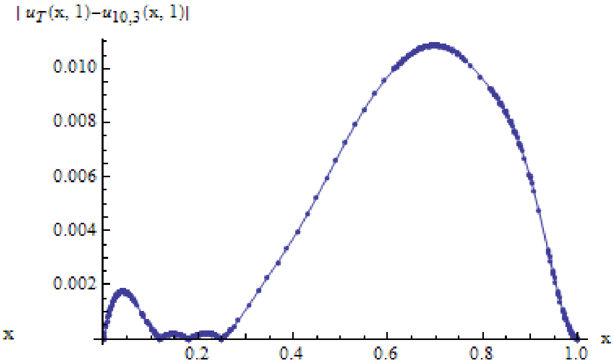

Fig.8 $|u_T(x,1)-u_{10,3}(x,1)|$ of Experiment 2

5. Conclusions

In this paper, it is the first time to solve GRLW equation by a new reproducing kernel method which avoid practising Gram-Schmidt orthonomalization. In order to show the accuracy of the present method, numerical comparison of two methods are presented in experiment 1, and it's obvious that the present method is better.

In experiment 2, some approximation solutions are given, and they have tiny errors compared with true solutions. So it can be considered that the present method is more simple and accurate for solving GRLW equation, and the method can be used by solving other equations.

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References

1. *Mohammadi M., Mokhtari R.,*'Solving the generalized regularized long wave equation on the basis of a reproducing kernel space', *J. Comput. Appl. Math.,* ***235*** *(2011), 4003 – 4014.*

2. *Peregrine D.H., Cui M.G.*, 'Calculations of the development of an undular bore', *J. Comput. Appl. Math.*, ***25*** *(1966), 321 – 330*
3. *Hamdi S., Enright H. W.*, 'Exact solutions and invariants of motion for general types of regularized long wave equations', *Math. Comput. Simul.*, ***65*** *(2004), 535 – 545.*
4. *Ramos J.I.*, 'Solitary wave interactions of the GRLW equation', *Chaos. Solitons. Fractals.*, ***33*** *(2007), 479 – 491.*
5. *Kaya D.*, 'A numerical simulation of solitary-wave solutions of the generalized regularized long wave equation', *Appl. Math. Comput.*, ***149*** *(2004), 833 – 841.*
6. *Kaya D., El-Sayed S.M.*, 'An application of the decomposition method for the generalized KdV and RLW equations', *Chaos. Solitons. Fractals.*, ***17*** *(2003), 869 – 877.*
7. *Roshan T.*, 'A Petrov-Galerkin method for solving the generalized regularized long wave (GRLW) equation', *Comput. Math. Appl.*, ***63*** *(2012), 943 – 956.*
8. *Wang Y.L., Su L.J.*, 'Using reproducing kernel for solving a class of singularly perturbed problems', *Comput. Math. Appl.*, ***61*** *(2011), 421 – 430.*