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Non-linear Stability Criterion in Numerical Approximation of Hyperelastic Rods

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Abstract
For various biomechanical and industrial problems, the stability analysis of an hyperelastic rod subjected to external load remains an interesting field of research recently studied. In most instances, a wide variety of methods have been proposed to overcome this problem. However, in large displacements and large deformations, we are faced with two serious problems: a numerical instability or the use a technical to solve this problem but that asks a hard implementation. In this work, a direct numerical method is proposed to analyze the nonlinear problem. We propose a new criterion based on a relative determinant in order to simplify the detection of the critical point without losing effectiveness. This method does not need an extra computation cost. In addition, an arclength method is used to control the external load applied to the rod. Finally, we exhibit some numerical results obtained from the application of the method.

Key Word and Phrases
Cosserat Rod, Stability Analysis, Relative Determinant, Critical Point, Continuation Method.

1. Introduction
Due to its industrial and biomechanical applications, the problem of modeling an elastic rod subjected to external forces is being an interesting research field. The reason for this interest can manifest itself in its use in many industrial applications and in biomechanical problems. We can cite, for example, pipelines in the oil industries, stents in the medical field, beams in civil constructions, and the DNA molecule in the biomechanical modeling [1], [2], [3]. Despite the works that have examined the modeling of elastic rods problem, the numerical calculation with large displacements and/or large deformations remains an immense subject. Especially if we want to analyze the new generations of metals used in medical areas such as NITINOL (Nickel Titanium Naval Ordnance Laboratory). Because these metals have very rich properties, it requires more difficulty in their modeling. For example, the study of the numerical aspect of superhelical turns combined with a phenomenon of supercoiling that can take place in the configuration of a DNA molecule [5]. It is important to note that the treatment of such problems can lead to several problems of stability. The stability analysis remains an interesting field of research recently studied. In most instances, a wide variety of methods have been proposed to overcome this problem. However, a numerical instability or the use a technical to overcome this problem, but that asks a hard implementation, challenges which face us.

This situation is due to the fact that tangent stiffness matrix is singular. This leads to the existence of several solutions. Besides, the stability problem requires an artificial symmetrization of the tangent stiffness matrix [6]. Our objective is to study the modeling, numerical aspects of the large displacements of an hyperelastic rod subject to external loads. In order to do so, a direct numerical method is proposed to analysis this nonlinear situation. We therefore propose, a new criterion based on applying the notion of a relative determinant. The goal is to simplify the detection of the critical point without losing effectiveness. In addition, an arclength method is used to control the external load applied at the rod. We finish our paper by presenting a numerical result of the proposed method.

2. Cosserat Rod

We make use of the director rod theory based on the model introduced by Cosserat brothers [7], used in the abstract framework proposed by Antman [4], and exploited in [8], [9]. Let R an elastic rod *having two dimensions, those of its* circular *cross section of* diameter 2ε, with respect the third its length L. The configuration of the rod is parameterized by an arclength s in $[0, L]$. We consider $\boldsymbol{r}(s)$ the position vector and $(\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3)$ is a right-handed triad of orthonormal directors which described as the directions of a rigid cross-section at s of R. The curve $\boldsymbol{r}(s)$ represents the central fiber of the rod, in the deformed configuration of R. We denote the $\boldsymbol{R} = \boldsymbol{d}_i \otimes \boldsymbol{e}_i \in SO(3)$, with $SO(3)$ is the group of all 3×3 orthogonal matrices with determinant 1, known as the rotation group. The orthogonal tensor $\boldsymbol{R}$ is a transition matrix from the material triad $(\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3)$ to a spatial triad $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$. With this Cosserat rods theory, we have a three-dimensional model by a one-dimensional rod parameterization. We noted throughout, the quantities with a superposed hat, e.g., $\hat{\boldsymbol{r}}$ that are associated with the unstressed configuration, and $\boldsymbol{r}$ in deformed configuration.

2.1. Strains Measures

Following Antman [4], we introduce the measures of deformation:

$$\boldsymbol{v} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{r}' \text{ and } \boldsymbol{u}^{\times} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{R}'. \tag{2.1}$$

where $\boldsymbol{u}^{\times}$ designed the skew matrix of the space $so(3)$ of all 3×3 skew matrices, and we have for all $\boldsymbol{v} \in \boldsymbol{lR^3}$, $\boldsymbol{u}^{\times}\boldsymbol{v} = \boldsymbol{u} \times \boldsymbol{v}$. The mechanical interpretation of these quantities is as follows:

- The components u_1 and u_2 of the vector $\boldsymbol{u}$ and describe the flexure, and u_3 is a measure of the twist of the rod.
- The components v_1 and v_2 of the vector $\boldsymbol{v}$ describe the shear deformations, and v_3 represents.

the extension of the rod.

2.2. Balance Equations and Constitutive Laws

The force and moment balance equations for the rod verify:

$$\boldsymbol{n}' + \boldsymbol{f}_{\boldsymbol{ext}} = \boldsymbol{0} \text{ and } \boldsymbol{m}' + \boldsymbol{r}' \times \boldsymbol{n} + \boldsymbol{t}_{\boldsymbol{ext}} = \boldsymbol{0}, \tag{2.2}$$

where $\boldsymbol{n}$ and $\boldsymbol{m}$, respectively, are the stress distribution vectors and moments over the cross-section at s, and:

$\boldsymbol{f}_{\boldsymbol{ext}}$ and $\boldsymbol{t}_{\boldsymbol{ext}}$ are the distributed external forces and external torques.

We assume that the rod is made of a hyperelastic material, i.e., there exists an elastic energy density:

$$\boldsymbol{m} = \boldsymbol{R}\frac{\partial W(s, \boldsymbol{u}-\hat{\boldsymbol{u}}, \boldsymbol{v}-\hat{\boldsymbol{v}})}{\partial \boldsymbol{u}} \quad \text{and} \quad \boldsymbol{n} = \boldsymbol{R}\frac{\partial W(s, \boldsymbol{u}-\hat{\boldsymbol{u}}, \boldsymbol{v}-\hat{\boldsymbol{v}})}{\partial \boldsymbol{v}}, \tag{2.3}$$

where $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{v}}$ are the strains measure in the relaxed configuration. We denote that this setting allows taking into account that rod can be initially curved/twisted rod.

2.3. Variational Formulation

Let $\delta\boldsymbol{r}$ and $\delta\boldsymbol{R}$ be kinematically admissible translational and rotational variations. They will be interpreted as incremental vectors of displacement and rotation. For these admissible variations, the increment in the elastic energy and the work of the external forces is:

$$\delta \boldsymbol{W}_{\boldsymbol{int}} = \int_{\boldsymbol{0}}^{\boldsymbol{L}} \left(\boldsymbol{0}, \boldsymbol{n}, \boldsymbol{r}^{\times\prime}, \boldsymbol{m} \right) \begin{pmatrix} \delta\boldsymbol{r} \\ \delta\boldsymbol{r}' \\ \delta\boldsymbol{R} \\ \delta\boldsymbol{R}' \end{pmatrix} \boldsymbol{ds}, \tag{2.4}$$

$$\delta \boldsymbol{W}_{\boldsymbol{ext}} = \int_{\boldsymbol{0}}^{\boldsymbol{L}} (\boldsymbol{f}_{\boldsymbol{ext}}, \boldsymbol{0}, \boldsymbol{t}_{\boldsymbol{ext}}, \boldsymbol{0}) \begin{pmatrix} \delta \boldsymbol{r} \\ \delta \boldsymbol{r}' \\ \delta \boldsymbol{R} \\ \delta \boldsymbol{R}' \end{pmatrix} \boldsymbol{ds}.$$

The variational formulation, which the principle of the virtual works is given, can be written:

$$\partial W(\boldsymbol{r}, \boldsymbol{R}; \delta\boldsymbol{r}, \delta\boldsymbol{R}) := \delta \boldsymbol{W}_{\boldsymbol{int}} - \boldsymbol{\lambda}\, \delta \boldsymbol{W}_{\boldsymbol{ext}} = \boldsymbol{0}, \tag{2.5}$$

with $\boldsymbol{\lambda}$ is a load factor. We denote that equation is non-linear; it can be solved using Newton's method.

3. Iterative Methods for Solving Stability Problems

To solve the equations governing the static state of a rod subjected to external loads presented in the previous sections does not a simple problem. The main difficulty is due to the non-linear nature of the weak formulation. In addition, the presence variety of SO(3) in the definition of kinematically admissible configuration space gives a non-linear structure. On the other hand, the choice of the energy densities is nonlinear to that taking into account the large displacements. These energy densities we use lead to coupled problems bending-torsion, which is physically natural in large displacements, but is difficult to treat numerically. The purpose of this section is to provide a resolution strategy for the nonlinear problem. We use a Newton-Raphson algorithm based on the linearization each of the terms of the formulations.

3.1 Newton-Raphson Method

The Newton-Raphson method is the efficient technique for solving equations numerically. It is an effective procedure for the numerical implementation for nonlinear equations. The principle of the method is the prediction of the solution by an approximate solution sequence. Starting with an estimated configuration $(\boldsymbol{r}^{\boldsymbol{n}}, \boldsymbol{R}^{\boldsymbol{n}})$ at an iteration $\boldsymbol{n}$, we find the admissible variations $(\boldsymbol{\Delta r}^{\boldsymbol{n}}, \boldsymbol{\Delta R}^{\boldsymbol{n}})$ by resolving the follow linearized problem:

$$\boldsymbol{F}(\boldsymbol{r}^{\boldsymbol{n}}, \boldsymbol{R}^{\boldsymbol{n}}; \boldsymbol{\delta r}^{\boldsymbol{n}}, \boldsymbol{\delta R}^{\boldsymbol{n}}) + \boldsymbol{\Delta F}(\boldsymbol{r}^{\boldsymbol{n}}, \boldsymbol{R}^{\boldsymbol{n}}; \boldsymbol{\delta r}^{\boldsymbol{n}}, \boldsymbol{\delta R}^{\boldsymbol{n}}; \boldsymbol{\Delta r}^{\boldsymbol{n}}, \boldsymbol{\Delta R}^{\boldsymbol{n}}) = \boldsymbol{0}. \tag{3.1}$$

3.2 Finite Element Method

In accordance with the works of Simo & Vu-Quoc [8], the finite element formulation of the theory presented and developed on the basis of equation **(5)**. We define a subdivision $(s_i)_{0\le i\le N}$ of the interval $[0, \mathrm{L}]$ and note by $\boldsymbol{h}$, the maximal length of the subintervals. All components of $\boldsymbol{r}, \boldsymbol{R}, \delta\boldsymbol{r}$ and $\delta\boldsymbol{R}$ are interpolated using the well-known cubic interpolation functions at node $\boldsymbol{a}$ noted $\boldsymbol{N}^{\boldsymbol{a}}$:

$$\phi^h = \sum_{a=1}^{n_{node}} \boldsymbol{N}^{\boldsymbol{a}} \phi_a,$$

with ϕ stands for $\boldsymbol{r}, \boldsymbol{R}, \delta\boldsymbol{r}$ or $\delta\boldsymbol{R}$.

In order to use the solution procedures of Newton type, one needs the linearised equilibrium equations, which can be obtained by the linearization of the principle of virtual work in its continuum form, see [5]. It should be noted that the update procedure of rotation $\boldsymbol{R}_{\boldsymbol{n}}^{\boldsymbol{h}}$ and the displacement $\boldsymbol{r}_{\boldsymbol{n}}^{\boldsymbol{h}}$ are obtained in two very distinct ways because $\boldsymbol{r}_{\boldsymbol{n}}^{\boldsymbol{h}} \in \boldsymbol{lR}^{\boldsymbol{3}}$**,** i. e. is a linear, but $\boldsymbol{R}_{\boldsymbol{n}}^{\boldsymbol{h}} \in \boldsymbol{SO}(\boldsymbol{3})$.

$$\boldsymbol{R}_{\boldsymbol{n+1}}^{\boldsymbol{h}} = \boldsymbol{exp}\left[\left(\boldsymbol{\delta R}_{\boldsymbol{n+1}}^{\boldsymbol{h}}\right)^{\times}\right] \boldsymbol{R}_{\boldsymbol{n}}^{\boldsymbol{h}} \text{ and } \boldsymbol{r}_{\boldsymbol{n+1}}^{\boldsymbol{h}} = \boldsymbol{r}_{\boldsymbol{n}}^{\boldsymbol{h}} + \boldsymbol{\delta r}_{\boldsymbol{n+1}}^{\boldsymbol{h}}, \tag{3.2}$$

with $\boldsymbol{n}$ is the Newton's iteration number.

3.3 Stability Analysis

We will use the direct stability criterion [10], to analyzing the stability of an elastic rod:

$$\boldsymbol{\Delta}[\boldsymbol{F}(\boldsymbol{r},\boldsymbol{R};\boldsymbol{p},\boldsymbol{q};\boldsymbol{\lambda})](\boldsymbol{p},\boldsymbol{q}) = (\boldsymbol{p},\boldsymbol{q})^{\boldsymbol{T}}\boldsymbol{lK}(\boldsymbol{r},\boldsymbol{R})(\boldsymbol{p},\boldsymbol{q}) > 0, \forall(\boldsymbol{p},\boldsymbol{q}) \tag{3.3}$$

where $\boldsymbol{lK} = \boldsymbol{Q_m} + \boldsymbol{Q_g} + \boldsymbol{Q_c}$ is the tangent matrix, and $\boldsymbol{Q_m}$, $\boldsymbol{Q_g}$ and $\boldsymbol{Q_c}$**,** respectively, is the material part, the geometrical part and the self-contact part of the tangent operator. According to this criterion, a balance state is stable if the tangent operator is positive definite, moreover the matrix K is symmetric. The equation $\det(\boldsymbol{lK}) = \boldsymbol{0}$ expressed the transition state between the stability and the instability. However, an alternative approach to constructing an augmented system for the stability calculation can be based on an eigenvalue problem. The constraint $\det(\boldsymbol{lK}) = \boldsymbol{0}$ implies that zero is an eigenvalue of that matrix $\boldsymbol{lK}$. This remains to the existence of an eigenvector $\boldsymbol{\psi}$ such $\boldsymbol{lK\psi} = \boldsymbol{0}$. Thus, the augmented system is given by:

$$\boldsymbol{F}(\boldsymbol{r},\boldsymbol{R};\boldsymbol{p},\boldsymbol{q};\boldsymbol{\lambda}) \coloneqq \begin{pmatrix} \boldsymbol{F}(\boldsymbol{r},\boldsymbol{R};\boldsymbol{p},\boldsymbol{q};\boldsymbol{\lambda}) \\ \boldsymbol{lK\psi} \\ \boldsymbol{f}(\boldsymbol{\psi}) \end{pmatrix} = \boldsymbol{0}. \tag{3.4}$$

where $\boldsymbol{f}(\boldsymbol{\psi})$ is chosen to find a limit point or a bifurcation point [10].

3.4 Stability Criterion

In typical problems, for checking whether the position of equilibrium is stable or not, one must check that the second derivative of the energy function is positive at the corresponding configuration. Since theoretically, this stability criterion is necessary and sufficient. But the application of this criterion in numerical approximation is not automatically. Because, for searching a zero of the determinant is not limited at its sign change only. Therefore, we need to follow the change of the monotonicity of the determinant especially when its value is less than a given value δ of accuracy. Due to the fact that, the loading dose is driven by the arclength method, despite its performance, we can to be faced with the problem unadjusted loading is imposed, particularly in a critical state. This is exactly the case where the rate of change of the deformation is abrupt and uncontrollable. In this situation, the determinant value can change quickly without detecting the zero value. Moreover, it is important to note that the coefficients of stiffness matrix can be larger when the rigidity coefficients become larger. We propose here a simple numerical treatment for determining stability, which consists to using a new stability criterion based on relative determinant. The use of the relative values of the determinant to adequately assess critical circumstances. To offer a more effective response, we propose:

$$\boldsymbol{det_{rel}}(\boldsymbol{lK_p}) = (-\boldsymbol{1})^{\boldsymbol{np}} \frac{|\ \boldsymbol{det}(\boldsymbol{lK_p})|}{|\ \boldsymbol{det}(\boldsymbol{lK_0})|}, \tag{3.5}$$

where $\boldsymbol{p}$ is a load step number, and $\boldsymbol{np}$ is the negative pivot numbers, respectively, the LU and the Crout decomposition method are used. A simple procedure to implement with no extra computation cost because these parameters are already computed by the decomposition method used for solving the linear system for each Newton iteration. In addition, the criterion (7) it can be effective for small displacements, because in large displacements and/or large deformations, a deformed configuration cannot be compared to a stress-free configuration. It is therefore appropriate for considering more efficient criterion defined by:

$$\boldsymbol{det_{rel}}(\boldsymbol{lK_p}) = (-\boldsymbol{1})^{\boldsymbol{np}} \frac{|\ \boldsymbol{det}(\boldsymbol{lK_p})|}{|\ \boldsymbol{det}(\boldsymbol{lK_{p-1}})|}, \tag{3.6}$$

To complete the stability study, we need to prescribe the assistance in the computation in finding an appropriate solution with regard to the critical load and the critical point. We adopted a technique that follows the sign change or the change of monotonicity of the relative determinant. In the first case, we use the bisection approach to find the critical load, take into account specific the sign change. In the second, we use a quadratic interpolation if we detect a change of monotonicity only the relative determinant is close to zero. We describe in the following algorithm, the principle of the method proposed to in the order to find a critical load.

Start: $\boldsymbol{det}(\boldsymbol{lK_{p-1}})$, $\boldsymbol{det}(\boldsymbol{lK_{p}})$, $\boldsymbol{det}(\boldsymbol{lK_{p+1}})$ and $\boldsymbol{np}$

If $(-1)^{\boldsymbol{np}} = -1$ (the sign of the determinant change)

Start the bisection approach to find the critical load

End

If $|\boldsymbol{det_{rel}}(\boldsymbol{lK_p})| \leq \delta$ (a prediction area for a critical state)

If $(\boldsymbol{det}(\boldsymbol{lK_{p+1}}) - \boldsymbol{det}(\boldsymbol{lK_p}))(\boldsymbol{det}(\boldsymbol{lK_p}) - \boldsymbol{det}(\boldsymbol{lK_{p-1}})) \leq 0$ (the monotonicity of the determinant is changed)

Start a quadratic interpolation to find the critical load

End

End

4. Numerical Examples

4.1 Initial curved Rod

We choose an example that serves to test the computation performance for the curved rod element in a 3D deformation problem. It's an example of nonlinear analysis. We adopt the following choice of stored elastic energy described in [2]:

$$W(\boldsymbol{r}, \boldsymbol{R}; \boldsymbol{r}', \boldsymbol{R}') = \int_{\boldsymbol{0}}^{\boldsymbol{L}} \left[\frac{\boldsymbol{GI}}{\boldsymbol{2}}\left(|\boldsymbol{d_1}'|^2 + |\boldsymbol{d_2}'|^2\right) + \frac{(\boldsymbol{E}-\boldsymbol{G})\boldsymbol{I}}{\boldsymbol{2}}|\boldsymbol{r}''|^2 - (\boldsymbol{f_{ext}}.\boldsymbol{r})\right]\boldsymbol{ds},$$

where $\boldsymbol{E}$ is Young's modulus, $\boldsymbol{G}$ is the shear modulus, and I is the principal moment of inertia of the rod's circular cross section.

Figure 2 shows the different configurations rod according to the load factor. This computation was reached after the detection a sign change of the determinant of the current tangent stiffness matrix. The computation of the eigenvalue $\lambda = -6.50 \times 10^{-3}$ claims that the critical state hypothesis. Then, a bisection method has been started to find the critical load and to fix the zero of the relative determinant at $\boldsymbol{Det_{rel}}\ (\boldsymbol{lK\_p}) = -8.2 \times 10^{-2}$. In addition, according to equation (9), the eigenvector φ verifies $\varphi^T \delta W_{ext} = 0.0691$; it is a bifurcation point.

We present an example, which is used to test the performance of the code. This is a non-linear analysis for a closed rod fixed at one end and subjected to a rotation about its central axis on the other end. The boundary conditions are:

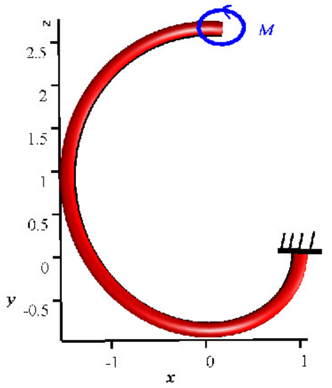

Fig. 1 Free-stress configuration rod

The rod is subject to an external torque M about the y–axis applied at its free end $\boldsymbol{r}$ (0) Figure 1. The other end of the rod $\boldsymbol{r}$ (L) is clamped. We impose the following boundary condition

$$\boldsymbol{r}(\mathbf{0}) = (\mathbf{0.200}, \mathbf{2.597}, \mathbf{0}),$$

$$\boldsymbol{r}(\boldsymbol{L}) = (\mathbf{1}, \mathbf{0}, \mathbf{0}), \boldsymbol{R}(\mathbf{0})(\boldsymbol{e_2}) = \boldsymbol{e_2} \ \boldsymbol{and} \ \boldsymbol{R}(\boldsymbol{L}) = \boldsymbol{I_3},$$

where $\boldsymbol{I_3}$ is the unit matrix of size 3.

The geometric and mechanical properties of the rod are in dimensionless units: the rod has a circular cross-section of radius $\boldsymbol{\varepsilon} = 0.08$; the length, $\boldsymbol{L} = 2\pi$; the flexural rigidity, $\boldsymbol{EI} = 1692$ and the torsional rigidity, $\boldsymbol{GI} = 2200$.

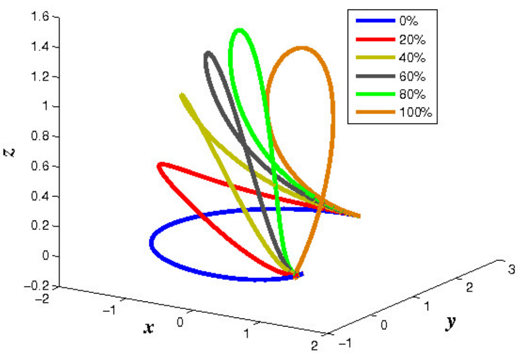

Fig. 2 Configurations rod according the load factor

4.2. Twist of a Closed Rod

We present here another example, which is used to test the performance of the stability. There is a non-linear analysis for a closed rod fixed at one end and subjected to a rotation about its central axis on the other end. The boundary conditions are:

$$\boldsymbol{r}(\mathbf{0}) = \boldsymbol{r}(\boldsymbol{L}), \boldsymbol{R}(\mathbf{0})\boldsymbol{e_3} = \boldsymbol{R}(\boldsymbol{L})\boldsymbol{e_3}, \boldsymbol{R}(\mathbf{0}) = \boldsymbol{I_3} \ \text{ and } \ \boldsymbol{e_1} . \boldsymbol{R}(\boldsymbol{L})\boldsymbol{e_1} = \boldsymbol{cos}(\boldsymbol{\alpha}),$$

where $\boldsymbol{I_3}$ is the unit matrix of size 3.

External loads are applied only at the point $\boldsymbol{r}(\boldsymbol{L})$ of the rod as it is explained in Figure 3. Per unit of measure, the length of the rod is $\boldsymbol{L} = 1$, the radius of the circular cross section is $\boldsymbol{\varepsilon} = 0.0772$, the flexural rigidity $\boldsymbol{EI} = 100$, the stiffness torsion is $\boldsymbol{GJ} = 200$, the stiffness to elongation is $\boldsymbol{EA} = 1200$ and the shear stiffness is $\boldsymbol{GA} = 500$. In FIG. 5.13, we show several balance shaft configurations according to the applied load values.

Based on a numerical reason, we are circumventing the problem of instability, by finding the branch of solution associated at the eigenvalue closest to zero. In our calculations, we found a first bifurcation point with a value load factor $\lambda = 1.717$ and a minimum eigenvalue equal to 1.602×10-5. Using 249 increments the load calculation converges. The finite elements with four nodes and discretization to thirty elements were sufficient to achieve good results (see: Figure 3). The relationship between the determinant of the matrix and the tangent loading factor are shown in the following table load values:

Critical Load	Load Iteration Number	Determinant	Eigenvalue	$\varphi.\delta W_{ext}$
1.717	249	-2.152×10^{-5}	-1.602×10^{-5}	1.978×10^{-5}
2.493	143	5.960×10^{-3}	3.481×10^{-5}	2.945×10^{-5}
3.937	172	1.036×10^{-2}	9.547×10^{-5}	-7.419×10^{-5}

We denote eqn (9), where $\boldsymbol{f}(\varphi) = \varphi.\delta W_{ext}$ is used to search the bifurcation points. By using further the energy criterion, we denote that the last column of the table above, the presence of three bifurcation point. For the former, we detected that the determinant of stiffness matrix changes the sign. The calculation was carried out in a zone of instability which explains the high number of increments made to converge. In this case, a method of bisection was used to find the critical load. Besides, the two other points, it has not detected negative values, but a small value of determinant at which we notice a change monotony of the determinant.

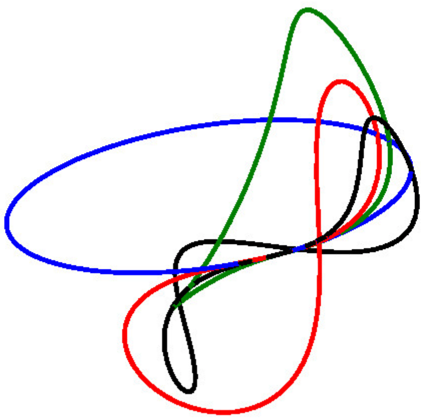

Fig. 3 Configurations according to the applied load values

5. Conclusions

Based on a stability criterion, this work proposes to investigate the numerical behavior of an elastic body in the critical state. The method developed in our study allows a simplification of the numerical implementation without any supplementary cost. The finite-element code developed allows us to include the steps necessary for the stability studies and to have a technical resolution adequate in order to identify the nature of the critical points, limit points and/or bifurcation points.

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