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Reproducing Kernel Method for PDEs arising from Downhole Temperature Field

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Abstract
Downhole temperature distribution with multiple pay zones in different distance is very important when oil wellbore producing. This paper presents a new method to solve a class of PDEs arising from heat transfer problem in the wellbore for the first time. By using reproducing kernel method, this equations with boundary conditions are investigated and the exact solution is represented in form of series in the reproducing kernel space. The numerical experiment and numerical simulation analyses are conducted to show that the numerical solutions have a good agreement with the exact solutions and the simulated result corresponds to general knowledge, which verify the precision and efficiency of the method in this work. The reproducing kernel method has the characteristics of simple and high precision, and it provides a new numerical method in fluid mechanics field, which is the theoretical basis for further observing oil field production condition.

Key Word and Phrases
Reproducing Kernel, Heat Transfer, Partial Differential Equations (PDEs), Numerical Simulation.

1. Mathematical Modeling

Being informed of downhole temperature distribution is very important in knowing producing development and producing condition. The study of dynamic temperature log has been a very useful information for oil production.

Y.J.Song and Y.Shi builded the temperature field model, the model of downhole temperature field in cylindrical coordinates and its boundary conditions as follows: [1]

$$\frac{\partial^2}{\partial r^2}(\lambda_l T)+\frac{1}{r}\frac{\partial}{\partial r}(\lambda_l T)+\frac{\partial^2}{\partial z^2}(\lambda_l T)=\frac{\partial}{\partial t}(c_l\rho_l T)+\frac{\partial}{\partial z}(c_l\rho_l v_z T) \tag{1.1}$$

where T is the temperature, $\lambda_1 = F_w\lambda_w + (1 - F_w)\lambda_0$ is the thermal conductivity of mixed fluid inside the oil wellbore, λ_w, λ_0 are the thermal conductivity of water and oil, F_w is water saturation. z is the depth of the oil wellbore, r is the radial distance. $\rho_1 c_1 = F_w\rho_w c_w + (1-F_w)\rho_0 c_0$, ρ_w, ρ_0 are the density of water and oil, c_w, c_0 are the specific heat of water and oil. v_z is the velocity of fluid inside the oil wellbore. The initial temperature of the formation and wellbore is at the geothermal condition which can be written as $T|_{t=0}=T_0 = a+bz$, a is the formation temperature when $z=0$, b is the geothermal gradient, the boundary of adiabatic condition is $\frac{\partial(\lambda_l T)}{\partial r}|_{r=0}=0$.

Y.J.Song and Y.Shi used the alternating direction implicit method (ADI) to discrete downhole temperature field model [1], [2]; M.Ren used the full implicit form of finite difference method to discrete and solve the model [3]; S.G.Li used the finite difference method [4] and M.Yang used the implicit finite difference method to solve the mathematical models [5]; Z.S.Xiao used the alternating direction implicit method (ADI) and speedup method (on the radial shaft adopts evenly spaced, in stratum adopts logarithmic interval) to discrete temperature field model in water injection [6].

By the current research we ignore the time t and replace T with u and r, z with x,y, for simplicity. The mathematical equation can be written as:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = F(x,y) \\ u(x,0) = 0, \ \frac{\partial u(x,0)}{\partial y} = 0, \\ u(0,y) = 0, \ \frac{\partial u(0,y)}{\partial x} = 0, \end{cases} \tag{1.2}$$

Besides, in this paper in order to solve equation (1.2) , we introduce the reproducing kernel method in Part 2.

2. Reproducing Kernel Method

2.1 Space $W_2^1[0,1] = \{u(x) \mid u \text{ is one-variable absolutely continuous function }, u' \in L^2[0,1]\}$

An inner product is defined:

$$\langle u(x), v(x) \rangle_{W_2^1} = u(0)v(0) + \int_0^1 u'(x)v'(x)dx, \quad u(x), v(x) \in W_2^1[0,1] \tag{2.1}$$

The reproducing kernel is:

$$R_x(t) = \begin{cases} 1+x, & t > x \\ 1+t, & x > t \end{cases} \tag{2.2}$$

2.2 Space $W_2^3[0,1] = \{u(x) \mid u, u', u'' \text{ are one-variable absolutely continuous functions, } u(0) = u'(0) = 0.\ u'' \in L^2[0,1]\}$

An inner product is defined:

$$< u(x), v(x) >_{W_2^3} = \sum_{i=0}^{2} u^{(i)}(0)v^{(i)}(0) + \int_0^1 u'''(x)v'''(x)dx, u(x), v(x) \in W_2^3[0,1] \tag{2.3}$$

The reproducing kernel is:

$$K_x(t) = \begin{cases} t^2(-5xt^2 + t^3 + 10x^2(3+t))/120, & t > x \\ x^2(x^3 - 5x^2t + 30t^2 + 10xt^2))/120, & x > t \end{cases} \tag{2.4}$$

3. Exact Solution and Approximate Solution

In order to solve (1.2), Let:

$$(Lu)(x,y) = F(x,y) \tag{3.1}$$

where L: $H(D) \to H_1(D)$ is a bounded linear operator[7], $D=[0,1]\times[0,1]$, L^{-1} is existent, $H(D)=W_2^3[0,1]\otimes W_2^3[0,1]$ and $H_1(D)=W_2^1[0,1]\otimes W_2^1[0,1]$ is the reproducing kernel of $\overline{K}_{(\xi,\eta)}(x,y)$ and $K_{(\xi,\eta)}(x,y)$. So, the solution of (3.1) is the solution of (1.2).

Let:

$$\varphi_i(x,y)=\overline{K}_{(x_i,y_i)}(x,y), \quad \psi_i(x,y)=L^*\varphi_i(x,y), \quad B\phi=b, \tag{3.2}$$

where $b=[\psi_1(x,y),\psi_2(x,y),...]^T$, $\phi=[\zeta_1,\zeta_2,...]^T$, $B=(L\psi_i(x,y)|_{(x,y)=(x_j,y_j)})_{i,j=1,2,...}$, L^* is the adjoint operator. If B^{-1} is existent, then $(L\zeta_j(x,y)|_{(x,y)=(x_i,y_i)})_{i,j=1,2,...}$ is an identity matrix. If $\{x_i,y_i\}_{i=1}^{\infty}$ is dense on D, $\psi_i\{x_i,y_i\}_{i=1}^{\infty}$ is a complete function system in $H(D)$, then an analytical solution of (3.1) is:

$$u(x,y)=\sum_{j=1}^{\infty}F(x_j,y_j)\zeta_j(x,y) \tag{3.3}$$

4. Numerical Experiment

Solve (1.2) with *F(x, y)*=2 *cosh x cosh y*, the exact solution *u(x, y)*= *cosh x cosh y*. The numerical results of exact solution, approximate solution and their absolute errors are tabulated in Table 1 and Fig.1-2.

Table 1. Exact solution, approximate solution and their absolute errors

(x,y)	Exact solution	Approximate solution	Absolute errors
(0.1,0.1)	1.01003	1.01003	3.16530E—07
(0.2,0.2)	1.04054	1.04053	6.65734E—06
(0.3,0.3)	1.09273	1.09272	1.53282E—05
(0.4,0.4)	1.16872	1.16872	5.30412E—06
(0.5,0.5)	1.27154	1.27165	1.07792E—04
(0.6,0.6)	1.40533	1.40566	3.35709E—04
(0.7,0.7)	1.57545	1.57611	6.64481E—04
(0.8,0.8)	1.78873	1.78965	9.18893E—04
(0.9,0.9)	2.05374	2.05435	6.12124E—04
(1.0,1.0)	Cosh[1]2	2.37998	1.11951E—03

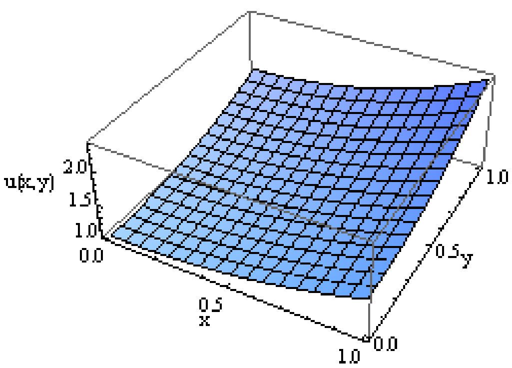

Fig. 1 Exact solution

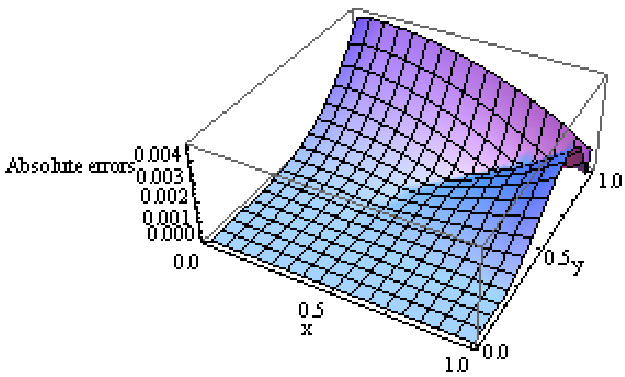

Fig. 2 Absolute errors

5. Numerical Simulation

We use the common body structure of vertical wells (there are two pay zones and three adjacent formations) to solve (1.1), by assuming that the fluid in the wellbore is the Newton fluid, the geothermal gradient is 0.025º*C/m*, ignoring the influence of casing and cement, the formation temperature is 15º*C* when the depth z=0*km*, the liquid producing capacity is 30m^3/d. By using further the common physical parameters [1]. The temperature field is compiled by reproducing kernel method. The numerical simulation analysis result is given in Fig.3.

6. Conclusions

From Figure 3, it can be seen that when the oil well bore production, then different radial distance make difference distribution of temperature field. In the well shaft, the temperature is higher than others because they are exchanged each other by the continuously heat fluid which come from underground, and we can not distinguish the fluid layer position. When r=0.07m and r=0.09m, there are two pay zones obviously, and when r = 0.07m, the temperature is higher. So, we can determine the pay zones through temperature data features, determine the thickness of pay zone and which position the fluid will into. The results of numerical simulation are reasonable and correspond to general knowledge, so reproducing kernel method can solve this problem feasibly.

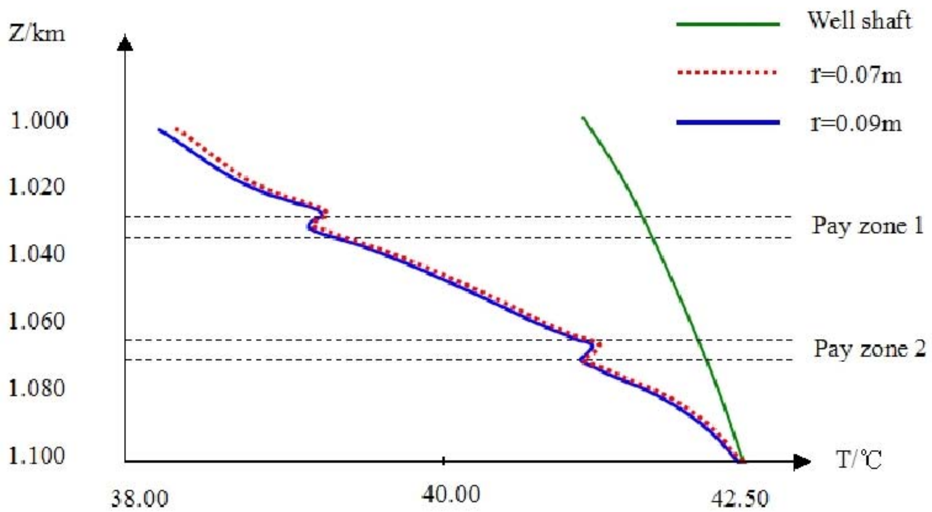

Fig. 3 Temperature distribution in the wellbore

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